## Statistical Reasoning \& Decision Making

## Decision Making

- Modern Take: Probably Not!
- Limited information
- Limited processing capacity
- However, people good at quick decisions under non-optimal conditions
- But first, how does human decision making deviate from economists' norms?


## Normative Rational Models

- Irrational Reasoning
- Reasoning processes that reach contradictory conclusions based on the same evidence

$$
\text { - A > B AND B > A } \leftarrow \text { Irrational }
$$

- Consistency important
- If John prefers a paper clip to a stereo, and a stereo to a free trip around the world, then John should prefer a paper clip to a free trip around the world


## Decision Making

- Is human decision making optimal?
- Guided by normative theories devised by economists and philosophers
- Traditional Assumption: Yes.
- Just need to figure out what's being optimized
- Study what people value
- If we know what people value, can predict their choices


## Heuristics \& Biases

- Visual Illusions
- Cognitive Illusions
- Situations where heuristics and strategies fail or are misleading
- Sub-optimal/Irrational decisions point to mechanisms
- Sub-optimal, but largely effective
- Analogous to vision
- Bounded Rationality (Simon)



## Normative Rational Models

- Prescriptive Models
- How we should perform
- Given assumptions about a person's goals, these models tell us what choices are optimal
- Provides norms for evaluating human decision making
- Descriptive Models
- How we do perform
- Sometimes differs from that prescribed by normative models

| Expected Value Theory |  |
| :--- | :--- |
| - Winning $\$ 40$ with <br> probability of .2 <br> Winning $\$ 30$ with <br> probability of .25 | - $\$ 40 \times .2=\$ 80 \times .25=\$ 7.50$ |
|  | - Expected Value $=$ <br> (Value of Outcome) $\times$ <br> (Probability of <br> Outcome) |
|  |  |

## Come on! Who takes EVT seriously?



## Paradoxes Generated by EVT

- Limitations of EVT revealed in paradoxes it produces
- Paradox - 2 inconsistent statements, both of which are intuitively true
- Resolution of a paradox can lead to changes in theory that gives rise to it
"Since OSHA valued a life at $\$ 3.5$ million, the regulation easily passed the costbenefit test. But the Office of Management and Budget, the administration's regulatory gatekeeper, stepped in with a new price on a construction worker's life - \$1 million, based on its own research - that stalled the rules for years." San Diego Union July 14, 1990


## Allais Paradox

- Propose 2 choice situations where people agree on the rational decision in each case
- Then show that these 2 decisions are inconsistent


## Allais Paradox: Choice One

- \$1,000 w/probability of 1.0
\$1,000 w/probability of .89
- \$5,000 w/probability of . 10
- \$0 w/probability of . 01


## Inconsistency

- Expected Value has not been maximized
- Expected Value has been maximized


## Same Choice

Choice 1

- $\$ 1,000 \mathrm{w} /$ probability of 1.0

Choice 2

- \$1,000 w/probability . 11$\}$
- \$0 w/probability . 89
- \$5,000 w/probability . 10
- \$0 w/probability 90


## Same Choice

Choice 1

- \$1,000 w/probability of 1.0
- $\$ 1,000 \mathrm{w} /$ probability of . 89
- $\$ 5,000 \mathrm{w} /$ probability of . 10
- $\$ 0 \mathrm{w} /$ probability of .01
.01

Choice 2

- \$1,000 w/probability . 11$\}$
- \$0 w/probability . 89
- \$5,000 w/probability .10
- \$0 w/probability . 90


## Responses to Allais Paradox

- Expected Value Theory should be revised to account for special status of "sure thing" options
- People don't calculate EV when confronted w/complicated choices


## Certainty Effect (Kahneman \& Tversky)

- $80 \%$ probability of losing 100 lives
- 100\% probability of losing 75 lives
- People prefer 80\% probability of losing 100 lives
- $10 \%$ chance to lose 75 lives
- 8\% chance to lose 100 lives
- People prefer 10\% chance to lose 75 lives
- But this choice is the same as the first, with probabilities reduced by a factor of 10


## Certainty Effect (Kahneman \& Tversky)

- $80 \%$ probability of losing 100 lives
- 100\% probability of losing 75 lives
- EVT says
- first choice loses 80 lives - second loses only 75 lives - second choice better
- Outcomes perceived with certainty are overweighted relative to uncertain outcomes
- $10 \%$ chance to lose 75 lives
- 8\% chance to lose 100 lives
- EVT says
- first choice loses 7.5 lives
- second loses 8 lives
- first choice (the one peopl choose) better
- When certainty doesn't cloud the picture, people choose in accordance with the normative theory


## Preference Reversals

- People make a distinction between how attractive a particular choice is and how much they're willing to pay for the chance to make the gamble
- Decision making theories typically consider these two factors equivalent measures of preference

- 11/12 chance to win 12 chips
- 1/12 chance to lose 24 chips
- Bet A chosen 50\% of time
- Bet A received a higher selling price $12 \%$ of time
- Bet B
- 2/12 chance to win 79 chips
- 10/12 chance to lose 5 chips
- Bet B chosen 50\% of time
- Bet B received a higher selling price $88 \%$ of time



## Framing Effects



- Irrational (inconsistent) decision making can be shown by presenting the same information in two different forms
- Changes in decision associated with different presentation forms are known as framing effects


## Framing Effects



- Imagine the US is preparing for an outbreak of disease which is expected to kill 600 people. 2 programs are proposed:
- Which would you choose?
- Program A: 200 people will be saved
- Program B:
- $1 / 3$ prob 600 people saved - 2/3 prob no people will be saved


## Framing Effect

- Program A and Program C identical
- Program B and Program D identical
- But people prefer
- A (save 200 people) over B (1/3 save 600) and
- D (2/3 600 die) over C (400 people die)
- Certainty affected decisions about lives lost differently from lives saved


## Alternative Framing



- Program C: 400 people will die
- Program D: $1 / 3$ probability no people will die, 2/3 probability 600 people will die
- Which would you choose?


## Sunk Cost Fallacy

- When past actions affect future choices in an irrational manner
- Walk out of a play when you've paid \$10/ticket, but not when you've paid \$50/ticket
- If you walk out, you will not get your money back, regardless of how much you paid!



## Framing Effects

- Change in decision associated w/different presentation forms
- Lives Saved versus Lives Lost
- Sunk Cost of \$10 vs. Sunk Cost of \$50
- Irrational because inconsistent
- But can lead to adaptive decisions in some circumstances
- E.g. sunk cost 'fallacy' adaptive in cases that require modest negatives followed by strongly positive outcomes
- learning tennis
- long-term investment in the stock market


## Expected Value Theory

- Winning $\$ 40$ with probability of .2
- Winning $\$ 30$ with probability of .25
- $\$ 40 \mathrm{x} .2=\$ 8$
- $\$ 30$ x $.25=\$ 7.50$
- Expected Value = (Value of Outcome) x (Probability of Outcome)


## Problems with EVT

- Generates Paradoxes
- EVT's best option doesn't always seem best
- Preference Reversals
- Attractiveness of gamble doesn't predict how much people willing to buy or sell
for
- Subjective vs. Objective Worth
- Diminishing returns
- Thresholds (e.g. tuition)


## Expected Utility Theory

- Utility - subjective value, not objective value
- People maximize expected utility rather than expected value


## Expected Utility

```
n
\sum P}\mp@subsup{\textrm{P}}{\textrm{i}}{2}\mp@subsup{W}{\textrm{i}}{}=\mp@subsup{\textrm{P}}{1}{}\mp@subsup{\textrm{W}}{1}{}+\mp@subsup{\textrm{P}}{2}{}\mp@subsup{\textrm{W}}{2}{}+\ldots\mp@subsup{\textrm{P}}{n}{}\mp@subsup{W}{\textrm{n}}{
i=1
W = subjective worth of consequences (utility)
P = probability of outcome
- Compute worth of each of the possible consequences and the
- Multiply each W by its P
```

- To calculate worth probability of each
- Sum the products


## Why people gamble...

- Expected Value
- Expected Utility
$=P(W) * V(W)+P(L) * V(L)$
$=P(W) \star U(W)+P(L) * U(L)$
$=1 / 6(\$ 4)+5 / 6(-\$ 1)$
$=1 / 6(\$ 4+\$ 2)+5 / 6(-\$ 1)$
$=-\$ 1 / 6$
$=+\$ 1 / 6$



## $\pi$ Function in Prospect Theory

- Low and High


Probabilities Overweighted

- $\mathrm{pi}(1)>2^{*} \mathrm{pi}(.5)$
- Probabilities are not additive


## Regret Theory

- People overweight anticipated feelings of regret when the difference between outcomes is large

|  | Ticket Numbers |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Option | $1-9$ | $10-21$ | $22-24$ |  |
| A | $\$ 24$ | $\$ 0$ | $\$ 0$ | $\leftarrow$ |
| B | $\$ 0$ | $\$ 16$ | $\$ 0$ |  |
|  | Ticket Numbers |  |  |  |
|  | $1-9$ | $10-12$ | $13-24$ |  |
| Option | $\$ 24$ | $\$ 0$ | $\$ 0$ |  |
| C | $\$ 16$ | $\$ 16$ | $\$ 0$ | $\leftarrow$ |
| D |  |  |  |  |

