Household Labor Supply and the Gains from Social Insurance*

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Abstract

The marginal gains from social income insurance programs, an integral component of any welfare analysis or policy design, are captured by the gap in the marginal utility of consumption across states of nature. The main approach in the modern literature on social insurance to identify this gap is based on studying consumption fluctuations. Recognizing the significant challenges involved in analyzing consumption data, recent influential work has developed new methods based on studying labor supply behavior; but, so far, these methods have been confined to the context of unemployment insurance. In this paper, we build on this recent work and offer a new labor-supply based approach that leverages household-level economic interactions and optimality conditions. Specifically, we demonstrate that, in frameworks of efficient household allocations, spousal labor supply responses to shocks have direct implications for the gains from more generous government benefits. By doing so, we show how labor market data can be used for assessing marginal welfare gains in a general class of social insurance schemes—including the large and important programs of disability insurance and survivors benefits. Hence, household labor supply behavior and responses to shocks—which are widely studied in theoretical and empirical work—hold valuable information for the optimal design of social insurance.

Keywords: Social Insurance; Evaluation of Welfare Gains; Household Labor Supply

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1 Introduction

Welfare analysis and the optimal design of any social insurance program requires identifying both the benefits and the costs associated with that program. The welfare costs of government transfers are captured by the impact of households' behavioral responses to the policy on the government’s budget, known as the “fiscal externality”. The welfare gains from more generous social insurance are captured by the gap in the marginal utility of consumption across states of nature. This gap, which is zero in the first-best allocation in which marginal utilities are smoothed across states of nature, measures market inefficiency and quantifies the potential benefit from additional government intervention. Estimating the welfare costs is conceptually straightforward with the appropriate policy variations in the data that identify the policy-specific elasticities (Hendren 2016). Estimating the welfare gains from social insurance is a more challenging task, and the main approach in the modern literature to tackle it is based on studying the consumption smoothing effects of government benefits (Chetty and Finkelstein 2013).

However, analyzing consumption across different states of nature involves significant challenges, particularly in the context of the household (Chetty and Finkelstein 2013; Bee et al. 2013; Pistaferri 2015). Consumption is very difficult to measure accurately due to noise and recall errors and is typically available for relatively small samples. Additionally, consumption measures are usually partial and cover a sub-set of goods, such as food expenditure. Focusing on limited aspects of expenditure can lead to misleading conclusions about actual consumption, e.g., in the presence of home production (as emphasized by Aguiar and Hurst 2005). Moreover, even comprehensive and accurate data on households’ overall expenditure across states of nature—which are largely uncommon—would require nontrivial assumptions or complex estimations to be translated into individuals’ consumption bundles, since in the household setting individual consumption is not directly assignable and observable. Doing so must carefully take into account consumption flows of durable goods, household public goods, and, importantly, economies of scale in the household’s consumption technology to account for the composition of the household and its evolution over the life-cycle and across states of nature (see, e.g., Blundell and Lewbel 1991, Browning et al. 2013, Blundell et al. 2013, and Low and Pistaferri 2015).

Recognizing these difficulties, recent influential studies have sought alternative techniques for recovering the gap in marginal utilities using information from the labor market. Specifically, Shimer and Werning (2007) show how to identify the gains from higher unemployment benefit levels using the comparative statics of reservation wages with respect to benefit increases. Chetty (2008) develops a method to recover the gains from higher benefit levels using liquidity and substitution effects in the search effort of the unemployed, and Landais (2015) provides a similar technique for the gains from

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1 Recent advances in the analysis of consumption partially deal with some of these limitations, either by using survey data that cover a wider set of non-durable and services expenditure (Blundell et al. 2015; Low and Pistaferri 2015), or by creating consumption measures from income and wealth registers (Browning and Leth-Petersen 2003; Koijen et al. 2014; Kreiner et al. 2014; Kostøl and Mogstad 2015; Autor et al. 2015; Kolsrud et al. 2016; De Giorgi et al. 2016).
longer benefit durations.\textsuperscript{2} These labor-supply based methods, however, have been so far limited to the context of unemployment shocks and unemployment insurance.

In this paper we build on this recent work and offer a new labor-supply based approach for welfare analysis, in which we leverage household-level economic interactions and optimality conditions. Exploiting the interplay between the consumption allocations and labor supply decisions of household members, our approach uses only the labor supply behavior of the indirectly-affected spouse. We show that, under the assumption of household efficiency, spousal labor supply responses to shocks can be mapped to predictions about the welfare gains from providing more generous government benefits. The simple logic that underlies our approach is as follows. In each state of nature spouses work to the point where their own marginal utility loss from working equals each household member’s valuation of the additional consumption from the spouses’ increased earnings. Hence, the sensitivity of spousal labor supply to shocks and economic incentives can reveal how the household’s valuation of additional consumption changes across different states, which captures the gains from insurance. Intuitively, the extent to which household labor supply responds to shocks as self-insure is directly related to the degree to which households lack formal insurance and hence to the scope for welfare-improving government benefits.

Our approach to welfare analysis, which explicitly allows for economic dependencies across household members, has two main contributions to the existing literature. First, compared to current methods that use labor market data, our method is applicable to a general class of shocks and social insurance schemes in a household setting.\textsuperscript{3} Importantly, by analyzing the behavior of spouses who are indirectly affected by shocks through the sharing of household resources, we allow these shocks to have arbitrary effects on the directly-affected household members’ working ability (e.g., their productivity or disutility from labor) and labor market opportunities (e.g., their employability).\textsuperscript{4} These include the prevalent setting of (fatal and non-fatal) severe health shocks, thereby allowing the welfare analysis of the large and important programs of disability insurance (DI) and survivors benefits using our framework. Other important applications include the assessment of the value of unemployment insurance for the long-term unemployed—whose long durations of unemployment significantly harm their employment prospects (as documented by, e.g., Kroft, Lange, and Notowidigdo 2013)—as well as any other setting in which the individuals who are directly impacted may be unresponsive to economic incentives or at a corner solution (of either working full-time or not working at all). For example, in the debate on the privatization of Social Security, the value of protecting against pension-wealth losses in the 401(k) account of a working individual can be recovered by the labor supply response of his or her spouse. Similarly, spousal labor supply can be

\textsuperscript{2}This is also in the spirit of arguments raised by Autor and Duggan (2007).

\textsuperscript{3}An obvious limitation of our approach is, therefore, its inapplicability to analyzing single individuals.

\textsuperscript{4}The labor supply behavior of directly-affected individuals in these cases may be unresponsive to shocks or economic incentives and hence cannot reveal the household’s consumption preferences and need for additional income insurance. If these attenuated behavioral responses were wrongly used in welfare analysis, they would be misleadingly interpreted as low valuations of additional income insurance, while being actually driven by the directly-affected individuals’ inability to adjust their labor supply accordingly.
also used to evaluate the welfare losses caused by the discontinuation of employee compensations, such as health insurance, life insurance, and employer matching in retirement savings.

Second, compared to welfare analysis that relies on consumption data, our approach has several significant advantages attributable to the reliance on data from the labor market. Labor supply data are typically more precise and widely available, in particular with researchers’ increasing access to register-based data on household earnings. Additionally, and of particular importance in the analysis of households, labor supply is directly assignable to individual household members. Therefore, unlike data on expenditure, information on labor supply does not require conversions to account for economies of scale and household composition (e.g., when the household composition changes as in spousal death), but is rather readily usable for welfare analysis. Moreover, in applications with significant spousal labor supply responses which mitigate the consumption drop the household would otherwise experience, focusing on consumption fluctuations could lead to notable underestimations of the gains from social insurance. In these settings, it is the utility cost of reduced leisure that captures the gains from additional insurance, which is accounted for in our method that analyzes spousal labor supply.\footnote{This is the case in Autor et al. (2015), who find that DI denials do not decrease the household income or consumption of married applicants due to significant spousal labor supply responses, and therefore conclude that their estimate for the welfare gains from DI receipt among married couples is due to increased leisure. It is also the case in Fadlon and Nielsen (2016), where we find large increases in spouses’ labor supply following fatal health shocks that attenuate the consumption drop they would otherwise experience. In that application, analyzing labor supply would account for the utility cost of reduced leisure while the analysis of consumption would under-estimate the gains from additional survivors benefits.}

Of course, compared to the consumption approach, our approach has its own limitations which we discuss in the paper. In particular, similar to other optimization-based approaches that map the marginal utility from consumption to the marginal utility from other arguments of the utility function, our approach requires that choices with respect to the analyzed argument are interior. Accordingly, our intensive-margin model requires that spouses’ optimization leads them to an interior solution in hours of work, and our alternative extensive-margin model requires the presence of marginal households.

Our propositions and formulas, which rely on household optimality conditions, identify labor supply moments and preference parameters that are important for welfare analysis of social insurance. As such, they do not take a stance regarding how the interested researcher or policy maker should go about implementing them empirically. Specifically, we believe that our results can be a useful guide in both structural and reduced-form studies that aim to make quantitative statements about welfare and optimal policy. For reduced-form studies, that wish to analyze marginal changes in given policy environments, the formulas indicate which treatment effects to analyze and highlight the quasi-experimental variation needed for their identification. For structural studies, that wish to analyze fundamental policy reforms and simulate alternative policies, our formulas emphasize welfare-relevant household behaviors and preference parameters that should be carefully modeled and identified. This can be useful in complex dynamic models in which full identification of all the model’s primitives is particularly challenging and where transparency of sources of identification...
is critical. For such cases, our formulas offer a set of key moments to match for identifying the structural model and a set of critical structural parameters the chosen estimation procedure should identify.

Besides the literature on social insurance, this paper relates to the large and recently revived work on spousal labor supply and its self-insurance role. This strand of the literature encompasses both structural and reduced-form studies in a variety of household shocks and social insurance settings. These include (and are not limited to) Ashenfelter (1980), Heckman and Macurdy (1980, 1982), Lundberg (1985), Maloney (1987, 1991), Spletzer (1997), Cullen and Gruber (2000), Stephens (2002), Blundell et al. (2015), Haan and Prowse (2015), and Wang (2016) in the context of wage and unemployment shocks and unemployment insurance, and Coile (2004), Meyer and Mok (2013), Autor et al. (2015), Olsson and Thoursie (2015), Dobkin et al. (2016), and Fadlon and Nielsen (2016) in the context of health and disability. Our paper emphasizes that the findings from these studies not only inform us of how households behave in response to shocks over the life-cycle, but also contain valuable normative content with direct implications for the design of social insurance in different settings. For example, in our empirical study of household responses to fatal and non-fatal severe health shocks (Fadlon and Nielsen 2016), we find increases in labor supply among survivors who experienced income losses when their spouses died, particularly when the deceased spouse had earned a significant share of the household income. Applying the formulas we develop here suggests large welfare gains from more generous survivors benefits and from conditioning benefits on the deceased spouse’s work history in the Danish setting.

The paper is organized as follows. We begin with Section 2 that sets a simple conceptual framework for the analysis of household labor supply and its responses to shocks. Then, in Section 3 we provide our core analysis and show how household labor supply can be normatively used to assess the welfare gains from social insurance. In Section 4 we discuss important extensions and generalization to the simple model and illustrate the generality of our results. Section 5 concludes.

2 A Stylized Model of Household Labor Supply

We start by analyzing a static model of household labor supply decisions. In this section, we lay out the conceptual framework and positively study the household’s behavior in response to shocks. Specifically, we formalize how spousal labor supply can be used as insurance against income shocks to the household. We consider both an intensive margin model and an extensive margin model. The extensive-margin model presumably better describes actual labor markets as it allows for common labor market frictions, such as hour requirements set by employers, which can limit employees’ ability to optimize on the intensive margin. We begin with the intensive-margin model, however,
for illustrative purposes. The intensive-margin model allows for the most immediate comparison between the consumption based representation of the welfare gains from social insurance and our proposed representation using household labor supply, and captures the intuition of our results in a transparent way.\footnote{The choice of the appropriate model should depend on the context. In applications where there is strong evidence for such frictions (e.g., the Danish context as documented by Chetty et al. 2011) a participation model would be more appropriate.}

The model that we study here—including the setup and preference specification—is the simplest possible model that demonstrates how household labor supply responses can be used to draw implications for the design of social insurance. Nonetheless, our qualitative arguments extend to much more general settings. After deriving the welfare formulas for the simple case, we discuss important generalizations to the highly-stylized static model as well as alternative assumptions about the household’s preference structure (including state dependence and complementarities) and the household’s behavior (including both the collective and the unitary approaches).

### 2.1 Intensive-Margin Model

#### Setup

We study labor supply decisions of a two-person household, which consists of individuals 1 and 2. We consider a world with two states of nature: a “good” state, state $g$, in which member 1 works; and a “bad” state, state $b$, in which member 1 experiences a shock—e.g., a severe health shock—and drops out of the labor force. We employ this extreme assumption regarding member 1’s labor supply for simplification, but any shock that leads to some degree of exogenous decline in this member’s labor supply can be readily analyzed within the same framework. Households spend a share of $\mu^g$ of their adult life in state $g$ and a share of $\mu^b$ in state $b$ (with $\mu^g + \mu^b = 1$). In what follows, the subscript $i \in \{1, 2\}$ refers to the household member and the superscript $s \in \{g, b\}$ refers to the state of nature.

#### Household Budget Constraint

Denote by $c^s_i$ and $l^s_i$ the individual consumption and labor supply of member $i$ in state $s$, respectively. Let $A^s$ denote the household’s state-contingent wealth and non-labor income—including transfers from any source of individually-purchased or employer-provided private insurance, transfers from relatives, and out-of-pocket expenses (such as medical bills). We denote by $\bar{z}^s_i(l^s_i)$ $i$’s net-of-tax labor income in state $s$, so that with a wage rate of $w_i$ and a linear labor-income tax rate of $\tau_i^s$ we have $\bar{z}^s_i(l^s_i) = z^s_i(1 - \tau_i^s)$, where $z^s_i = w_i l^s_i$ are gross earnings. Finally, let $B^s$ represent benefits from the government in state $s$. It is possible to allow for income-testing in government benefits, but with some added analytical complication. We therefore choose to abstract from modeling this feature here, and focus on the state-contingent aspect of benefits which is at the core of our analysis of insurance against different states of nature. With this notation, the household’s overall income in state $s$, $y^s$, satisfies $y^s = A^s + \bar{z}^s_1(l^s_1) + \bar{z}^s_2(l^s_2) + B^s$.

#### Preferences

Let $U(c^s_1, c^s_2; l^s_1, l^s_2)$ represent the household’s utility as a function of consumption and labor supply of each member in each state. For simplicity, we assume for now (and relax later) that $U(c^s_1, c^s_2; l^s_1, l^s_2) = u_1(c^s_1) + u_2(c^s_2) - v_1(l^s_1) - v_2(l^s_2)$, where $u_i(c^s_i)$ is member $i$’s utility from consumption and $v_i(l^s_i)$ represents member $i$’s disutility from labor (including the utility loss from...
direct work costs and the opportunity costs of lost home production). We employ the normalization $u_1(0) = v_1(0) = 0$. This lets the model incorporate the case in which the bad state is a fatal health shock (in which case $c_i^0 = l_i^1 = 0$), so that the household preferences reduce to the utility from member 2’s allocation: $u_2(c_2^0) - v_2(l_2^1)$. Additionally, we assume that the consumption utility and the labor disutility functions are well-behaved—i.e., that $u_i'(c_i^0) > 0$, $u_i''(c_i^0) < 0$, $v_i'(l_i^1) > 0$, and $v_i''(l_i^1) > 0$.

**Household Behavior.** In the baseline static model where the household consumes its entire disposable income in each state of nature, there are no savings decisions involved, which we introduce in the dynamic extension to the model. Hence, in the current setting, the household’s choices reduce to the labor supply and consumption allocation decisions. Formally, in each state $s$ the household solves the problem:

$$\max U(c_1^s, c_2^s; l_1^s, l_2^s) \text{ s.t. } c_1^s + c_2^s = y^s.$$ 

Optimal consumption allocation across spouses must satisfy $u_1'(c_1^s) = u_2'(c_2^s)$. Additionally, in the intensive-margin model, the first-order condition with respect to the (interior) labor supply choice of the indirectly-affected member 2 satisfies: $u_2'(c_2^s) = \frac{v_2'(l_2^s)}{w_2(1-\tau_2^s)}$. Put together, the two optimality conditions imply that

$$u_1'(c_1^s) = u_2'(c_2^s) = \frac{v_2'(l_2^s)}{w_2(1-\tau_2^s)}.$$ 

This simple combination of optimality conditions with respect to consumption and labor supply choices will prove powerful for welfare analysis—it is the key source of the ability to map consumption utility to spousal labor disutility and hence of the identification of the gains from income insurance based on the indirectly-affected member’s labor supply. Importantly, since we want to allow the directly-affected member 1 to be at a corner solution in state $b$ ($l_1^b = 0$) due to, e.g., a severe disability that affects his or her ability to work, our analysis relies only on the labor supply responses of the indirectly-affected spouse.

**Spousal Labor Supply as Self-Insurance.** At this point it is easy to see the self-insurance role of spousal labor supply responses to shocks. Define $y_{2-1}^b$ as the household’s resources excluding those directly attributed to 2’s labor supply decision—i.e., $y_{2-1}^b \equiv A^s + \bar{z}_1^s(l_1^1) + B^s$—such that the (exogenous) income loss from the shock is $L \equiv y_{2-1}^b - y_{2-2}^b$ (the gap in the spouse’s unearned income across the two states). The household optimization conditions imply that the spouse’s labor supply response to the shock $\frac{\nu_{2}^b}{l_2^1}$ is greater whenever the imposed income loss $L$ is larger:

$$\frac{\partial \left( \frac{\nu_{2}^b}{l_2^1} \right)}{\partial L} = -\frac{u_2''(c_2^0) \partial c_2^0}{l_2^1 w_2(1-\tau_2^s)} > 0,$$

when consumption in the bad state is a normal good ($\frac{\partial c_2^0}{\partial c_2^b} > 0$). Intuitively, when individuals experience shocks that cause them to decrease their labor supply and earn less income, their spouses can compensate for the associated income loss by increasing their own labor supply. Since the relative increase in spousal labor supply in response to shocks increases with the income loss, it can reveal
the extent to which the household lacks formal insurance and needs to self-insure.\footnote{Empirical work that finds evidence in support of the self-insurance role of spousal labor supply includes, among other studies, Stephens (2002) and Blundell et al. (2015) who find that wives’ labor supply is an important consumption insurance device against permanent shocks to husbands’ wages; Cullen and Gruber (2000) who study whether spousal labor supply is crowded out by unemployment insurance benefits and find a large crowd-out effect; Autor et al. (2015) who find similar crowd-out effects in the context of disability insurance; and Fadlon and Nielsen (2016), where we find a significant increase in survivors’ labor supply following their spouse’s death which is entirely driven by households that experience substantial income losses due to the shock.}

2.2 Extensive-Margin Model

In the labor force participation version of our model \( l_i^s = 1 \) if \( i \) works and \( l_i^s = 0 \) otherwise. We adjust the household resource constraint so that \( \bar{z}_i^s(l_i^s) = z_i^s \times (1 - \tau_i^s) \times l_i^s \), where \( z_i^s \) are gross earnings conditional on working, and \( \tau_i^s \) are average tax rates. We also let \( B^s(l_2^s) \) represent benefits from the government in state \( s \) as a function of \( l_2^s \). In this model we let benefits differ by \( 2 \)'s participation since it allows the welfare analysis to focus on the value from insurance across different states (rather than across different spousal employment statuses within states) in an analytically and conceptually simple way. Specifically, it enables us to analyze the optimality of insurance generosity across states of the world for given spousal employment. In the household utility function we replace \( v_i(l_i^s) \) with \( v_i \times l_i^s \), so that \( U(c_1^s, c_2^s; l_1^s, l_2^s) = u_1(c_1^s) + u_2(c_2^s) - v_1 \times l_1^s - v_2 \times l_2^s \), where \( v_i \) represents each member \( i \)'s disutility from labor. The couple’s disutilities from labor \((v_1, v_2)\) are drawn from a continuous distribution defined over \([0, \infty) \times [0, \infty)\). We denote the marginal probability density function of \( v_2 \) by \( f(v_2) \) and its cumulative distribution function by \( F(v_2) \).\footnote{The simple participation model of this section is most closely related to Kleven et al. (2009) and Immervoll et al. (2011), who study optimal taxation of couples with extensive-margin labor supply responses.}

**Household Behavior.** Recall that in the simple model member 1 works in state \( g \) and does not work in state \( b \), so that the household’s overall income in each state \( s \) depends on member 2’s participation decision, \( y_i^*(l_i^s) \). In each of member 2’s potential employment statuses, consumption is efficiently allocated across spouses, such that the consumption bundles \( c_1^s(l_2^s) \) and \( c_2^s(l_2^s) \) are solutions to

\[
V(y_i^*(l_2^s)) \equiv \max_{c_1^s, c_2^s} u_1(c_1^s) + u_2(c_2^s) \tag{2}
\]

\[
s.t. c_1^s + c_2^s = y^*(l_2^s),
\]

where \( V(y_i^*(l_2^s)) \) is the household’s “consumption utility” for any level of household income. The indirectly-affected spouse works in state \( s \) if and only if

\[
v_2 < \bar{v}_2^s \equiv V(y_i^*(1)) - V(y_i^*(0)). \tag{3}
\]

That is, the spouse works if the household’s valuation of the additional consumption coming from his or her labor income compensates for his or her utility loss from working.

**Spousal Labor Supply as Self-Insurance.** For each state \( s \), denote the spouse’s probability of participation (or the participation rate in the population) by \( e_{2}^s \equiv F(\bar{v}_2^s) \). Define \( y_{-2}^s \) as the household’s resources excluding those directly attributed to 2’s labor supply decision—i.e., \( y_{-2}^s \equiv A^s + \bar{z}_1^s(l_1^s) \)—
such that the (exogenous) income loss from the shock is $L \equiv y_g^b - y_b^b$. Similar to the intensive-margin case, the spouse’s labor supply response to the shock $\frac{e_g}{e_b}$ increases in the income loss $L$:

$$\frac{\partial \left( \frac{e_g}{e_b} \right)}{\partial L} = \frac{f(\tilde{v}_g)}{F(\tilde{v}_b)} \left[ u'_i(c_i^b(0)) - u'_i(c_i^b(1)) \right] > 0,$$

as long as $[u'_i(c_i^b(0)) - u'_i(c_i^b(1))] > 0$.

3 Welfare Analysis: Implications for the Gains from Social Insurance

Using this setup, we provide in this section our main result and show how the gains from social insurance can be represented using only moments of spousal labor supply within our stylized framework. We do so in three steps. First, we derive the general formula for the gains from social income insurance—namely, the gap in marginal utilities of consumption across states of nature. Second, we briefly describe the consumption based approach to identifying this gap. Third, we provide our alternative representation for these gains using spousal labor supply responses. To gain intuition, we begin by deriving the results within the intensive-margin model, where the comparison to the consumption based approach is most straightforward, and then derive the counterpart formulas in the more realistic extensive-margin case. We conclude this section with a discussion of the identifying assumptions that underlie our approach. Important extensions and generalizations to the stylized model are considered in the next section.

In the design of optimal policies, the planner must weigh the gains against the associated costs. On the cost side, for example, transferring $1$ across states generates fiscal externalities that households impose on the government budget through their within-state behavioral responses to this policy change. In our case, the government’s revenue could decrease since more generous social insurance will lead to decreases in spousal labor supply in the bad state (see Fadlon and Nielsen 2015 for details and for exact marginal cost formulas in the stylized model). Identifying marginal costs is conceptually straightforward and much of the social insurance literature has focused on their estimation in different contexts. Therefore, we abstract from their analysis in this paper and focus only on the challenging task of identifying the gains from social insurance.

3.1 Intensive-Margin Model

Planner’s Problem. Denote the vector of tax rates on labor income by $T \equiv (\tau_g^b, \tau_g^g, \tau_b^b, \tau_b^g)$, and the vector of state-contingent benefits by $B \equiv (B_g^b, B_g^g)$. For the purpose of simplifying our formulas we analyze the case in which $\tau_g^b = \tau_b^b$, but the analysis readily extends to other cases. Let $W^s$ denote the household’s value function in state $s$ such that $W^s \equiv \max U(c_1^s, c_2^s; l_1^s, l_2^s)$ s.t. $c_1^s + c_2^s = y^s$. Therefore, the household’s expected utility is $J(B, T) \equiv \mu^b W^b + \mu^g W^g$. The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility subject to the requirement that expected benefits paid, $E(B^*) \equiv \mu^b B^b + \mu^g B^g$, equal expected taxes collected,
\[ E\left( \sum_{i=1}^{2} w_i \tau_i^b l_i^b \right) = \mu^g (w_1 \tau_1^g l_1^g + w_2 \tau_2^g l_2^g) + \mu^b (w_1 \tau_1^b l_1^b + w_2 \tau_2^b l_2^b). \] Hence, the planner chooses the benefit levels \( B \) and taxes \( T \) that solve
\[ \max_{B,T} J(B,T) \quad \text{s.t.} \quad E(B^*) = E\left( \sum_{i=1}^{2} w_i \tau_i^g l_i^g \right). \tag{5} \]

**Welfare Gains from Social Insurance.** What is the welfare gain from providing more generous benefits when the bad state occurs? To answer this question, consider transferring resources from the good state \( g \) to the bad state \( b \) through a small increase in, e.g., the tax rate \( \tau_1^g \) to finance a balanced-budget increase in benefits in the bad state \( B^b \).

The social gain from this perturbation consists of the household’s valuation of additional insurance. To construct a measure for this valuation, consider first the household’s utility loss from the marginal increase in \( \tau_1^g \) that finances the additional insurance. This loss is captured by \[ \frac{\partial J(T,B)}{\partial \tau_1^g} = \mu^g z_1^g u_i'(c_i^g), \] as the household’s income in state \( g \) is reduced by \( z_1^g \) dollars, which are valued at \( u_i'(c_i^g) \) per dollar. Partially differentiating the government’s budget, this marginal increase in \( \tau_1^g \) allows a balanced-budget increase in \( B^b \) of the amount \[ \frac{\partial B^b}{\partial \tau_1^g} = \frac{\mu^g z_1^g}{\mu^b}. \] The household’s valuation per $1 increase in \( B^b \) is given by \[ \frac{\partial J(T,B)}{\partial B^b} = \mu^b u_i'(c_i^b), \] where it produces a value of \( u_i'(c_i^b) \) and is transferred to the household with probability \( \mu^b \). The utility gain from the increase in benefits when the shock occurs is, therefore, \[ \frac{\partial J(T,B)}{\partial \tau_1^g} \times \frac{\partial B^b}{\partial \tau_1^g} = \mu^g z_1^g u_i'(c_i^b). \tag{6} \]

Put together, the welfare benefits from a (balanced-budget) increase in \( B^b \) financed by an increase in \( \tau_1^g \) are \[ \frac{\partial J(T,B)}{\partial B^b} \times \frac{\partial B^b}{\partial \tau_1^g} - \frac{\partial J(T,B)}{\partial \tau_1^g} = \mu^g z_1^g \left( u_i'(c_i^b) - u_i'(c_i^g) \right). \] To gain cardinal interpretation for this expression, we follow the recent social insurance literature and normalize it by the welfare gain from decreasing the labor income tax rate in the good state, \( \tau_1^g \) (Chetty and Finkelstein 2013). Overall, the normalized welfare benefit from our policy change is
\[ MB = \frac{\frac{\partial J(T,B)}{\partial B^b} \times \frac{\partial B^b}{\partial \tau_1^g} - \frac{\partial J(T,B)}{\partial \tau_1^g}}{\frac{\partial J(T,B)}{\partial \tau_1^g}} = \frac{u_i'(c_i^b) - u_i'(c_i^g)}{u_i'(c_i^g)}. \tag{6} \]

That is, the marginal welfare gain is captured by the insurance value of transferring resources from the good to the bad state, which is measured by the gap in marginal utilities of consumption across the two states. This “rate of return” on shifting funds, which is zero in the first-best allocation in which marginal utilities are smoothed across states of nature, measures market inefficiency and quantifies the potential gain from government intervention. This expression mirrors the benefit side of Baily’s (1978) and Chetty’s (2006a) formula for the optimal level of social insurance applied to our setting.

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10The partial differentiation of the government’s budget allows us to focus on the gains from social insurance. To include the costs, we would analyze the total derivative \( \frac{db^b}{dx} \) which takes into account not only the required mechanical adjustments, but also the households’ behavioral responses to the policy change that have an impact on the government’s budget (the “fiscal externality”). We illustrate that in our analysis of the dynamic participation model in the appendix.

11Our analysis resembles the derivation of individuals’ willingness to pay for additional unemployment insurance in Hendren (2015). Deriving these welfare benefits can be also achieved by characterizing the first-order conditions of the planner’s problem as in Chetty (2006a), Chetty and Finkelstein (2013), and Fadlon and Nielsen (2015).
Consumption-Based Method for Identifying Welfare Gains. The main approach in the modern public finance literature for assessing the welfare gains from social insurance in (6) studies the consumption smoothing effects of government transfers. It can be transparently illustrated using the method that was developed by Baily (1978) and Chetty (2006a) and was first implemented by Gruber (1997) in the context of unemployment insurance. Specifically, this method takes a quadratic approximation to the utility function to represent the gap in marginal utilities as
\[ MB \simeq \gamma_i \times \left( \frac{c_i^g - c_i^b}{c_i^g} \right), \]
where \( \gamma_i \) is \( i \)'s coefficient of relative risk aversion evaluated at \( c_i^g \). This formula evaluates fluctuations in the consumption of goods across states, \( c_i^g - c_i^b \)—which measure the degree of lack of consumption smoothing—with the rate of change in the utility from marginal dollars, captured by the curvature of the utility function (\( \gamma \))—which measures the utility cost of not smoothing consumption. Put differently, the benefits from insurance can be evaluated by the analysis of “quantity” fluctuations in consumption, which are then “priced” in utility terms.

Labor Supply Representation of Welfare Gains. We show next that simple, yet powerful, implications of the household’s labor supply decisions allow us to rewrite the marginal benefit in (6) in terms of the indirectly-affected spouse’s labor supply. By doing so, we show that the gains from additional insurance can be alternatively measured by evaluating changes in the consumption of the spouse’s leisure instead of changes in the household members’ consumption of goods. The following proposition summarizes this welfare result. We provide a simple proof and then discuss the intuition behind the welfare formula.

**Proposition 1.** The marginal benefit from raising \( B^b \) can be represented by
\[ MB \simeq \varphi \times \left( \frac{l_{gb}^2 - l_{gb}^g}{l_{gb}^g} \right), \tag{7} \]
where \( \varphi = \frac{\psi_2'(l_{gb}^g)}{\psi_2'(l_{gb}^g) l_{gb}^g} \).

**Proof.** The household’s optimality conditions imply that each household member’s marginal utility from consumption can be mapped to the spouse’s marginal disutility from labor, since \( u_i'(c_i^g) = u_i'(c_i^b) = \frac{\psi_2'(l_{gb}^g)}{w_2(1-\tau_2^g)} \). This allows us to represent the marginal benefit from social insurance by
\[ MB = \frac{\psi_2'(l_{gb}^g) - \psi_2'(l_{gb}^g)}{\psi_2'(l_{gb}^g)} \simeq \varphi \times \left( \frac{l_{gb}^2 - l_{gb}^g}{l_{gb}^g} \right). \]
Intuitively, we use the household’s optimality conditions to represent the degree to which households are able to smooth the marginal utility from consumption, \( u_i'(c_i^g) - u_i'(c_i^b) \), using the degree to which they are able to smooth the marginal disutility from the spouse’s labor, \( \frac{\psi_2'(l_{gb}^g) - \psi_2'(l_{gb}^g)}{\psi_2'(l_{gb}^g)} \). A quadratic approximation to member 2’s labor disutility function around \( l_{gb}^g \) yields the result.\(^{12}\)

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\(^{12}\)Recall that we analyze the case of \( \tau_2^g = \tau_2^b \) which simplifies the formula. The formula can be readily adjusted to other cases using the same equalities as described in this proof.

\(^{13}\)When the third order terms of spousal labor disutility are not small, the labor supply representation requires an additional term, analogous to the additional term that involves the coefficient of relative prudence in the consumption smoothing representation (Chetty 2006a).
Intuition. We saw that the benefits from extra dollars of income insurance are measured by the relative utility gain from additional consumption of goods in the bad state compared to the good state. This additional insurance also decreases the need to compensate for the income loss associated with the shock through spousal labor supply as a self-insurance mechanism. In turn, it allows us to alternatively express the benefits from more generous social insurance by using the relative utility gain from additional consumption of spousal leisure. Put differently, the formula assesses the benefits of incrementally smoothing labor supply across states as a result of additional formal social insurance that reduces costly self-insurance.

Similar to the consumption representation, our approach evaluates benefits by multiplying the change in the “quantity” of spousal labor supply in response to shocks, \(\frac{\partial l^g_2}{\partial y^g_2}\), by the rate of change in the spouse’s disutility from additional work, \(\varphi\), which captures the utility “price” of the labor supply quantity fluctuations across the two states. According to the formula, the welfare gains from additional benefits are higher whenever spousal responses to shocks are larger—that is, whenever the household’s baseline ability to smooth the spouse’s consumption of leisure across states of nature is lower. In the comparative statics of our model in equation (1) we saw that this quantity term increases with income losses and captures the self-insurance role of spousal labor supply. Therefore, intuitively, larger spousal labor supply responses—which correspond to a stronger need to self-insure—imply a greater scope for welfare-improving social insurance due to lack of adequate formal insurance. Similarly, the welfare gains from additional benefits are higher whenever \(\varphi\) is larger, as it implies that self-insurance through spousal labor supply (and the lack of smoothing spousal leisure) is more costly.

Analogous to analyzing fluctuations in the marginal utility of consumption that requires calibrating or estimating the utility parameter \(\gamma_i\), analyzing fluctuations in the spouse’s marginal disutility from labor requires calibrating or estimating the utility parameter \(\varphi\). This can be done using the variety of tools that the literature has developed for identifying preference parameters (in the lab, using quasi-experiments, or estimating structural models). One possible mapping that the researcher may choose to exploit in any of these techniques is that of \(\varphi\) (the curvature of the labor disutility function) to labor supply elasticities, comparable to the mapping of the coefficient of relative risk aversion \(\gamma_i\) (the curvature of the consumption utility function) that was introduced by Chetty (2006b). The intuition is that the extent to which an individual responds to changes in economic incentives (wages and income) is directly linked to the rate at which preferences change (over either consumption or labor). For details see Appendix A.\(^{14}\)

### 3.2 Extensive-Margin Model

**Planner’s Problem.** Let \(W^s(v_2)\) denote the household’s value function in state \(s\) such that

\[\varphi = \frac{1+\varepsilon(l^g_2, y^g_2)\frac{\partial y^g_2}{\partial y^g_2} - \varepsilon(l^g_2, w_2)\frac{\partial w_2}{\partial y^g_2}}{\varepsilon(l^g_2, w_2) - \varepsilon(l^g_2, y^g_2)^2\frac{\partial w_2}{\partial y^g_2}},\]

where \(\varepsilon(l^g_2, y^g_2) = \frac{\partial y^g_2}{\partial y^g_2}\), \(\varepsilon(l^g_2, w_2) = \frac{\partial y^g_2}{\partial w_2}\), \(w_2\) is the amount of the household’s wealth and non-labor income received by 2.

\(^{14}\)Specifically, we show that \(\varphi\)
\[
W^s(v_2) \equiv \begin{cases} 
V(y^s(1)) - v_1 \times l^s_1 - v_2 & \text{if } v_2 < \bar{v}_2^s \\
V(y^s(0)) - v_1 \times l^s_1 & \text{if } v_2 \geq \bar{v}_2^s.
\end{cases}
\]

Therefore, the household’s expected utility is \( J(B, T) \equiv \mu^g \int_0^\infty W^g(v_2)f(v_2)dv_2 + \mu^b \int_0^\infty W^b(v_2)f(v_2)dv_2. \) The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility subject to the requirement that expected benefits paid, \( E(B^g(l_2^g)) \equiv \mu^g[I_2^gB^g(1) + (1 - e_2^g)B^g(0)] + \mu^b[I_2^bB^b(1) + (1 - e_2^b)B^b(0)] \), equal expected taxes collected, \( E \left( \sum_{i=1}^2 z_i^s \tau_i^s l_i^s \right) = \mu^g(z_1^s \tau_1^s + e_2^g z_2^s \tau_2^s) + \mu^b(z_2^s \tau_2^s). \) Hence, the planner chooses the benefit levels \( B \) and taxes \( T \) that solve

\[
\max_{B,T} J(B, T) \quad \text{s.t.} \quad E(B^s(l_2^s)) = E \left( \sum_{i=1}^2 z_i^s \tau_i^s l_i^s \right). \tag{8}
\]

**Welfare Gains from Social Insurance.** As in the intensive-margin model, we ask here what is the welfare gain from providing more generous benefits when the bad state occurs? Recall that government benefits to households, \( B^g(l_2^s) \), can depend on both the state of nature and on spousal employment. To focus on the value of insurance across states (as opposed to insurance within a state) we consider transferring resources from the good state \( g \) to the bad state \( b \) for a given choice of spousal employment. Specifically, we analyze a small decrease in the benefit \( B^g(0) \) to finance a balanced-budget increase in \( B^b(0). \)

As before, the social gain from this perturbation consists of the household’s valuation of additional insurance. To construct a measure for this valuation, consider first the household’s utility loss from a $1 reduction in \( B^g(0) \) to finance the additional insurance. This loss is captured by

\[
\frac{\partial J(T,B)}{\partial B^g(0)} = \mu^g \frac{\partial}{\partial B^g(0)} \left( \int_0^\infty W^g(v_2)f(v_2)dv_2 \right) = \mu^g(1 - e_2^g)u'_i(c_i^0(0)),
\]

since every dollar taken produces a value of \( u'_i(c_i^0(0)) \) and was transferred to household with probability \( \mu^g(1 - e_2^g) \). Partially differentiating the government’s budget, this reduction in \( B^g(0) \) allows a balanced-budget increase in \( B^b(0) \) of the amount

\[
\left| \frac{\partial B^b(0)}{\partial B^g(0)} \right| = \frac{\mu^g(1 - e_2^g)}{\mu^b(1 - e_2^b)}. \tag{8}
\]

The household’s valuation per $1 increase in \( B^b(0) \) is given by

\[
\frac{\partial J(T,B)}{\partial B^b(0)} = \frac{\partial}{\partial B^b(0)} \left( \int_0^\infty W^b(v_2)f(v_2)dv_2 \right) = \mu^b(1 - e_2^b)u'_i(c_i^b(0)),
\]

with probability \( \mu^b(1 - e_2^b) \). The utility gain from the increase in benefits when the shock occurs is, therefore,

\[
\frac{\partial J(T,B)}{\partial B^b(0)} \times \left| \frac{\partial B^b(0)}{\partial B^g(0)} \right| = \mu^g(1 - e_2^g)u'_i(c_i^b(0)).
\]

Put together, the welfare benefits from a (balanced-budget) increase in \( B^b(0) \) financed by a $1 decrease in \( B^g(0) \) is

\[
\frac{\partial J(T,B)}{\partial B^b(0)} \times \left| \frac{\partial B^b(0)}{\partial B^g(0)} \right| - \frac{\partial J(T,B)}{\partial B^g(0)} = \mu^g(1 - e_2^g) \left( u'_i(c_i^b(0)) - u'_i(c_i^0(0)) \right). \tag{9}
\]

As in the simple model, this perturbation concerns the distribution of benefits to low-income households across different states of nature. Other perturbations to the system will follow the steps of the analysis conducted below. We focus on this particular aspect of the policy since it captures the essence of insuring households against shocks in a mathematically simple way.
Labor Supply Representation of Welfare Gains. Proposition 2 summarizes our welfare result for the extensive-margin model.

**Proposition 2.** The marginal benefit from raising $B^b(0)$ can be represented by

$$MB = \Phi \times \left( \frac{e^b_2}{e^s_2} \right) - 1,$$

where $\Phi \equiv \phi^b/\phi^s$, $\phi^s \equiv \frac{|(e^s_2 B^s(0))|}{B^s(0) \times f(\bar{v}_2)}$, and $\varepsilon(e^s_2, B^s(0))$ is the spouse’s participation elasticity with respect to the policy tool $B^s(0)$.

**Proof.** The steps of this proof follow those of Proposition 1 for the intensive-margin model. We first represent the marginal gains by mapping the marginal utility from consumption into utility losses from spousal labor supply, and then map those into moments of spousal labor supply behavior. Recall that the indirectly-affected spouse works when the value of additional consumption from his or her labor income, $\bar{v}^2_2 \equiv V(y^s(1)) - V(y^s(0))$, outweighs his or her disutility from labor, $\nu_2$. This decision rule reveals the household’s consumption value of an additional dollar, $V'(y^s(0)) = u'_i(e^i_2(0))$, through the change in the critical labor-disutility threshold below which the spouse works ($\bar{v}^2_2$) in response to an increase in benefits, since it implies that $$\left| \frac{\partial e^2_2}{\partial B^s(0)} \right| = V'(y^s(0)) = u'_i(e^i_2(0)).$$ This equality allows us to rewrite the marginal benefit from social insurance using the change in the marginal entrant’s disutility from labor $MB = \left| \frac{\partial e^2_2}{\partial B^s(0)} \right|$.

That is, similar to using $\nu'_2(t^2_2)$ in the intensive-margin model for identification of marginal utilities from consumption, we use $\left| \frac{\partial e^2_2}{\partial B^s(0)} \right|$ in the extensive-margin model. Next, we use the equality $e^s_2 = F(\bar{v}^s_2)$ that links the marginal entrant’s labor disutility to labor market outcomes, which also implies the semi-elasticity $\varepsilon(e^s_2, B^s(0))/B^s(0) = (v^s_2) \frac{\partial e^2_2}{F(\bar{v}^s_2) \partial B^s(0)}$. These equalities allow us to represent $MB$ with moments of spousal labor supply and yield the result. \(\blacksquare\)

**Intuition.** Both Propositions 1 and 2 are based on capturing the value of additional social insurance using the value of additional spousal leisure. In Proposition 1 this value is given by the foregone welfare cost from the disutility of marginal hours of work, and in Proposition 2 it is given by the foregone welfare cost from the disutility of the marginal worker. Proposition 2 shows how the marginal net benefit from social insurance can be expressed using different moments of the spouse’s

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10This is isomorphic to differentiating the first-order condition for the spouse’s search effort in a search model of labor force participation (see Appendix B.2).

11Conceptually, extensive-margin responses can be used for welfare analysis of marginal policy changes for the following reason: extensive-margin responses are governed by the labor disutility of the marginal entrant; and the labor disutility of the marginal entrant can be mapped into consumption preferences through the household’s optimization. Within a state, marginal entrants are always indifferent regarding whether to work or not; it is the cross-state analysis that entails information on the benefits from additional insurance. For other applications in which the identification of household preferences and welfare evaluations rely on discrete choices similar to labor force participation responses see Chetty (2006b) and Chetty (2009).
labor supply in a model of labor force participation. The leading term, $\Phi \times \left( \frac{\epsilon_b}{\epsilon_s} \right)$, which captures the gross marginal benefit, has two parts: $\frac{\epsilon_b}{\epsilon_s}$ that consists of a cross-state response; and $\Phi$ that consists of within-state responses. In line with the result for the intensive-margin case in Proposition 1, the extensive-margin formula in Proposition 2 evaluates the gains by multiplying the change in the quantity of spousal labor supply in response to shocks, $\frac{\epsilon_b}{\epsilon_s}$, by the change in the relative price of labor force participation across the two states, $\Phi$. As before, the potential welfare gains from additional benefits are larger whenever spousal self-insurance in responses to shocks is greater, and whenever the utility cost of this self-insurance is higher.

Note that in the extensive-margin model, the formula directly maps the price component $\Phi$ to labor supply elasticities. To see how $\Phi$ captures the relative price, or utility cost, of spousal labor supply, let us consider its components. Within a state $s$, the extent to which spousal labor force participation responds to a policy variation in the benefits $B^s(0)$ is captured by $\varepsilon(e_2^s, B^s(0))/B^s(0)$, the percent change in within-state participation. This measure is proportional to $\bar{\partial}_{v_2} e_2^s/B^s(0)$, the change in the labor disutility of the marginal entrant, which identifies the cost of additional labor supply on the margin in the extensive-margin model. Hence, $\varepsilon(e_2^s, B^s(0))/B^s(0)$ reveals information on the social marginal cost of spousal labor supply in each state. However, this semi-elasticity is also proportional to the share of marginal households, $f(v_2^s)$. Therefore, we must normalize it by $f(v_2^s)$ to achieve a within-state price change that is attributable to preferences only, which yields the within-state measure $\phi^s \equiv \varepsilon(e_2^s, B^s(0))/B^s(0)$. The relative change in prices across states of nature, required for our purposes of evaluating cross-state transfers, is then $\Phi \equiv \phi^b/\phi^g$.

The cross-state term $\frac{\epsilon_b}{\epsilon_s}$ in Proposition 2 is not policy specific and is present whenever the planner considers additional insurance by transferring resources from the good to the bad state. However, the price term, $\Phi$, adjusts according to the policy change and involves within-state semi-elasticities with respect to the specific policy tools that the planner considers changing. In the extensive-margin model the price term also requires calibrating the labor disutility distribution (specifically, the ratio $\frac{f(v_2^s)}{f(v_2^b)}$), which can be done in a variety of different ways.

3.3 Discussion

Household Efficiency. The key identifying assumption that underlies our analysis is that household decisions are Pareto efficient. While individual rationality on its own implies that a household member’s disutility from labor is equated to his or her marginal utility from consumption, household-level efficiency implies additionally that marginal utilities from consumption are equated across household members. Hence, on the margin, all members of the household exhibit the same

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18 For example, if in a specific empirical application spousal participation across states of nature is consistent with $v_2^b$ and $v_2^s$ being within a small region of the support $[0, \infty)$, one can use a locally linear approximation of $F$ in the threshold region $(v_2^b, v_2^s)$, so that $\frac{f(v_2^s)}{f(v_2^b)} \approx 1$. This approximation is isomorphic to a second-order approximation to the search effort function in a search model of participation that we analyze in Appendix B.2. Alternatively, if one wishes to avoid approximations, one can accompany the analysis with assumptions regarding the family of distributions to which $F$ belongs, and then calibrate its structural parameters using participation rates observed in the data as the identifying moments.
returns from the consumption of additional resources, and any member not at a corner solution can reveal through labor supply responses the consumption preferences of each member of the household.

The assumption of household Pareto efficiency relies on the premise that when spouses have symmetric information about each other’s preferences and consumption (because they interact on a regular basis) we would expect them to find ways to exploit any possibilities of Pareto improvements. Importantly, as emphasized by Browning, Chiappori, and Weiss (2014), this does not preclude the possibility of power issues such that the allocation of resources within the household can depend on its members’ respective Pareto weights. It simply assumes that no resources are left on the table.¹⁹

Note that for a given household composition any approach to modeling the household’s behavior and preferences which assumes efficiency would yield our results. Importantly, this includes the two leading approaches to modeling household behavior: the collective approach (Chiappori 1988, 1992; Apps and Rees 1988) and the unitary approach.²⁰

Use of Optimality Conditions. Both the analysis of fluctuations in marginal utility of consumption (through the use of the envelope theorem) and the analysis of fluctuations in spousal marginal disutility from labor rely on the assumption that households makes optimal choices. In fact, with this assumption, multiple representations for the gains from social insurance can be recovered using the marginal utilities of any single argument of the utility function since they are linked through the household’s optimality conditions.²¹ This flexibility allows researchers to use the representation most applicable given the available data and research tools (Chetty 2009; Chetty and Finkelstein 2013; Finkelstein et al. 2015). A main advantage of our proposed approach is the availability of large-scale and accurate data on labor market outcomes and the wide array of research tools the literature has developed for the analysis of household labor supply.

Compared to other optimization-based approaches, assessing the utility cost of consumption fluctuations does not preclude household members from being at a corner with respect to other choice variables. However, Proposition 1 for the intensive-margin model requires that spouses’ optimization leads them to an interior solution in hours of work, so that the marginal disutility from spousal

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¹⁹There are some cases in which the efficiency assumption fails (see discussion in Browning, Chiappori, and Weiss 2014). To model these cases, one would need to specify the underlying model of household decision making in order to identify one spouse’s preferences from the other spouse’s behavior.

²⁰Our simple model can be viewed as unitary with household preferences \( U(c^1_1, c^2_1; l^1_1, l^2_1) \), or as collective with equal Pareto weights across spouses and with individual utility functions \( U_i(c^i_1, l^i_1) = u_i(c^i_1) - v_i(l^i_1) \) in the intensive-margin specification and \( U_i(c^i_1, l^i_2) = u_i(c^i_1) - v_i \times l^i_2 \) in the extensive-margin specification. More generally using the collective approach, household decisions can be characterized as solutions to the maximization of \( \beta_1 U_1(c^1_1, l^1_1) + \beta_2 U_2(c^2_1, l^2_1) \), where \( \beta_1 \) and \( \beta_2 \) are the Pareto weights on members 1 and 2, respectively. Setting \( \beta_1 = \beta_2 = 1 \) is without loss of generality if the spouses’ relative bargaining power is stable across states of nature. Browning, Chiappori, and Weiss (2014) discuss the important distinction in the collective model between ex-post realizations of different states of nature, which do not affect the spouses’ relative bargaining power under efficient risk sharing, and ex-ante distributions of income shocks, which may affect the Pareto weights. Similar to Chiappori (1992), baseline weights do not affect our welfare results for a given household composition. When the bargaining power does change across states, or when the shock changes the household’s composition (e.g., when state \( b \) is member 1’s death), appropriate adjustments are required to reflect the new weights on the household members’ utilities.

²¹For example, in the context of health insurance, Finkelstein et al. (2015) develop a health-based approach for welfare analysis of Medicaid coverage that relies on evaluating the marginal health returns to out-of-pocket medical spending in different states of nature.
labor is linked to the marginal utility from consumption through an equality (see Finkelstein et al. 2015 for a related discussion). Allowing for violations of this assumption, Proposition 2 provides the corresponding formula for labor force participation decisions. However, it is not assumption free: the requirement for identification in the stylized extensive-margin case is that the marginal entrant’s value of labor disutility is interior to the support of the labor disutility distribution.\footnote{In a search model of labor force participation (as in Appendix B.2), this is equivalent to having spouses at an interior solution for search effort.} Hence, identification would not be achieved for applications in which all spouses never work (e.g., due to significant labor market frictions) or work full-time prior to the shock.

4 Extensions and Generalizations

The stylized model that we analyzed is the simplest possible model that demonstrates our normative findings. However, as we mentioned earlier, the qualitative arguments that we made so far extend to much more general settings. In this section, we discuss some main extensions to the simple model.

*Dynamics over the Life-Cycle and General Choice Variables.* In the context of social insurance over the life-cycle, it is important to consider households’ self-insurance through ex-ante mechanisms such as precautionary savings. In Appendix B, we analyze a fully-dynamic life-cycle model. This model allows for endogenous savings, as well as private and informal insurance arrangements.

Generally speaking, our formulas extend to this model with the adjustment that spousal labor supply in different states of nature are replaced with their averages taken over the periods households spend in each state (analogous to the dynamic consumption formula in Chetty 2006a and Chetty and Finkelstein 2013).\footnote{The dynamics of the life-cycle analysis likewise enter the marginal costs of social insurance. A household in state \( g \) not only decreases its labor supply due to higher taxes in the present, but also in response to increased benefits in the hitherto unencountered state \( b \). The prospect of higher benefits in the case that the household experiences a shock lowers its need to save for that scenario, which translates into a decrease in labor supply in state \( g \).} Note that even in the presence of ex-ante responses in expectation of shocks, these are still the ex-post responses to shocks that assess the gains from social insurance policies that condition benefits on shock realizations. The intuition behind this result is as follows: when forward-looking households make adjustments in anticipation of shocks (according to their expectations), their responses after the shock is experienced recover its residual uninsured risk. This leftover risk assesses the insurance gap that the government can potentially fill. Empirically, when ex-ante responses are possible, one has to carefully choose research designs that cleanly recover the causal ex-post impact of shocks (on either consumption or spousal labor supply), which are the moments required for identifying households’ willingness to pay for benefits (see discussions on these issues in Chetty 2008 and Hendren 2015).\footnote{Extending the underlying logic of the dynamic generalization, it is possible to also allow for heterogeneity across households in, e.g., the income loss that they experience and their degree of insurance. In this case, both our intensive and the extensive margin formulas for changes in the generosity of universal benefits would adjust to include the population average responses.}
Both the static and the dynamic models can additionally incorporate a general class of arbitrary choice variables—such as time investment in home production (similar to Chetty 2006a, Chetty and Finkelstein 2013, and Finkelstein et al. 2015). The generality of our analysis to inclusion of additional choice variables and response margins stems from the fact that our results are derived using optimality conditions, which map each member's consumption utility to spousal labor disutility, and using the envelope theorem. Since these conditions hold in more complex models that maintain the efficiency assumption, the economic forces that underlie the assessment of the gains from social insurance using spousal labor supply remain similar in more general settings.25

**State-Dependent Preferences.** Next, we show how to extend our approach to allow for state-dependent utility as household members’ preferences can differ in several dimensions across states of nature. Consumption utility state dependence would not alter our formulas since we mapped the identification of welfare gains from consumption to labor supply. Similarly, state-dependence in the directly-affected member’s labor disutility—e.g., due to a severe health shock—would not affect the results which rely on spousal labor supply. This is, indeed, one of the main motives for focusing on the indirectly-affected spouses. The type of state-dependence that would affect the formulas is with respect to the indirectly-affected spouse’s disutility from labor.

To illustrate how this sort of state dependence enters our formulas, let us analyze the intensive-margin model with state-dependent preferences of the form \( U^s(c_1^s, c_2^s; l_1^s, l_2^s) = u_1^s(c_1^s) + u_2^s(c_2^s) - v_1^s(l_1^s) - v_2^s(l_2^s) \). We provide an illustration for the extensive-margin model in Appendix C. With these preferences, the formula in Proposition 1 becomes: \( MB \equiv \theta_t \times \phi \times \left( \frac{b_l}{b_t} \right) \) + \( \theta_t - 1 \), where \( \phi = \frac{v_2^{r^s}(l_2^s)}{v_2^{r^s}(l_2^s)} \) similar to before, and \( \theta_t = \frac{v_1^u(l_1^s)}{v_2^u(l_2^s)} \) measures the degree of state dependence by evaluating the extent to which the marginal cost of spousal labor supply (or the marginal value of spousal leisure) varies across states of nature starting from equal levels of labor supply. Evidently, similar adjustments to the welfare formulas are required in the consumption-smoothing approach.

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25More precisely, when the additional arguments enter the household’s utility in a way that is separable from labor supply the same formulas go through. When there are utility function dependencies between these arguments and labor supply, spousal marginal labor disutility still directly maps into marginal utilities from consumption, but the mapping to labor supply behavior requires adjustments. These adjustments are identical to those described below when utility is state dependent. Related analyses and alternative assumptions in the inclusion of general choice variables can be found in Chetty (2006a) in the context of welfare evaluations of social income insurance using consumption and in Finkelstein et al. (2015) in the context of welfare evaluations of health insurance using various optimization-based methods.
(see Chetty and Finkelstein 2013).\footnote{For completeness, we report the adjusted consumption-based formula in the context of our model. In the intensive-margin model the marginal gains from social insurance become \( MB = \left[ \theta c_i \times \gamma_i \times \left( \frac{c_i^g - c_i^b}{c_i^g - c_i^f} \right) \right] + [\theta c_i - 1], \) where \( \gamma_i = \frac{u_{n_i}^{g_i}(c_i^g)}{u_{n_i}^{f_i}(c_i^f)} \) and \( \theta c_i = \frac{u_{i}^{b_i}(c_i^b)}{u_{i}^{f_i}(c_i^f)} \). Chetty and Finkelstein (2013) provide an equivalent version of this formula when state dependence is defined differently as \( \frac{u_{i}^{b_i}(c_i^b) - u_{i}^{g_i}(c_i^g)}{u_{i}^{f_i}(c_i^f)} \).

Compared to Proposition 1, the first bracketed term adjusts the “price” component of self-insurance through spousal labor supply from \( \varphi \) to \( \theta t \times \varphi \). To gain intuition, consider the case in which \( \theta t > 1 \) so that spousal leisure is more valuable in the bad state. Since the formula assesses benefits from social insurance by evaluating the gains from the consumption of leisure, this would make the transfer of resources from state \( g \) to state \( b \) more socially desirable. The reason is that it would allow for more leisure in the bad state, in which it is valued more highly, by decreasing the need to compensate for the associated income loss through spousal labor supply. The additional component in the second bracketed term captures welfare considerations that are beyond income losses, and may be present even when households are well-insured and there is no self-insurance \( \left( \frac{L^2 - L^1}{L^1} = 0 \right) \). For example, when \( \theta t > 1 \) the planner would want to allow for more leisure in the bad state simply because it is valued more highly on the margin in that state (and the opposite when \( \theta t < 1 \)). That is, this term captures the value of insurance against “utility shocks” rather than income shocks that are associated with state transitions. Note that \( \theta t \) is a composite parameter that includes direct changes in spouses’ marginal disutility from labor as a result of a shock but also indirect changes—for example, in the presence of consumption-leisure utility complementarities, the marginal disutility from labor can vary across states of nature due to changes in consumption and the utility dependence between consumption and leisure.\footnote{Examples for state dependence are as follows. When the bad state is member 1’s sickness, spousal labor supply may become more costly if 2 places greater value on time spent at home—e.g., to take care of his or her sick spouse. When the bad state is 1’s death, working may become less desirable if the surviving spouse experiences depression and has difficulties working, or conversely, working may become more desirable if the surviving spouse feels lonely and wishes to seek social integration. When the bad state is member 1’s unemployment that is accompanied by consumption drops, state-dependence would be present if there are consumption-leisure complementarities. Some recent empirical studies, however, provide evidence consistent with consumption-leisure separability—e.g. Browning and Crossley (2001) (who find that unemployment shocks have little impact on consumption for individuals with high levels of assets), Aguila et al. (2011) (who find no change in consumption, defined as non-durable expenditure, around retirement), and Low and Pistaferri (2015).

Overall, state dependence affects any analysis that aims to make quantitative welfare statements, whether it relies on analyzing consumption or analyzing labor supply and whether it uses reduced-form or structural estimation techniques. There is no consensus in the literature on the magnitude (or even sign) of utility state dependence in the context of the significant shocks that social insurance programs aim to protect against (such as disability and unemployment). In practice, many empirical normative studies ignore this important aspect since the identification of state dependence has proven very challenging. We share Chetty and Finkelstein’s (2013) view that this is an important area for future work.\footnote{A partial list of studies that offer identification strategies and estimate the degree of state dependence in different applications includes Viscusi and Evans (1990), Evans and Viscusi (1991), Lillard and Weiss (1997), Rust and Phelan (1997), Sloan et al. (1998), De Nardi et al. (2006), Edwards (2008), Finkelstein et al. (2013), Low and Pistaferri (2015), and Fadlon and Nielsen (2015).}

**Household Resource Constraint.** Our model accommodates income streams that can depend on
the state of nature and on employment choices, and can therefore flexibly incorporate any state
and employment contingent private or social insurance payments. In a similar fashion, it is also
possible to allow for income-tested (and age-dependent) transfers and taxes and more complex
taxation schemes. For example, one can account for non-linear income taxation and differential tax
rules for joint filing by studying a general state-dependent function that maps labor supply into
household-level net-of-tax earnings: \( z^s(l^1_s, l^2_s) \).

So far, however, we have abstracted from potentially important economies of scale in the house-
hold’s consumption technology. To allow for household public goods and economies of scale in a
straightforward and general way, we follow Browning et al. (2013) and introduce an arbitrary tech-
nology, \( G_s \), that transforms income into consumption.\(^{29}\) That is, we can rewrite the household
state-specific resource constraint as \( c^s_1 + c^s_2 = G^s(y^s) \), where each member’s \( c^s_i \) is measured in (unob-
servable) “private good equivalent” units.\(^{30}\) Since \( G^s \) is state specific, it also accounts for potential
household composition changes as a result of the shock (e.g., when state \( b \) is represents a fatal health
shock), as well as any post-shock changes in the household’s technology (e.g., in the degree or nature
of home production). With this specification, the formula for the welfare gains from a $1 increase
in insurance becomes \( MB = \frac{G^b(y^b)u^s(c^b_1) - G^g(y^g)u^s(c^g_1)}{G^g(y^g)u^s(c^g_1)} \), since the benefit from a $1 transfer must be
scaled by the amount of consumption units that it produces. This highlights one of the challenges
in the analysis of consumption: the researcher must identify the function \( G^s \) in order to trans-
late income or overall expenditure into individual consumption in household settings, which entails
strong assumptions or complex estimations (Browning et al. 2013). The labor supply representation,
however, remains the same since we can still write the marginal gains as \( MB = \frac{\nu^s(l^1_b) - \nu^s(l^2_b)}{\nu^s(l^2_b)} \), which
tremendously simplifies the analysis when economies of scale are considered.

5 Conclusion

This paper develops a new approach to welfare analysis of social insurance in general household
settings using information from the labor market. We have shown how when households make
optimal choices, household labor supply behavior can be used to draw implications for the gains
from social insurance. Intuitively, our analysis have illustrated that the degree to which households
self-insure through spousal labor supply in response to shocks and differentially respond to economic
incentives across states of nature reveal their lack of formal insurance and, hence, the scope for more
generous government benefits in bad states of the world. Doing so, we have also highlighted the
important welfare relevance of the prevalent empirical work on household labor supply and its self-

\(^{29}\) This is also similar to Becker’s (1965) model of household production and more recent applications and extensions to his
framework, such as Kleven (2004) and Kleven and Kreiner (2007).

\(^{30}\) One important and widely used element involved in converting expenditure data into private consumption is equivalence
scales. However, despite their practical importance and the sensitivity of welfare analysis to them, the main equivalence scale
estimates are ad-hoc and not theoretically based (such as the modified OECD equivalence scale of 0.67 and the square-root scale
of 0.71). A recent example for model-based estimates for adult equivalence scales, is Browning, Chiappori, and Lewbel (2013),
who find non-negligible differences across genders. Under the assumption of equal sharing of income among the two spouses,
their scale estimates are 0.80 for males and 0.72 for females.
insurance role in different settings.

Using labor supply rather than consumption data for welfare evaluations involves significant advantages due to the wide availability of large-scale and precise data on household labor market outcomes and due to the ability to directly assign labor supply behavior to individual members of a household. Our approach offers a way to exploit these advantages, using either reduced-form or structural estimation techniques, in a general class of shocks and social insurance schemes. These include important applications such as fatal and non-fatal severe health shocks and long run unemployment. They also include less traditional but increasingly important applications, such as assessing the value of protecting against pension-wealth losses in private savings accounts (relevant for the debate on the privatization of Social Security) and evaluating the welfare losses caused by discontinuation of employee compensations, such as health insurance, life insurance, and employer matching in retirement savings.
References


Appendix A: Identification of $\varphi$

In this section we derive a relationship between $\varphi$, as defined in Section 3.1, and observable labor supply elasticities in our stylized intensive-margin model. The analysis uses a similar strategy as that introduced by Chetty (2006b) to recover risk aversion—i.e., we recover the curvature of the labor disutility function in the same way that Chetty (2006b) recovers the curvature of the consumption utility function. The intuition for the method is that the extent to which an individual responds to changes in economic incentives (wages and income) is directly linked to the rate at which preferences change (over consumption or labor). To conduct the analysis at the individual level, we exploit Chiappori’s (1992) “sharing-rule” interpretation of the collective model. That is, we assume that wealth and non-labor income in state $s$, denoted by $A^s$, are shared between the members such that $y^g_2 \equiv \pi^g_2(w_1, w_2, A^s)$ is the amount received by 2 and $y^g_1 \equiv A^* - \pi^g_2(w_1, w_2, A^*)$ is the amount received by 1. We abstract from explicitly modeling the labor-income tax schedule from Section 3.1, but this can be done by letting $w_i$ represent net-of-tax wages in this appendix. With these definitions, one can write 2’s program in state $g$ as

$$\max_{c^g_2, l^g_2} u_2(c^g_2) - v_2(l^g_2)$$

s.t. $c^g_2 = y^g_2 + w_2 l^g_2$.

The first-order conditions of this program imply that $w_2 u_2'(y^g_2 + w_2 l^g_2) = v_2'(l^g_2)$. Partially differentiating the latter equation with respect to $y^g_2$ and $w_2$ yields $\frac{\partial v_2}{\partial y_2} = -\frac{w_2 u_2''}{(w_2 y^g_2 + w_2 l^g_2)}$ and $\frac{\partial v_2}{\partial w_2} = -\frac{w_2 u_2'' + w_2 l^g_2 u_2''}{(w_2 y^g_2 + w_2 l^g_2)}$. It follows that $\varphi = \frac{v_2'(l^g_2)}{v_2'(l^g_2)} = \frac{1 + (l^g_2, y^g_2)^{\frac{w_2}{w_2}} v_2''}{\varepsilon(l^g_2, w_2) - \varepsilon(l^g_2, y^g_2) \frac{w_2 y^g_2}{w_2}}$, where $\varepsilon(l^g_2, y^g_2) = \frac{\partial v_2}{\partial y_2} \frac{y^g_2}{l^g_2}$, $\varepsilon(l^g_2, w_2) = \frac{\partial v_2}{\partial w_2} \frac{w_2}{l^g_2}$.

Appendix B: Dynamic Models of Household Labor Supply

In this appendix we analyze the dynamic extensions to our stylized static models.

Appendix B.1: Intensive-Margin Model

Setup. We consider a discrete-time setting in which households live for $T$ periods $\{0, ..., T - 1\}$ (where $T$ is allowed to go to infinity) and set both the interest rate and the agents’ time discount rate to zero for simplicity. Households consist of two individuals, members 1 and 2. We assume that at time 0 households are in the “good” state, state $g$, in which member 1 works. In each period, the household transitions with probability $p_t$ to the “bad” state, state $b$, in which member 1 experiences a shock and drops out of the labor force. In what follows, the first subscript $i \in \{1, 2\}$ refers to the spouse, the second subscript $t \in \{0, ..., T - 1\}$ indexes time, and the superscript $s \in \{g, b\}$ refers to the state of nature.

Household Budget Constraint. Denote by $c^g_i$ and $l^g_i$ the individual consumption and labor supply of member $i$ in period $t$ at state $s$, respectively. Let $I^g_t$ denote the household’s time-state contingent non-labor income or expenses—including transfers from any source of individually-purchased or employer-provided private insurance, transfers from relatives, and out-of-pocket expenses (such as medical bills)—and let $A_t$ denote household assets in the beginning of period $t$. We denote by $z^g_{it}$, $i$’s labor income in period $t$ at state $s$, so that with a wage rate of $w_i$ we have $z^g_{it} = w_i l^g_{it}$. Since the analysis here becomes more complicated due to dynamics, we simplify the tax-and-benefit system so that benefits from the government in the bad state $B^b$ are financed through a lump-sum tax $\tau$ in the good state. It is possible to analyze other financing schemes but with some added analytical complication.

Preferences. Let $U(c^g_{1t}, c^g_{2t}; l^g_{1t}, l^g_{2t})$ represent the household’s flow utility at time $t$ as a function of consumption and labor supply of each member in each period and state. We assume that $U(c^g_{1t}, c^g_{2t}; l^g_{1t}, l^g_{2t}) = u_1(c^g_{1t}) + u_2(c^g_{2t}) - v_1(l^g_{1t}) - v_2(l^g_{2t})$, where $u_i(c^g_{it})$ is member $i$’s utility from consumption and $v_i(l^g_{it})$ represents member $i$’s disutility from labor. We employ the normalization $u_1(0) = v_1(0) = 0$. This lets the model incorporate the case in which the bad state is a fatal health shock (in which case $c^g_{1t} = l^g_{1t} = 0$), so that the household preferences reduce to the utility from member 2’s allocation: $u_2(c^g_{2t}) - v_2(l^g_{2t})$. Additionally, we

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1Note the subtlety that we focus on partial derivatives of the spouse’s behavior with respect to $y^g_2$ and $w_2$. In particular, $y^g_2$ is held fixed when we change $w_2$. 

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Lemma B.1. The normalized welfare gain from raising $B^b$ is

$$M_B = \frac{\frac{\partial J_0(B^b, \tau, A_0)}{\partial B^b} \frac{\partial B^b}{\partial \tau} + \frac{\partial J_0(B^b, \tau, A_0)}{\partial \tau} \frac{\partial B^b}{\partial B^b}}{-\frac{\partial J_0(B^b, \tau, A_0)}{\partial \tau}}}{E\left(\frac{u'_b(c^b_{it})}{u'_b(c^b_{it})}\right)}.$$  

Assume that the consumption utility and the labor disutility functions are well-behaved—i.e., that $u'_b(c^b_{it}) > 0$, $u''_b(c^b_{it}) < 0$, $v'_b(l^b_{it}) > 0$, and $v''_b(l^b_{it}) > 0$.

Household Behavior. We now turn to describe the household’s decision making in each state of nature.

In the good state, the household solves the following problem

$$V^g_t(B^b, \tau, A_t) \equiv \max U(c^g_{1t}, c^g_{2t}, I^g_{1t}, I^g_{2t}) + \rho_{t+1} V^g_{t+1}(B^b, \tau, A_{t+1}) + (1 - \rho_{t+1}) V^g_{t+1}(B^b, \tau, A_{t+1})$$

subject to:

$$c^g_{1t} + c^g_{2t} + A_{t+1} = A_t + I^g_{1t} + z^g_{1t} + z^g_{2t} - \tau,$$

where $V^g_{t+1}(B, T, A_{t+1})$ is the value of entering period $t+1$ in the bad state with assets $A_{t+1}$, which happens with probability $\rho_{t+1}$. In the good state, the first-order conditions and the envelope theorem imply that

$$u'(c^g_{1t}) = u'(c^g_{2t}) = \frac{\partial V^g_{t+1}}{\partial A_{t+1}} = \rho_{t+1} \frac{\partial V^g_{t+1}}{\partial A_{t+1}} + (1 - \rho_{t+1}) \frac{\partial V^g_{t+1}}{\partial A_{t+1}}.$$

When in the bad state, the household’s maximization problem becomes

$$V^b_t(B^b, \tau, A_t) \equiv \max U(c^b_{1t}, c^b_{2t}, I^b_{1t}, I^b_{2t}) + V^b_{t+1}(B^b, \tau, A_{t+1})$$

subject to:

$$c^b_{1t} + c^b_{2t} + A_{t+1} = A_t + I^b_{1t} + z^b_{1t} + z^b_{2t} + B^b.$$

The first-order conditions and the envelope theorem imply that

$$u'_1(c^b_{1t}) = u'_2(c^b_{2t}) = \frac{\partial V^b_{t+1}}{\partial A_{t+1}} = \frac{\partial V^b_{t+1}}{\partial A_{t+1}}.$$

Recall that in the bad state we want to allow member 1 to be at a corner solution with respect to labor supply, so that member 1’s optimality condition need not hold with equality.

Planner’s Problem. Define $J_t(B^b, \tau, A_t) \equiv \rho_t V^b_0(B^b, \tau, A_t) + (1 - \rho_t) V^g_0(B^b, \tau, A_t)$ as the value of entering period $t$ from the good state and with household assets $A_t$. The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility at time $t$.

Define $J_t(B^b, \tau, A_t)$ and subject to balancing the government’s budget. The budget constraint equates the expected transfers to expected revenues at time 0, $J_0(B^b, \tau, A_0)$, subject to the government’s budget. The budget constraint equates the expected transfers to expected revenues at time 0, $E_0(B^b, \tau, A_0)$, subject to the government’s budget. The budget constraint equates the expected transfers to expected revenues at time 0, $E_0(B^b, \tau, A_0)$, subject to the government’s budget. The budget constraint equates the expected transfers to expected revenues at time 0, $E_0(B^b, \tau, A_0)$, subject to the government’s budget.

Welfare Gains from Social Insurance. Similar to the static model, we ask what is the welfare gain from providing more generous benefits when the bad state occurs? To answer this question, consider transferring resources from the good state to the bad state $b$ through a small increase in the tax $\tau$ to finance a balanced-budget increase in benefits in the bad state $B^b$.

The welfare gain from this perturbation is

$$\frac{\partial J_0(B^b, \tau, A_0)}{\partial B^b} \frac{\partial B^b}{\partial \tau} + \frac{\partial J_0(B^b, \tau, A_0)}{\partial \tau} \frac{\partial B^b}{\partial B^b} = \left(\rho_0 \frac{\partial V^b_0}{\partial B^b} + (1 - \rho_0) \frac{\partial V^g_0}{\partial B^b}\right) \frac{\partial B^b}{\partial \tau} + \left(\rho_0 \frac{\partial V^b_0}{\partial \tau} + (1 - \rho_0) \frac{\partial V^g_0}{\partial \tau}\right).$$

The normalized welfare benefit from our policy change, which has a direct cardinal interpretation, is presented in the following lemma. This lemma essentially replicates the results in Chetty (2006a) applied to our setting.

Lemma B.1. The normalized welfare gain from raising $B^b$ is

$$MB = \frac{\frac{\partial J_0(B^b, \tau, A_0)}{\partial B^b} \frac{\partial B^b}{\partial \tau} + \frac{\partial J_0(B^b, \tau, A_0)}{\partial \tau} \frac{\partial B^b}{\partial B^b}}{-\frac{\partial J_0(B^b, \tau, A_0)}{\partial \tau}} = \frac{E \left(\frac{u'_b(c^b_{it})}{u'_b(c^b_{it})}\right) - E \left(\frac{u'_b(c^b_{it})}{u'_b(c^b_{it})}\right)}{E \left(\frac{u'_b(c^b_{it})}{u'_b(c^b_{it})}\right)},$$

$$\text{starting the planning problem from the good state, we get, e.g., that } D^g = E_0[n] = \sum_{t=0}^{T-1} \Pr(n_s > t) = \sum_{t=0}^{T-1} \prod_{t=0}^{T-1} (1 - \rho_t).$$
where $E(u'_i(c^s_i))$ is i’s average marginal utility of consumption while in state $s$.

**Proof:** The general logic of the proof is to characterize the derivatives of the value functions in their sequential problem representation—that is, as a sum of derivatives over time and over different states of nature. To do so, we work backwards from period $T-1$ to period 0.

Let us begin with the first term on the right-hand side of equation (4), and derive a formula for $\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T}$. Using the fact that $\frac{\partial V^b}{\partial T} = u'_i(c^b_{it}) + \frac{\partial V^b}{\partial T} + 1$ and $\frac{\partial V^g}{\partial T} = u'_i(c^g_{T-1})$, we can work backwards to get $\frac{\partial V^b}{\partial T} = u'_i(c^g_{T-1}) + \sum_{m=1}^{T-1} u'_i(c^g_{m})$. Note that throughout the analysis marginal utilities are evaluated at the optimal allocation conditional on reaching time $t$ in state $s$ and with a specific history. In a similar way, with $\frac{\partial V^g}{\partial T} = \rho_t \frac{\partial V^b}{\partial T} + (1-\rho_t) \frac{\partial V^g}{\partial T}$ we get $\frac{\partial V^g}{\partial T} = \rho_t \frac{\partial V^b}{\partial T} + \sum_{m=t+2}^{T-1} (\prod_{l=t+1}^{m-1}(1-\rho_l)) \rho_m \frac{\partial V^b}{\partial T}$.

Together, $\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T} = \rho_0 \frac{\partial V^b}{\partial T} + \sum_{m=0}^{T-1} \prod_{l=0}^{m-1}(1-\rho_l) \rho_m \frac{\partial V^b}{\partial T}$. As a notation we define: $\prod_{i=a}^{b} x_i = 1$ whenever $b < a$, which allows us to simplify the latter summation to $\sum_{m=0}^{T-1} \prod_{l=0}^{m-1}(1-\rho_l) \rho_m \frac{\partial V^b}{\partial T}$. The first bracketed term captures the probability of a household in the good state transitioning to the bad state exactly in period $m$. The second bracketed term then sums the marginal utility of consumption in each future period $r$, where the household is in the bad state from period $m$ until $T-1$. Putting together the two terms in brackets, we get the overall expected sum of the marginal utility of consumption in the bad state for a household that is in the good state at period 0. To get the average, we divide the expected sum by the number of periods a household is expected to be in the bad state, $D^b$. With this definition we obtain $\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T} = D^b E(u'_i(c^b_{it}))$. Differentiating the budget constraint in (3) yields $\frac{\partial B^b}{\partial T} = \frac{D^b}{D^g}$, so that the first term in he first term in equation (4) can be written as

$$
\left( \rho_0 \frac{\partial V^b}{\partial B^b} + (1-\rho_0) \frac{\partial V^g}{\partial B^g} \right) \frac{\partial B^b}{\partial T} = D^g E(u'_i(c^b_{it})).
$$

Let us turn to the second term on the right-hand side of equation (4), $\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T}$. First note that $\frac{\partial V^g}{\partial T} = 0$. Then, since $\frac{\partial V^b}{\partial T} = -u'_i(c^g_{it}) + (1-\rho_t) \frac{\partial V^b}{\partial T} + 1$ and $\frac{\partial V^g}{\partial T} = -u'_i(c^g_{t-1})$, we can work backwards and show that $\frac{\partial V^b}{\partial T} = -\sum_{m=1}^{T-1} (\prod_{l=t+1}^{m}(1-\rho_l)) u'_i(c^g_{m})$. Put together, we have $\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T} = (1-\rho_0) \frac{\partial V^b}{\partial T} = -\sum_{m=1}^{T-1} (\prod_{l=m+1}^{T}(1-\rho_l)) u'_i(c^g_{m})$. As before, we define $E(u'_i(c^g_{m})) = \frac{1}{\prod_{l=m}^{T-1}(1-\rho_l)} \sum_{m=0}^{T-1} (\prod_{l=0}^{m-1}(1-\rho_l)) u'_i(c^g_{m})$ to be the average marginal utility of consumption in the good state. Therefore, we have shown that

$$
\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T} = -D^g E(u'_i(c^g_{it})).
$$

Hence, combining equations (6) and (7) we can express (4) by $D^g E(u'_i(c^g_{it})) - D^g E(u'_i(c^g_{it}))$. Since this expression has no cardinal interpretation, we normalize it by the welfare gain from decreasing taxes in the good state, $-\frac{D^b}{D^g} = (\rho_0 \frac{\partial V^b}{\partial T} + (1-\rho_0) \frac{\partial V^g}{\partial T}) = D^g E(u'_i(c^g_{it}))$. Putting these parts together yields the result, which completes the proof. ■

**Labor Supply Representation of Welfare Gains.** Next, we rewrite the marginal benefit in (5) in terms of the indirectly-affected spouse’s labor supply.

**Proposition B.1.** The marginal benefit from raising $B^b$ can be represented by

$$
MB \cong \varphi \times \left( \frac{D^b}{D^g} \right),
$$

(8)
where $l^{*}_2$ is the average labor supply of spouse 2 is state $s$ and $\varphi = \frac{v''(l^{*}_2)}{v''(l^{*}_2)l^{*}_2}$.

Proof. Using the household’s optimality conditions in (1) and (2) we can rewrite (5) as $MB = \frac{E(v'_2(l^{*}_2)) - E(v'_2(l^{*}_2))}{E(v''(l^{*}_2))}$.

The rest of the proof closely follows that for the consumption-based formula in Chetty (2006a) applied to spousal labor supply. A quadratic approximation to member 2’s labor disutility function around $l^{*}_2$ implies that $v'_2(l^{*}_2) \equiv v'_2(l^{*}_2) + v''(l^{*}_2) (l - l^{*}_2)$ and $E(v''(l^{*}_2)) = v''(l^{*}_2)$, and hence $MB \equiv \frac{v'_2(l^{*}_2)}{v''(l^{*}_2)}$.

Employing a similar quadratic approximation again around $l^{*}_2$ we get $v'_2(l^{*}_2) \equiv v'_2(l^{*}_2) + v''(l^{*}_2) (l - l^{*}_2)$ which yields the result.  

Appendix B.2: Extensive-Margin Model

Following the normative literature on unemployment insurance, we analyze dynamic household labor force participation decisions using a standard dynamic search model extended to households of two individuals.\(^3\)

Setup. The setup of the problem is the same as in the dynamic intensive-margin model in Appendix B.1 above.

Household Budget Constraint. In this model $l^{*}_2 = 1$ if $i$ works and $l^{*}_2 = 0$ otherwise. Compared to the dynamic intensive-margin model in Appendix B.1, we adjust the household resource constraint so that $z_t (l^{*}_2) = z_t \times l^{*}_2$, where $z_t$ are gross earnings conditional on working. We also let $B^*(l_2^{*})$ represent benefits from the government in state $s$ as a function of $l^{*}_2$, and denote the vector of benefits by $B$.

Preferences. Let $U(c^1_t, c^2_t) = u_1(c^1_t) + u_2(c^2_t)$ represent the household’s flow consumption utility at time $t$ as a function of consumption of each member in each period and state. We employ the normalization $u_1(0) = 0$ and assume that the individual consumption utility functions are well-behaved—i.e., that $u'_1(c^1_t) > 0$ and $u''_1(c^1_t) < 0$. We denote member 2’s utility cost of search effort at time $t$ when unemployed by $\kappa(e^{2}_t)$, which we assume to be strictly increasing and convex.

Household Behavior. At the beginning of each period $t$ in state $s$, the household chooses the individual consumption flows, $c^*_t$, as well as member 2’s search effort, $e^{2}_t$, if 2 is unemployed and enters period $t$ without a job. We normalize $e^{2}_t$ to equal the probability of finding a job in the same period, and assume that at the beginning of the planning period member 2 does not work. If 2 finds a job in period $t$, the job begins at time $t$ and is assumed to last until the end of the planning period.

The value function for households in state $s$ who enter period $t$ when 2 is without a job and with household assets $A_t$ is

$$V^{s,0}_t(B, \tau, A_t) \equiv \max \left\{ e^{2}_t \left( U(c^1_t(1), c^2_t(1)) + W^{s,1}_{t+1}(B, \tau, A_{t+1}^{s}) \right) \right.$$

$$\left. + (1 - e^{2}_t) \left( U(c^1_t(0), c^2_t(0)) + W^{s,0}_{t+1}(B, \tau, A_{t+1}^{s}) \right) - \kappa(e^{2}_t) \right\},$$

where the budget constraints satisfy

$$c^1_t(l^{*}_2) + c^2_t(l^{*}_2) + A_{t+1}^{s} (l^{*}_2) = A_t + y^1_t(l^{*}_2),$$

$$y^2_t(l^{*}_2) = I^2_t + z_t + z^2_t (l^{*}_2) + B^g (l^{*}_2) - \tau,$$

and $W^{s,1}_{t+1}(B, \tau, A_{t+1}^{s})$ are the continuation value functions which depend on whether the job search was successful or not in time $t$. The continuation functions are defined by

$$W^{s,0}_{t+1}(B, \tau, A_{t+1}^{s}) \equiv (1 - \rho_{t+1}) V^{s,0}_{t+1}(B, \tau, A_{t+1}^{s}) + \rho_{t+1} V^{s,1}_{t+1}(B, \tau, A_{t+1}^{s}),$$

and

$$W^{s,1}_{t+1}(B, \tau, A_{t+1}^{s}) \equiv V^{s,1}_{t+1}(B, \tau, A_{t+1}^{s}).$$

where $V^{s,1}_{t}(B, \tau, A_t)$ is the value of entering period $t$ when 2 is employed in state $s$, which is defined by

$$V^{s,1}_{t}(B, \tau, A_t) \equiv \max \left\{ U(c^1_t(1), c^2_t(1)) + W^{s,1}_{t+1}(B, T, A_{t+1}^{s}) \right\}.$$

\(^3\)One can also use this model to analyze the static case as an alternative to our stylized extensive-margin model.
The optimal search effort is chosen according to the first-order condition
\[
\left(U(e_{1t}^s(1), e_{2t}^s(1)) + W_{t+1}^s(B, T, A_{t+1}(1))\right) - \left(U(e_{1t}^s(0), e_{2t}^s(0)) + W_{t+1}^s(0)(B, T, A_{t+1}(0))\right) = \kappa'(e_{2t}^s), \tag{9}
\]
which implies that the effect of a $1 increase in the benefit level \(B^s(0)\) on search intensity in state \(s\) is
\[
\frac{\partial e_{2t}^s}{\partial B^s(0)} = -\frac{1}{\kappa''(e_{2t}^s)} \left(u'_s(e_{2t}^s(0)) + \frac{\partial W_{t+1}^s(0)}{B^s(0)}\right). \tag{10}
\]

**Planner’s Problem.** We define the household’s expected utility at the beginning of the planning period by \(J_0(B, \tau, A_0) \equiv \rho_0 V_0^{b,h}(B, \tau, A_0) + (1 - \rho_0) V_0^{g,h}(B, \tau, A_0)\). The social planner’s objective is to choose the tax-and-benefit system that maximizes the household’s expected utility at time 0 subject to a balanced-budget constraint that equates the expected revenues to expected transfers. Let \(D^g\) denote the expected share of the household’s life-time in state \(s\). With this notation, the expected average revenue collected from each household is \(D^g\).

Next, let \(\bar{e}_t^s\) denote the conditional probability of member 2 being employed if observed in state \(s\). To construct the benefit side of the budget constraint, consider randomly choosing a household at a random point in its life-cycle. The probability of choosing a household in state \(s\) is \(D^s\) and, hence, the probability of choosing a household in state \(s\) in which 2 is unemployed is \(D^s(1 - \bar{e}_t^s)\). Accordingly, the expected value of average benefits transferred to a household, which we denote by \(E(B)\), is equal to
\[
E(B) \equiv \sum_{s \in \{g, h\}} D^s [(1 - \bar{e}_t^s) B^s(0) + \bar{e}_t^s B^b(1)].
\]
Using this notation, we can write the planner’s maximization problem as
\[
\text{max } J_0(B, \tau, A_0) \quad \text{s.t. } E(B) = D^g.
\]

**Welfare Gains from Social Insurance.** What is the welfare gain from providing more generous benefits when the bad state occurs? Similar to the static model, we consider transferring resources from the good state \(g\) to the bad state \(b\) (for a given choice of spousal employment) through a small increase in the benefit \(B^b(0)\) financed by a balanced-budget decrease in \(B^g(0)\). As we mention in footnote 10, we derive in the analysis here the net welfare gain including both benefits and costs for illustrative purposes. Since we focus on \(B^b(0)\), we assume that \(B^b(1) = 0\). This decreases the number of terms that involve behavioral responses in the cost side of the formula and simplifies the algebra, but we can readily conduct the same analysis without this assumption.

The net welfare gain from this perturbation is
\[
\frac{dJ_0(B, \tau, A_0)}{dB^b(0)} = S_1 + S_2 \frac{dB^g(0)}{dB^b(0)},
\tag{11}
\]
where \(S_1 \equiv \rho_0 \frac{\partial V_0^{b,h}}{\partial B^b(0)} + (1 - \rho_0) \frac{\partial V_0^{g,h}}{\partial B^g(0)}\) and \(S_2 \equiv \rho_0 \frac{\partial V_0^{b,h}}{\partial B^b(0)} + (1 - \rho_0) \frac{\partial V_0^{g,h}}{\partial B^g(0)}\). The following proposition provides an approximated formula for a normalized version of this gain, which has a direct cardinal interpretation.

**Proposition B.2.**

The normalized net marginal gain from raising \(B^b(0)\), which we denote by \(M_w\), can be represented by
\[
M_w \equiv MB - MC,
\]
under a locally quadratic approximation to the effort function around \(e_{20}^s\), with

1. \(MB \equiv \Phi \times \left(\frac{e_{20}^s}{e_{20}^s}\right) - 1\), where \(\Phi \equiv \frac{|e(e_{20}^s, B^b(0))/B^b(0)|/e(e_{20}^s, B^b(0))/B^b(0)}{\varepsilon(x, y)} \equiv \frac{\partial x}{\partial y} \cdot e_{20}^s\) is 2’s participation probability at the beginning of the planning period, and \(e_{20}^s\) is 2’s mean participation rate in households that transition to state \(b\),

2. \(MC \equiv \beta_0 + \beta_1 e(1 - e_{20}^s, B^b(0)) + \beta_2 e(1 - e_{20}^s, B^b(0))\), where the coefficients \(\beta_0, \beta_1, \text{ and } \beta_2\) are functions of the transition probabilities, average participation rates and benefits, and \(\varepsilon(x, y) \equiv \frac{\partial x}{\partial y} \cdot e^4\).

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4Specifically, \(\beta_0 \equiv \frac{\sigma B^b(1 - e_{20}^s) - D^g(1 - e_{20}^s)}{D^g(1 - e_{20}^s)}\), \(\beta_1 \equiv \sigma B^g(1 - e_{20}^s)\), and \(\beta_2 \equiv \sigma B^b(1 - e_{20}^s)\), where \(\sigma \equiv (1 - \rho_0)(1 - e_{20}^s)/\rho(1 - e_{20}^s)\) and \(\rho \equiv \sum_{m = 0}^{\tau - 1} (1 - \rho_1) \rho_m\). Note that the elasticities in \(MC\) consist of the total effect of increasing \(B^b(0)\), which takes into
Proof. Similar to the proof of Lemma B.1, the general logic of the proof is to characterize the derivatives of the value functions in their sequential problem representation—that is, as a sum of derivatives over time and over different states of nature. To do so, we work backwards from period $T - 1$ to period 0.

We begin by providing expressions for $\frac{\partial V^t_{b,0}}{\partial B^t(0)}$ and $\frac{\partial V^t_{e,0}}{\partial B^t(0)}$ in order to characterize $S_1$. First, we have that

$$\frac{\partial V^t_{b,0}}{\partial B^t(0)} = (1 - e^{gb}_{2t}) \left(u'(c^b_{it}(0)) + \frac{\partial W^t_{b,0}}{\partial B^t(0)}\right)$$

and

$$\frac{\partial V^t_{e,0}}{\partial B^t(0)} = \frac{\partial V^t_{e,0}}{\partial B^t(0)},$$

which imply that

$$\frac{\partial V^t_{b,0}}{\partial B^t(0)} = (1 - e^{gb}_{2t}) \left(u'(c^b_{it}(0)) + \frac{\partial V^t_{b,0}}{\partial B^t(0)}\right).$$

Working backwards one can show that

$$\frac{\partial V^{T-1}_{b,0}}{\partial B^{T-1}(0)} = (1 - e^{gb}_{2t}) \left[u'(c^b_{iT}(0)) + \sum_{m = t+1}^{T-1} \left(\Pi_{m = t+1}^{m-1} (1 - e^g_{2m}) \left[u(c^b_{m}(0))\right]\right)\right].$$

Next, since

$$\frac{\partial W^{T-1}_{b,0}}{\partial B^{T-1}(0)} = 0,$$

we obtain

$$\frac{\partial V^{T-1}_{e,0}}{\partial B^{T-1}(0)} = (1 - e^{gb}_{2t}) \frac{\partial V^{T-1}_{e,0}}{\partial B^{T-1}(0)},$$

where

$$\frac{\partial V^{T-1}_{e,0}}{\partial B^{T-1}(0)} = (1 - e^{gb}_{2t}) + (1 - e^{gb}_{2t}) + \frac{\partial V^{T-1}_{b,0}}{\partial B^{T-1}(0)},$$

which implies by working backwards from period $T - 1$ to period 0 that

$$\frac{\partial V^t_{b,0}}{\partial B^t(0)} = (1 - e^{gb}_{2t}) \sum_{m = t+1}^{T-1} \left(\Pi_{m = t+1}^{m-1} (1 - e^g_{2m})(1 - \rho_m)\right) \rho_m \frac{\partial V^t_{b,0}}{\partial B^t(0)}.$$

Putting the terms together, it follows that

$$S_1 = \rho s \frac{\partial V^t_{b,0}}{\partial B^t(0)} + (1 - \rho) \frac{\partial V^t_{b,0}}{\partial B^t(0)} \sum_{m = t+1}^{T-1} \left(\Pi_{m = t+1}^{m-1} (1 - e^g_{2m})(1 - \rho_m)\right) \frac{\partial V^t_{b,0}}{\partial B^t(0)}.$$

Using equation (10) and $\frac{\partial V^t_{b,0}}{\partial B^t(0)} = (1 - e^{gb}_{2t}) \left(u'(c^b_{iT}(0)) + \frac{\partial V^t_{b,0}}{\partial B^t(0)}\right)$, we get that

$$\frac{\partial V^t_{b,0}}{\partial B^t(0)} = -\kappa'(e^b_{2t}) \frac{\partial V^t_{b,0}}{\partial B^t(0)} (1 - e^{gb}_{2t}).$$

Plugging this expression into (12) yields the following result

$$S_1 = -\sum_{m = 0}^{T-1} \left(\Pi_{m = t}^{m-1} (1 - e^g_{2m})(1 - \rho_m)\right) (1 - e^g_{2m}) \kappa'(e^b_{2m}) \frac{\partial V^t_{b,0}}{\partial B^t(0)}.$$

To understand the meaning of this formula let us break it down into its components. First, note that it is a weighted sum of a function of the change in effort (or participation rate), $\frac{\partial V^t_{b,0}}{\partial B^t(0)}$. The weight, the term in brackets, is the probability of reaching period $m$ with 2 unemployed and transitioning to state $b$ exactly in that period. For households that transition to state $b$ in period $m$ when 2 is employed, the change in effort and participation rates is zero (because they stay employed and do not engage in search effort). Therefore, dividing the probability weights by the chance of transitioning to state $b$ at some point throughout the planning horizon, $\rho \equiv \sum_{m = 0}^{T-1} \left(\Pi_{m = t}^{m-1} (1 - \rho_m)\right)$, and rewriting (13) in terms of elasticities (with $\varepsilon(x, y) \equiv \frac{\partial x}{\partial y}$) yield

$$S_1 = \rho E_b \left(1 - e^b_{2t}\right) \kappa'(e^b_{2t}) \left|\varepsilon(e^b_{2t}, B^t(0))\right| \frac{\partial V^t_{b,0}}{\partial B^t(0)} \right| \rho \equiv \rho E_b (g(e^b_{2t}))$$

where $e^b_{2t}$ denotes participation in the period the household transitions to state $b$ and $E_b$ is the expectation operator conditional on transitioning to state $b$ at some point in the life-cycle. By expanding $g(c)$ around 2’s average participation in households that transition to the bad state—which we denote by $e^b_{2t}$ (as it is the expected value from the perspective of time 0)—such that $g(c) \cong g(e^b_{2t}) + g'(e^b_{2t})(c - e^b_{2t})$, we approximate $E_b(g(e^b_{2t})) \cong E_b(g(e^b_{2t})) = g(e^b_{2t})$ and obtain the approximation

$$S_1 \cong \rho \left(1 - e^b_{2t}\right) \kappa'(e^b_{2t}) \left|\varepsilon(e^b_{2t}, m^b)\right| \frac{e^b_{2t}}{B^t(0)}.$$

We now turn to provide expressions for $\frac{\partial V^t_{b,0}}{\partial B^t(0)}$ and $\frac{\partial V^t_{e,0}}{\partial B^t(0)}$ in order to characterize $S_2$. Since households that transition to state $b$ stay in that state, we have that $\frac{\partial V^t_{b,0}}{\partial B^t(0)} = 0$. In addition, $\frac{\partial V^t_{e,0}}{\partial B^t(0)} = (1 - e^g_{2t}) \left(u'(c^b_{it}(0)) + \frac{\partial W^t_{b,0}}{\partial B^t(0)}\right)$, which combined with equation (10) yields

$$\frac{\partial V^t_{e,0}}{\partial B^t(0)} = -(1 - e^g_{2t}) \kappa'(e^g_{2t}) \frac{\partial V^t_{b,0}}{\partial B^t(0)}.$$

Put together, we get that

$$S_2 = (1 - \rho)(1 - e^g_{2t}) \kappa'(e^g_{2t}) \left|\varepsilon(e^b_{2t}, B^t(0))\right| \frac{e^g_{2t}}{B^t(0)}.$$

To complete the proof we need to calculate $\frac{dB^t(0)}{\partial B^t(0)}$. Total differentiation of the simplified budget con-
straint \(D^g (1 - \epsilon_2^g) B^g(0) + D^b (1 - \epsilon_2^b) B^b(0) = D^g \tau\) with respect to \(B^b(0)\) gives us

\[
\frac{d B^b(0)}{d B^b(0)} = \frac{- D^b (1 - \epsilon_2^b, B^b(0))}{D^b (1 - \epsilon_2^b) B^b(0)} - \frac{D^b (1 - \epsilon_2^b, B^b(0))}{D^b (1 - \epsilon_2^b)}.
\]  

(16)

where \(\epsilon(x, y) \equiv \frac{dy}{dx} \). Plugging (14), (15), and (16) into (11), and using a quadratic approximation to the effort function around \(\epsilon_2^g, 0\), we obtain the approximated formula for our normalized welfare gain \(M_w \equiv \frac{\partial u_0(B^{1,0},A^{1,0})}{\partial x(1 - \epsilon_2^g)\partial x(1 - \epsilon_2^g)}/\rho(1 - \epsilon_2^g)\) that is stated in the proposition, which completes the proof.

Appendix C: State-Dependent Preferences in the Extensive-Margin Model

In this appendix we show how one can extend our stylized extensive-margin framework to allow for state-dependent utility.

Let \(U^s(c_1^s, c_2^s; l_1^s, l_2^s) = u_1^s(c_1^s) + u_2^s(c_2^s) - v_1^s \times l_1^s - v_2^s \times l_2^s\) represent the household’s utility as a function of consumption and labor force participation of each member in each state. \(u_i^s(c_i^s)\) and \(v_i^s\) are member \(i\)’s utility from consumption and disutility from labor in state \(s\), respectively.\(^5\) For simplicity, we model this type of state dependence as \(v_2^g = v_2\) and \(v_2^b = \theta_l \times v_2^g\), such that \(\theta_l\) captures the percent change in the utility cost of spousal labor compared to the baseline state \(g\).\(^6\) The next proposition summarizes how the formula in Proposition 2 adjusts to account for state-dependence.

**Proposition C.1.** With state-dependent preferences, the marginal benefit from raising \(B^b(0)\) can be represented by

\[
MB = \theta_b \times \Phi \times \left(\frac{\epsilon_2^g}{\epsilon_2^b}\right) - 1,
\]

(17)

where \(\Phi \equiv \phi^b/\phi^g\), \(\phi^s \equiv \frac{|\epsilon(c_1^s, B^s(0))|}{B^s(0) \times \bar{f}(c_2^s)}\), and \(\epsilon(c_2^s, B^s(0))\) is the spouse’s participation elasticity with respect to the policy tool \(B^s(0)\).

**Proof.** The proof follows the proof of Proposition 2 with the adjustment of using the state-dependent preference structure. Note that in this case the “price” component, \(\Phi\), adjusts to \(\theta_b \times \Phi\), since the relative cost of spousal labor supply across states of nature becomes larger by a factor of \(\theta_b\).

**References**


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\(^5\)Since the additively-separable component of spousal labor disutility is used for identification of welfare gains in Proposition C.1, it is possible to allow for flexible consumption-leisure complementarities in \(u_i^s(c_i^s)\) or to let each member’s consumption utility depend on the other member’s consumption and leisure without changing the formula.

\(^6\)In the appendix for the dynamic search model in Fadlon and Nielsen (2015) we show that this is a simplification and that it is not necessary to define such a global parameter for our theoretical results, by illustrating how it can be locally and non-parametrically defined.