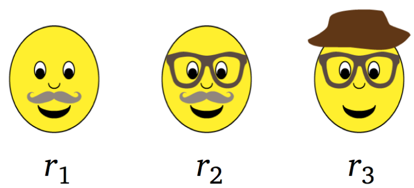
Linguistics 247
Homework 2

Linguistics 247, Roger Levy

Handed out 29 October 2014 – Due 5 November 2014

1. Consider the following scenario (due to Chris Potts):



Without thinking too deeply about it, distribute $100 in bets on who a speaker who uses the word *glasses* to refer to one of the faces is referring to, if the speaker can only use one word. (Ask a friend or two to do this too, if you have a chance, and report & compare your results.)

2. For this three-faces scenario, compute the IBR model results assuming that the utterance set is {`hat`,`glasses`,`mustache`}, equal utterance costs, and a uniform referent prior. How many levels of recursion are necessary before the scalar implicature emerges that "glasses" implies "not hat"? Can you give an informal explanation of why?

Solution: First, let's load the IBR model from Homework 1:

```
> Best.Response <- function(EU) {
+   for(i in 1:nrow(EU)) {
+     br <- EU[i,]==max(EU[i,])
+     #cat(i,":",br,"\n")
+     EU[i,br] <- 1/sum(br)
+     EU[i,!br] <- 0
+   }
+   return(EU)
+ }
> sigma.0 <- function(lexicon) Best.Response(t(lexicon))
> rho <- function(sigma,prior) Best.Response(t(prior*sigma))
> sigma <- function(rho) Best.Response(t(rho))
```

Now we'll set up the lexicon and run the model:

```
> lexicon <- matrix(c(0,0,1,0,1,1,1,1,0),3,3,
+ dimnames=list(c("hat","glasses","mustache"),c("r1","r2","r3")))
> uniform.prior <- rep(1/3,3)
> print(lexicon)

         r1 r2 r3
hat       0  0  1
glasses   0  1  1
mustache  1  1  0

> print(sigma0 <- sigma.0(lexicon))

   hat glasses mustache
r1 0.0     0.0      1.0
r2 0.0     0.5      0.5
r3 0.5     0.5      0.0

> print(rho1 <- rho(sigma0,uniform.prior))

         r1  r2  r3
hat       0 0.0 1.0
glasses   0 0.5 0.5
mustache  1 0.0 0.0

> print(sigma1 <- sigma(rho1))

   hat glasses mustache
r1   0       0        1
r2   0       1        0
r3   1       0        0

> print(rho2 <- rho(sigma1,uniform.prior))

         r1 r2 r3
hat       0  0  1
glasses   0  1  0
mustache  1  0  0

> print(sigma2 <- sigma(rho2))

   hat glasses mustache
r1   0       0        1
r2   0       1        0
r3   1       0        0
```

The scalar implicature of "glasses" implicating "not hat" emerges only at the σ_1 level—from the listener's perspective, only at the ρ_2 level. Unlike ordinary scalar implicature (e.g., *some*→*not all*), this takes an extra level to emerge. The reason for this is that the model has to first infer that "mustache" is not the best way to signal r_2—because "mustache" is uniquely well-suited to signal r_1—before there is compelling pressure for the context-specific meaning of "glasses" to specialize to r_2.

3. Compute RSA's $L(S(L_0))$ for the standard *some/all* language game—that is, with lexicon

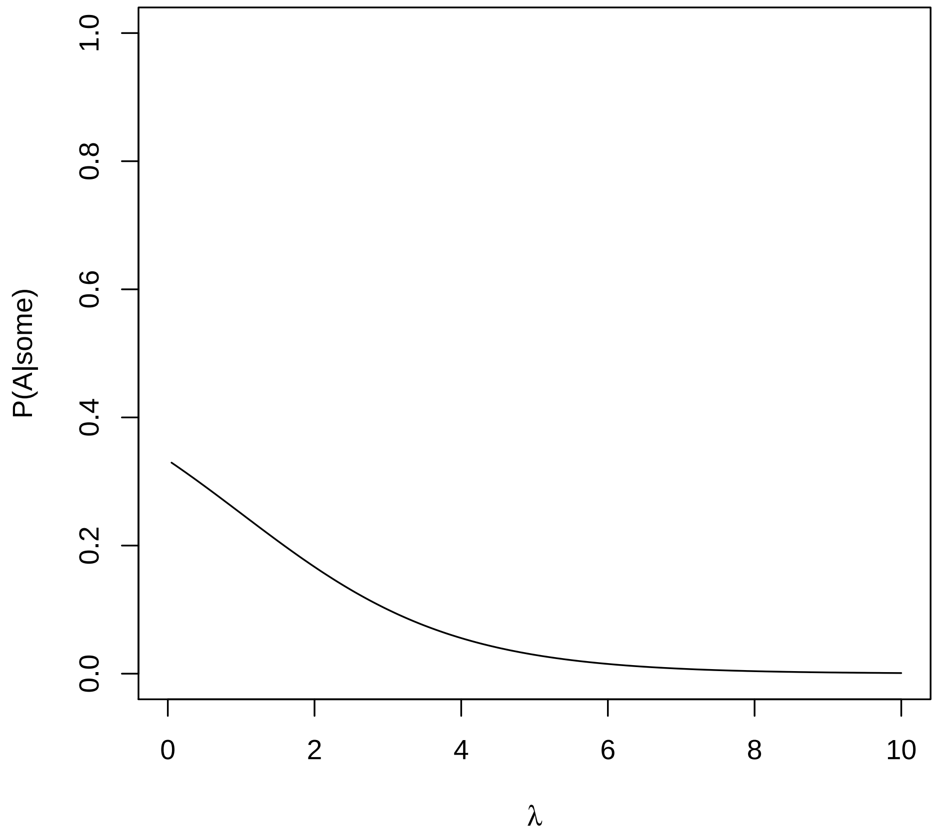
	$\exists\neg\forall$	$\forall$
some	1	1
all	0	1

and uniform prior. Plot $P(\forall|\text{some})$ in $L(S(L_0))$ as a function of the "greedy rationality" parameter λ ranging from 0 to 10 (note that Frank and Goodman called this parameter α but I'm going to call it λ). How dependent is the *qualitative* pattern of scalar implicature on the value of λ?

Solution:

```
> row.normalize <- function(matr) {
+   Z <- apply(matr,1,sum)
+   return(1/Z * (matr))
+ }
> column.normalize <- function(matr) t(row.normalize(t(matr)))
> L0 <- function(lexicon) row.normalize(lexicon)
> S <- function(listener,alpha) {
+   U <- log(listener)
+   Q <- exp(alpha * U)
+   return(row.normalize(t(Q)))
+ }
> L <- function(speaker, prior) {
+   Q <- prior * speaker
+   return(row.normalize(t(Q)))
+ }
> lexicon <-
+ matrix(c(1,0,1,1),2,2,dimnames=list(c("some","all"),c("E~A","A")))
> lambdas <- seq(0,10,by=0.05)
> uniform.prior <- rep(1/2,2)
> p.A.some <- sapply(lambdas,function(lambda)
+ L(S(L0(lexicon),lambda),uniform.prior)[1,2])

> plot(lambdas,p.A.some,type="l",
+      xlab=expression(lambda),ylab="P(A|some)",ylim=c(0,1))
```



Recall that $P(\forall|\text{some}) = \frac{1}{2}$ for L_0. Regardless of the precise value of λ, we can see that $P_{L(S(L_0))}(\forall|\text{some}) < P_{L_0}(\forall|\text{some})$. Thus the *qualitative* pattern of scalar implicature is highly robust and not sensitive to the choice of λ.

4. Consider an extended *some/all* game in which a third alternative utterance, *some but not all*, is available. The "lexicon" is now:

	$\exists\neg\forall$	$\forall$
some	1	1
all	0	1
some but not all	1	0

Now extend your implementation of RSA to include utterance costs as defined in Equation (S2) of Frank and Goodman (2012). Let $D(\text{some}) = D(\text{all}) = 0$, and $D(\text{some but not all}) > 0$. For $\lambda = 1$, plot $P(\forall|\text{some})$ for $L(S(L_0))$ as a function of the cost $D(\text{some but not all}) > 0$. What is the relationship between the asymptotic value this game of $P_{L(S(L_0))}(\forall|\text{some})$ as this cost increases and the value of this probability in the game from Problem 3 when $\lambda = 1$?

Solution:

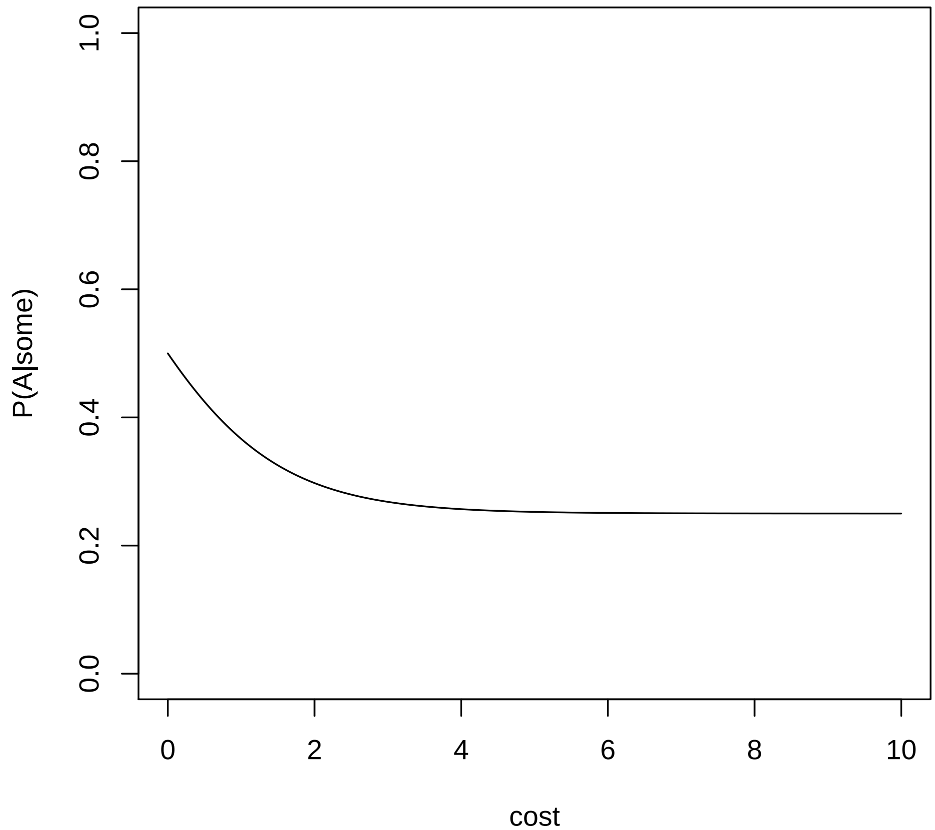
Here is the computation:

```
> S <- function(listener,costs,alpha) {
+   U <- log(listener)-costs
+   Q <- exp(alpha * U)
+   return(row.normalize(t(Q)))
+ }
> lexicon <- matrix(c(1,0,1,1,1,0),3,2,
+                   dimnames=list(c("some","all","some but not all"),
+                   c("E~A","A")))
> costs <- c(0,0,1)
> print(L(S(L0(lexicon),costs,alpha=1),uniform.prior))

                       E~A         A
some             0.6334782 0.3665218
all              0.0000000 1.0000000
some but not all 1.0000000 0.0000000

> costs <- seq(0,10,by=0.05)
> p.A.some <- sapply(costs,function(cost)
+ L(S(L0(lexicon),c(0,0,cost),alpha=1),uniform.prior)[1,2])

> plot(costs,p.A.some,type="l",xlab="cost",ylab="P(A|some)",ylim=c(0,1))
```



As the cost D(some but not all) grows, the posterior probability in question asymptotes to the same posterior probability that we saw in Problem 3 for $\lambda = 1$. That is, in this game high utterance cost really does render the utterance ignorable for purposes of computation of scalar implicature.

References

Frank, M. C. and Goodman, N. D. (2012). Predicting pragmatic reasoning in language games. *Science*, 336(6084):998.