The intersection of a weighted finite-state grammar and a weighted context-free grammar

Roger Levy

UC San Diego
Department of Linguistics

Computational Psycholinguistics
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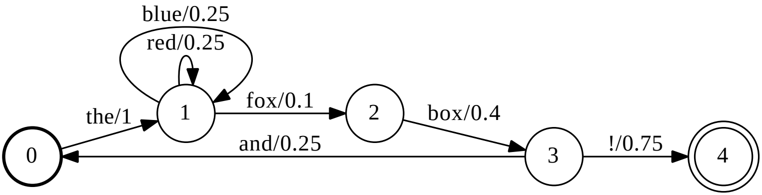
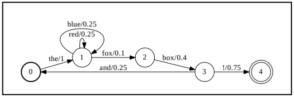
Weighted Finite-State Automata (WFSAs)

- An example of a WFSA:



Weighted Finite-State Automata (WFSAs) Take Two



A WEIGHTED FINITE-STATE AUTOMATON (WFSA) consists of a tuple (Q, V, S, R) such that:

- Q is a finite set of STATES $q_0 q_1 \ldots q_N$, with q_0 the designated START STATE;
- Σ is a finite set of terminal symbols;
- $F \subseteq Q$ is the set of FINAL STATES;
- Δ is a finite set of TRANSITIONS each of the form $q \overset{i}{\leadsto} q'$, meaning that "if you are in state q and see symbol i you can consume it and move to state q'";
- λ is a function mapping transitions to real numbers (weights);
- ρ is a function mapping final states to real numbers (weights).

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- ρ is a function mapping final states to real numbers (weights).

- $w_{1\ldots N} \in \Sigma^N$ is ACCEPTED or RECOGNIZED by an automaton iff there is a PATH of transitions $\underset{1\ldots N}{\rightsquigarrow}$ to a final state $q^* \in F$ such that

$$q_0 \underset{1}{\overset{w_1}{\rightsquigarrow}} \underset{2}{\overset{w_2}{\rightsquigarrow}} \ldots \underset{N-1}{\overset{w_{N-1}}{\rightsquigarrow}} \underset{N}{\overset{w_N}{\rightsquigarrow}} q^*$$

- The WEIGHT of such a path $\underset{1\ldots N}{\rightsquigarrow}$ is the product of the weights of each of the transitions, together with the weight of the final state:

$$P(q_0 \underset{1}{\overset{w_1}{\rightsquigarrow}} \underset{2}{\overset{w_2}{\rightsquigarrow}} \ldots \underset{N-1}{\overset{w_{N-1}}{\rightsquigarrow}} \underset{N}{\overset{w_N}{\rightsquigarrow}} q^*) = \rho(q^*) \prod_{i=1}^{N} \lambda(\underset{i}{\rightsquigarrow}) \tag{1}$$

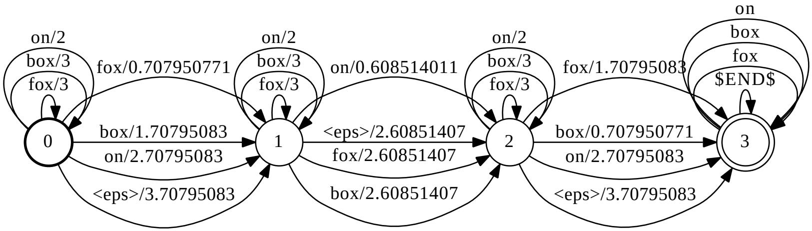
Integrating uncertain input and grammatical knowledge

Here's a toy PCFG – eight rules which we'll call our "grammatical knowledge"

Probabilities				Surprisals (negative log-probs, in nats)			
1	ROOT	$\rightarrow$	NP	0	ROOT	$\rightarrow$	NP
0.5	NP	$\rightarrow$	N	0.6931472	NP	$\rightarrow$	N
0.3	NP	$\rightarrow$	N N	1.203973	NP	$\rightarrow$	N N
0.2	NP	$\rightarrow$	NP PP	1.609438	NP	$\rightarrow$	NP PP
1	PP	$\rightarrow$	P NP	0	PP	$\rightarrow$	P NP
1	P	$\rightarrow$	on	0	P	$\rightarrow$	on
0.8	N	$\rightarrow$	box	0.2231436	N	$\rightarrow$	box
0.2	N	$\rightarrow$	fox	1.609438	N	$\rightarrow$	fox

Integrating uncertain input and grammatical knowledge

Here's a toy weighted finite-state automaton for uncertain input corresponding to *fox on box*...



Integrating uncertain input and grammatical knowledge

- Beautiful property of (weighted) context-free grammars: *closure under intersection with (weighted) finite-state automata* (**??**)
- This means that for any CFG G and any FSA A, a CFG $G' = G \cap A$ can be constructed such that every string and only strings accepted by both G and A are accepted by G'
- If the grammars are weighted, the weight assigned to a string by G' is the combined weight assigned by G and A

Intersecting weighted FSAs and CFGs

Constructive procedure for $G' = G \cap A$ (assuming G is at most binary branching):

- If ROOT is the root category of G and q_0 is the start state of A, then call ROOT the root category of G' and for every final state q_i in A with weight p construct a rule ROOT $\rightarrow$ ROOT$_{[0,i]}$ with weight p

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- For every binary rule $X \to Y\ Z$ with weight p in G and state triple (q_i, q_j, q_k) in A, construct a rule $X_{[i,k]} \to Y_{[i,j]}\, Z_{[j,k]}$ with weight p

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- For every unary rule $X \rightarrow Y$ with weight p in G and state pair (q_i, q_j) in A, construct a rule $X_{[i,j]} \rightarrow Y_{[i,j]}$ with weight p

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- For every unary rule $X \rightarrow Y$ with weight p in G and state pair (q_i, q_j) in A, construct a rule $X_{[i,j]} \rightarrow Y_{[i,j]}$ with weight p
- For every transition $q_i \overset{w}{\leadsto} q_j$ with weight p in A, construct a rule $w_{[i,j]} \rightarrow w$ with weight p

Intersecting weighted FSAs and CFGs

Probabilities				Surprisals (negative log-probs, in nats)			
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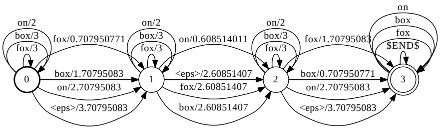

Rules that go into the intersection:

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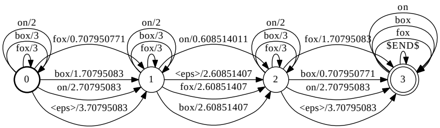

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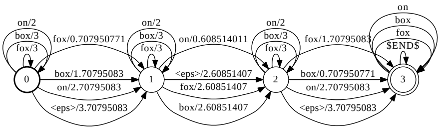

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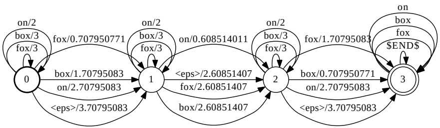

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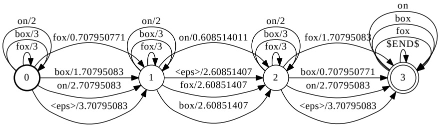

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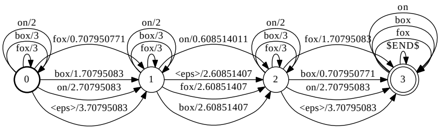

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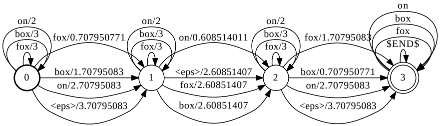

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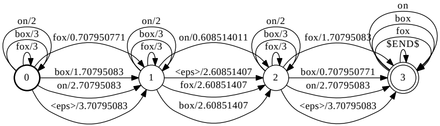

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0	$\text{ROOT}_{[0,1]}$	$\rightarrow \text{NP}_{[0,1]}$
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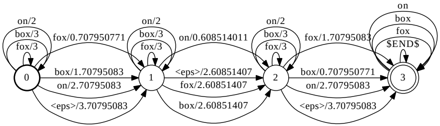

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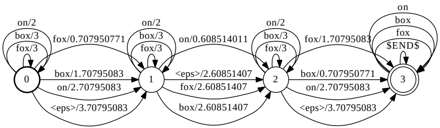

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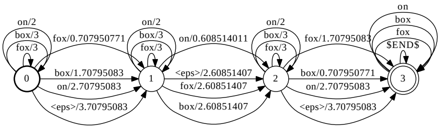

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...		

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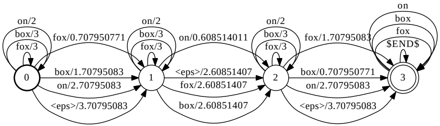

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0	$\text{ROOT}_{[0,2]}$	$\rightarrow \text{NP}_{[0,2]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,3]}\ \text{PP}_{[3,0]}$
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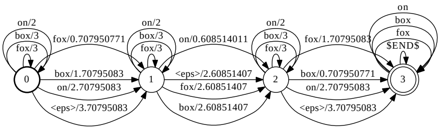
0.2	N	$\rightarrow$	fox	1.609438	N	$\rightarrow$	fox



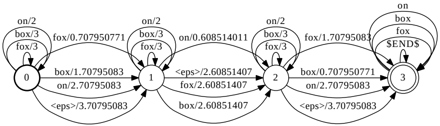
Rules that go into the intersection:

0	ROOT	$\rightarrow$	$\text{ROOT}_{[0,3]}$
0	$\text{ROOT}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,0]}$
0	$\text{ROOT}_{[0,1]}$	$\rightarrow$	$\text{NP}_{[0,1]}$
0	$\text{ROOT}_{[0,2]}$	$\rightarrow$	$\text{NP}_{[0,2]}$
0	$\text{ROOT}_{[0,3]}$	$\rightarrow$	$\text{NP}_{[0,3]}$
0	$\text{ROOT}_{[1,3]}$	$\rightarrow$	$\text{NP}_{[1,3]}$
...			

1.609438	$\text{NP}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,0]}\ \text{PP}_{[0,0]}$
1.609438	$\text{NP}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,1]}\ \text{PP}_{[1,0]}$
1.609438	$\text{NP}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,2]}\ \text{PP}_{[2,0]}$
1.609438	$\text{NP}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,3]}\ \text{PP}_{[3,0]}$
1.609438	$\text{NP}_{[0,0]}$	$\rightarrow$	$\text{NP}_{[0,0]}\ \text{PP}_{[0,1]}$

Intersecting weighted FSAs and CFGs

Probabilities				Surprisals (negative log-probs, in nats)		
1	ROOT	$\rightarrow$NP	0	ROOT	$\rightarrow$NP
0.5	NP	$\rightarrow$N	0.6931472	NP	$\rightarrow$N
0.3	NP	$\rightarrow$N N	1.203973	NP	$\rightarrow$N N
0.2	NP	$\rightarrow$NP PP	1.609438	NP	$\rightarrow$NP PP
1	PP	$\rightarrow$P NP	0	PP	$\rightarrow$P NP
1	P	$\rightarrow$on	0	P	$\rightarrow$on
0.8	N	$\rightarrow$box	0.2231436	N	$\rightarrow$box
0.2	N	$\rightarrow$fox	1.609438	N	$\rightarrow$fox



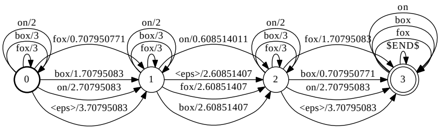
Rules that go into the intersection:

0	ROOT	$\rightarrow \mathrm{ROOT}_{[0,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,0]}$	$\mathrm{PP}_{[0,0]}$
0	$\mathrm{ROOT}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,0]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,1]}$	$\mathrm{PP}_{[1,0]}$
0	$\mathrm{ROOT}_{[0,1]}$	$\rightarrow \mathrm{NP}_{[0,1]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,2]}$	$\mathrm{PP}_{[2,0]}$
0	$\mathrm{ROOT}_{[0,2]}$	$\rightarrow \mathrm{NP}_{[0,2]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,3]}$	$\mathrm{PP}_{[3,0]}$
0	$\mathrm{ROOT}_{[0,3]}$	$\rightarrow \mathrm{NP}_{[0,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,0]}$	$\mathrm{PP}_{[0,1]}$
0	$\mathrm{ROOT}_{[1,3]}$	$\rightarrow \mathrm{NP}_{[1,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\rightarrow \mathrm{NP}_{[0,1]}$	$\mathrm{PP}_{[1,1]}$

...

Intersecting weighted FSAs and CFGs

Probabilities				Surprisals (negative log-probs, in nats)			
1	ROOT	→	NP	0	ROOT	→	NP
0.5	NP	→	N	0.6931472	NP	→	N
0.3	NP	→	N N	1.203973	NP	→	N N
0.2	NP	→	NP PP	1.609438	NP	→	NP PP
1	PP	→	P NP	0	PP	→	P NP
1	P	→	on	0	P	→	on
0.8	N	→	box	0.2231436	N	→	box
0.2	N	→	fox	1.609438	N	→	fox

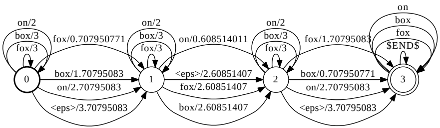


Rules that go into the intersection:

0	ROOT	$\to \mathrm{ROOT}_{[0,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,0]}\ \mathrm{PP}_{[0,0]}$
0	$\mathrm{ROOT}_{[0,0]}$	$\to \mathrm{NP}_{[0,0]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,1]}\ \mathrm{PP}_{[1,0]}$
0	$\mathrm{ROOT}_{[0,1]}$	$\to \mathrm{NP}_{[0,1]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,2]}\ \mathrm{PP}_{[2,0]}$
0	$\mathrm{ROOT}_{[0,2]}$	$\to \mathrm{NP}_{[0,2]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,3]}\ \mathrm{PP}_{[3,0]}$
0	$\mathrm{ROOT}_{[0,3]}$	$\to \mathrm{NP}_{[0,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,0]}\ \mathrm{PP}_{[0,1]}$
0	$\mathrm{ROOT}_{[1,3]}$	$\to \mathrm{NP}_{[1,3]}$	1.609438	$\mathrm{NP}_{[0,0]}$	$\to \mathrm{NP}_{[0,1]}\ \mathrm{PP}_{[1,1]}$
...			...		

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	$\rightarrow$ NP	0	ROOT	$\rightarrow$ NP
0.5	NP	$\rightarrow$ N	0.6931472	NP	$\rightarrow$ N
0.3	NP	$\rightarrow$ N N	1.203973	NP	$\rightarrow$ N N
0.2	NP	$\rightarrow$ NP PP	1.609438	NP	$\rightarrow$ NP PP
1	PP	$\rightarrow$ P NP	0	PP	$\rightarrow$ P NP
1	P	$\rightarrow$ on	0	P	$\rightarrow$ on
0.8	N	$\rightarrow$ box	0.2231436	N	$\rightarrow$ box
0.2	N	$\rightarrow$ fox	1.609438	N	$\rightarrow$ fox

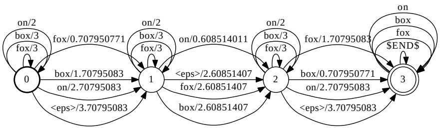


Rules that go into the intersection:

0	ROOT	$\rightarrow \text{ROOT}_{[0,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}\ \text{PP}_{[0,0]}$			
0	$\text{ROOT}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,1]}\ \text{PP}_{[1,0]}$	2	$\text{on}_{[0,0]}$	$\rightarrow$ on
0	$\text{ROOT}_{[0,1]}$	$\rightarrow \text{NP}_{[0,1]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,2]}\ \text{PP}_{[2,0]}$			
0	$\text{ROOT}_{[0,2]}$	$\rightarrow \text{NP}_{[0,2]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,3]}\ \text{PP}_{[3,0]}$			
0	$\text{ROOT}_{[0,3]}$	$\rightarrow \text{NP}_{[0,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}\ \text{PP}_{[0,1]}$			
0	$\text{ROOT}_{[1,3]}$	$\rightarrow \text{NP}_{[1,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,1]}\ \text{PP}_{[1,1]}$			
...			...					

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	$\to$NP	0	ROOT	$\to$NP
0.5	NP	$\to$N	0.6931472	NP	$\to$N
0.3	NP	$\to$N N	1.203973	NP	$\to$N N
0.2	NP	$\to$NP PP	1.609438	NP	$\to$NP PP
1	PP	$\to$P NP	0	PP	$\to$P NP
1	P	$\to$on	0	P	$\to$on
0.8	N	$\to$box	0.2231436	N	$\to$box
0.2	N	$\to$fox	1.609438	N	$\to$fox

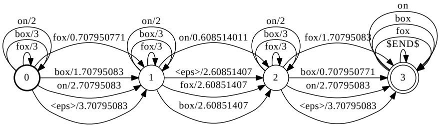


Rules that go into the intersection:

0	ROOT	$\to$ROOT$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,0]}$ PP$_{[0,0]}$			
0	ROOT$_{[0,0]}$	$\to$NP$_{[0,0]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,1]}$ PP$_{[1,0]}$	2	on$_{[0,0]}$	$\to$on
0	ROOT$_{[0,1]}$	$\to$NP$_{[0,1]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,2]}$ PP$_{[2,0]}$	3	box$_{[0,0]}$	$\to$on
0	ROOT$_{[0,2]}$	$\to$NP$_{[0,2]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,3]}$ PP$_{[3,0]}$			
0	ROOT$_{[0,3]}$	$\to$NP$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,0]}$ PP$_{[0,1]}$			
0	ROOT$_{[1,3]}$	$\to$NP$_{[1,3]}$	1.609438	NP$_{[0,0]}$	$\to$NP$_{[0,1]}$ PP$_{[1,1]}$			
...			...					

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	→ NP	0	ROOT	→ NP
0.5	NP	→ N	0.6931472	NP	→ N
0.3	NP	→ N N	1.203973	NP	→ N N
0.2	NP	→ NP PP	1.609438	NP	→ NP PP
1	PP	→ P NP	0	PP	→ P NP
1	P	→ on	0	P	→ on
0.8	N	→ box	0.2231436	N	→ box
0.2	N	→ fox	1.609438	N	→ fox

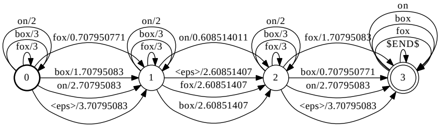


Rules that go into the intersection:

0	ROOT	$\rightarrow \text{ROOT}_{[0,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}\ \text{PP}_{[0,0]}$				
0	$\text{ROOT}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,1]}\ \text{PP}_{[1,0]}$	2		$\text{on}_{[0,0]}$	$\rightarrow$ on
0	$\text{ROOT}_{[0,1]}$	$\rightarrow \text{NP}_{[0,1]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,2]}\ \text{PP}_{[2,0]}$	3		$\text{box}_{[0,0]}$	$\rightarrow$ on
0	$\text{ROOT}_{[0,2]}$	$\rightarrow \text{NP}_{[0,2]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,3]}\ \text{PP}_{[3,0]}$	3		$\text{fox}_{[0,0]}$	$\rightarrow$ on
0	$\text{ROOT}_{[0,3]}$	$\rightarrow \text{NP}_{[0,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,0]}\ \text{PP}_{[0,1]}$				
0	$\text{ROOT}_{[1,3]}$	$\rightarrow \text{NP}_{[1,3]}$	1.609438	$\text{NP}_{[0,0]}$	$\rightarrow \text{NP}_{[0,1]}\ \text{PP}_{[1,1]}$				
...			...						

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	$\rightarrow$NP	0	ROOT	$\rightarrow$NP
0.5	NP	$\rightarrow$N	0.6931472	NP	$\rightarrow$N
0.3	NP	$\rightarrow$N N	1.203973	NP	$\rightarrow$N N
0.2	NP	$\rightarrow$NP PP	1.609438	NP	$\rightarrow$NP PP
1	PP	$\rightarrow$P NP	0	PP	$\rightarrow$P NP
1	P	$\rightarrow$on	0	P	$\rightarrow$on
0.8	N	$\rightarrow$box	0.2231436	N	$\rightarrow$box
0.2	N	$\rightarrow$fox	1.609438	N	$\rightarrow$fox

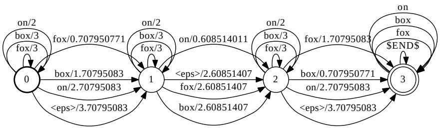


Rules that go into the intersection:

0	ROOT	$\rightarrow$ROOT$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$ PP$_{[0,0]}$			
0	ROOT$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$ PP$_{[1,0]}$	2	on$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,1]}$	$\rightarrow$NP$_{[0,1]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,2]}$ PP$_{[2,0]}$	3	box$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,2]}$	$\rightarrow$NP$_{[0,2]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,3]}$ PP$_{[3,0]}$	3	fox$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,3]}$	$\rightarrow$NP$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$ PP$_{[0,1]}$	0.70795	fox$_{[0,1]}$	$\rightarrow$fox
0	ROOT$_{[1,3]}$	$\rightarrow$NP$_{[1,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$ PP$_{[1,1]}$			
...			...					

Intersecting weighted FSAs and CFGs

Probabilities				Surprisals (negative log-probs, in nats)			
1	ROOT	→	NP	0	ROOT	→	NP
0.5	NP	→	N	0.6931472	NP	→	N
0.3	NP	→	N N	1.203973	NP	→	N N
0.2	NP	→	NP PP	1.609438	NP	→	NP PP
1	PP	→	P NP	0	PP	→	P NP
1	P	→	on	0	P	→	on
0.8	N	→	box	0.2231436	N	→	box
0.2	N	→	fox	1.609438	N	→	fox



Rules that go into the intersection:

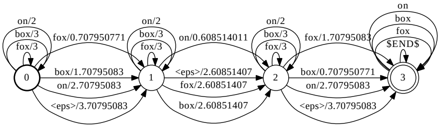
0	ROOT	→	$\text{ROOT}_{[0,3]}$
0	$\text{ROOT}_{[0,0]}$	→	$\text{NP}_{[0,0]}$
0	$\text{ROOT}_{[0,1]}$	→	$\text{NP}_{[0,1]}$
0	$\text{ROOT}_{[0,2]}$	→	$\text{NP}_{[0,2]}$
0	$\text{ROOT}_{[0,3]}$	→	$\text{NP}_{[0,3]}$
0	$\text{ROOT}_{[1,3]}$	→	$\text{NP}_{[1,3]}$
...			

1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,0]}$ $\text{PP}_{[0,0]}$
1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,1]}$ $\text{PP}_{[1,0]}$
1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,2]}$ $\text{PP}_{[2,0]}$
1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,3]}$ $\text{PP}_{[3,0]}$
1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,0]}$ $\text{PP}_{[0,1]}$
1.609438	$\text{NP}_{[0,0]}$	→	$\text{NP}_{[0,1]}$ $\text{PP}_{[1,1]}$
...			

2	$\text{on}_{[0,0]}$	→	on
3	$\text{box}_{[0,0]}$	→	on
3	$\text{fox}_{[0,0]}$	→	on
0.70795	$\text{fox}_{[0,1]}$	→	fox
...			

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	$\rightarrow$NP	0	ROOT	$\rightarrow$NP
0.5	NP	$\rightarrow$N	0.6931472	NP	$\rightarrow$N
0.3	NP	$\rightarrow$N N	1.203973	NP	$\rightarrow$N N
0.2	NP	$\rightarrow$NP PP	1.609438	NP	$\rightarrow$NP PP
1	PP	$\rightarrow$P NP	0	PP	$\rightarrow$P NP
1	P	$\rightarrow$on	0	P	$\rightarrow$on
0.8	N	$\rightarrow$box	0.2231436	N	$\rightarrow$box
0.2	N	$\rightarrow$fox	1.609438	N	$\rightarrow$fox



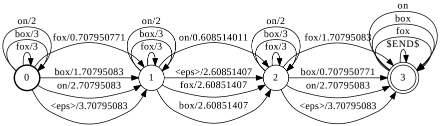
Rules that go into the intersection:

0	ROOT	$\rightarrow$ROOT$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$	PP$_{[0,0]}$				
0	ROOT$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$	PP$_{[1,0]}$	2	on$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,1]}$	$\rightarrow$NP$_{[0,1]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,2]}$	PP$_{[2,0]}$	3	box$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,2]}$	$\rightarrow$NP$_{[0,2]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,3]}$	PP$_{[3,0]}$	3	fox$_{[0,0]}$	$\rightarrow$on
0	ROOT$_{[0,3]}$	$\rightarrow$NP$_{[0,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$	PP$_{[0,1]}$	0.70795	fox$_{[0,1]}$	$\rightarrow$fox
0	ROOT$_{[1,3]}$	$\rightarrow$NP$_{[1,3]}$	1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$	PP$_{[1,1]}$	...		
...			...						

...As you can see, there's a lot of waste here!

Intersecting weighted FSAs and CFGs

Probabilities			Surprisals (negative log-probs, in nats)		
1	ROOT	$\rightarrow$NP	0	ROOT	$\rightarrow$NP
0.5	NP	$\rightarrow$N	0.6931472	NP	$\rightarrow$N
0.3	NP	$\rightarrow$N N	1.203973	NP	$\rightarrow$N N
0.2	NP	$\rightarrow$NP PP	1.609438	NP	$\rightarrow$NP PP
1	PP	$\rightarrow$P NP	0	PP	$\rightarrow$P NP
1	P	$\rightarrow$on	0	P	$\rightarrow$on
0.8	N	$\rightarrow$box	0.2231436	N	$\rightarrow$box
0.2	N	$\rightarrow$fox	1.609438	N	$\rightarrow$fox



Rules that go into the intersection:

0	ROOT	$\rightarrow$ROOT$_{[0,3]}$
0	ROOT$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$
0	ROOT$_{[0,1]}$	$\rightarrow$NP$_{[0,1]}$
0	ROOT$_{[0,2]}$	$\rightarrow$NP$_{[0,2]}$
0	ROOT$_{[0,3]}$	$\rightarrow$NP$_{[0,3]}$
0	ROOT$_{[1,3]}$	$\rightarrow$NP$_{[1,3]}$
...		

1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$ PP$_{[0,0]}$
1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$ PP$_{[1,0]}$
1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,2]}$ PP$_{[2,0]}$
1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,3]}$ PP$_{[3,0]}$
1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,0]}$ PP$_{[0,1]}$
1.609438	NP$_{[0,0]}$	$\rightarrow$NP$_{[0,1]}$ PP$_{[1,1]}$
...		

2	on$_{[0,0]}$	$\rightarrow$on
3	box$_{[0,0]}$	$\rightarrow$on
3	fox$_{[0,0]}$	$\rightarrow$on
0.70795	fox$_{[0,1]}$	$\rightarrow$fox
...		

...As you can see, there's a lot of waste here!

Test question: how many rules are there in G'?

Intersecting weighted FSAs and CFGs

How many rules are there in G'?

- A has 4 states (one of which is final) and 25 arcs
- G has 3 binary rules and 5 unary rules

Recap of constructive procedure:

- If ROOT is the root category of G and q_0 is the start state of A, then call ROOT the root category of G' and for every final state q_i in A with weight p construct a rule ROOT $\rightarrow$ ROOT$_{[0,i]}$ with weight p $\rightarrow$1 rule in G'

Intersecting weighted FSAs and CFGs

How many rules are there in G'?

- A has 4 states (one of which is final) and 25 arcs
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Recap of constructive procedure:

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- For every binary rule $X \rightarrow Y\ Z$ with weight p in G and state triple (q_i, q_j, q_k) in A, construct a rule $X_{[i,k]} \rightarrow Y_{[i,j]}\ Z_{[j,k]}$ with weight p $\rightarrow 3 \times 4^3 = 192$ rules in G'

Intersecting weighted FSAs and CFGs

How many rules are there in G'?

- A has 4 states (one of which is final) and 25 arcs
- G has 3 binary rules and 5 unary rules

Recap of constructive procedure:

- If ROOT is the root category of G and q_0 is the start state of A, then call ROOT the root category of G' and for every final state q_i in A with weight p construct a rule ROOT $\rightarrow$ ROOT$_{[0,i]}$ with weight p $\rightarrow$1 rule in G'
- For every binary rule $X \rightarrow Y\ Z$ with weight p in G and state triple (q_i, q_j, q_k) in A, construct a rule $X_{[i,k]} \rightarrow Y_{[i,j]}\ Z_{[j,k]}$ with weight p $\rightarrow 3 \times 4^3 = 192$ rules in G'
- For every unary rule $X \rightarrow Y$ with weight p in G and state pair (q_i, q_j) in A, construct a rule $X_{[i,j]} \rightarrow Y_{[i,j]}$ with weight p $\rightarrow 4 \times 4^2 = 80$ rules in G'

Intersecting weighted FSAs and CFGs

How many rules are there in G'?

- A has 4 states (one of which is final) and 25 arcs
- G has 3 binary rules and 5 unary rules

Recap of constructive procedure:

- If ROOT is the root category of G and q_0 is the start state of A, then call ROOT the root category of G' and for every final state q_i in A with weight p construct a rule ROOT $\rightarrow$ ROOT$_{[0,i]}$ with weight p $\rightarrow$1 rule in G'
- For every binary rule $X \rightarrow Y\ Z$ with weight p in G and state triple (q_i, q_j, q_k) in A, construct a rule $X_{[i,k]} \rightarrow Y_{[i,j]}\ Z_{[j,k]}$ with weight p $\rightarrow 3 \times 4^3 = 192$ rules in G'
- For every unary rule $X \rightarrow Y$ with weight p in G and state pair (q_i, q_j) in A, construct a rule $X_{[i,j]} \rightarrow Y_{[i,j]}$ with weight p $\rightarrow 4 \times 4^2 = 80$ rules in G'
- For every transition $q_i \overset{w}{\rightsquigarrow} q_j$ with weight p in A, construct a rule $w_{[i,j]} \rightarrow w$ with weight p $\rightarrow$ 25 rules in G'

Intersecting weighted FSAs and CFGs

How many rules are there in G'?

- A has 4 states (one of which is final) and 25 arcs
- G has 3 binary rules and 5 unary rules

Recap of constructive procedure:

- If ROOT is the root category of G and q_0 is the start state of A, then call ROOT the root category of G' and for every final state q_i in A with weight p construct a rule ROOT $\rightarrow$ ROOT$_{[0,i]}$ with weight p $\rightarrow$1 rule in G'
- For every binary rule $X \rightarrow Y\ Z$ with weight p in G and state triple (q_i, q_j, q_k) in A, construct a rule $X_{[i,k]} \rightarrow Y_{[i,j]}\ Z_{[j,k]}$ with weight p $\rightarrow 3 \times 4^3 = 192$ rules in G'
- For every unary rule $X \rightarrow Y$ with weight p in G and state pair (q_i, q_j) in A, construct a rule $X_{[i,j]} \rightarrow Y_{[i,j]}$ with weight p $\rightarrow 4 \times 4^2 = 80$ rules in G'
- For every transition $q_i \overset{w}{\rightsquigarrow} q_j$ with weight p in A, construct a rule $w_{[i,j]} \rightarrow w$ with weight p $\rightarrow$ 25 rules in G'

So we're talking about $1 + 192 + 80 + 25 = 298$ rules in G'!

Intersecting weighted FSAs and CFGs

- As you have seen, however, many of the rules in G' cannot actually be used
- Fortunately, there are tools for finding a *minimal* equivalent representation for G':
  - Perform ϵ-removal (find an equivalent WFSA with no ϵ-transitions
  - Parse the WFSA bottom-up and then top-down to get only the necessary rules
  - Find the PARTITION FUNCTIONS (also called NORMALIZING CONSTANTS; **??**) for all symbols in G' and use them to find new rule probabilities (**?**)
- This G' is actually a *probabilistic* CFG—rule weights sum to 1!

Intersecting weighted FSAs and CFGs

- In our example, a minimal G' has only 96 rules:

```
ROOT -> ROOT[0,1] 1.0
ROOT[0,1] -> NP[0,1] $END$[1,1] 0.96840954
ROOT[0,1] -> NP[0,2] $END$[2,1] 0.013195342
ROOT[0,1] -> NP[0,3] $END$[3,1] 0.018306052
NP[0,0] -> N[0,0] N[0,0] 0.02898563
NP[0,0] -> N[0,0] 0.9703199
N[0,0] -> fox[0,0] 0.19999999
N[0,0] -> box[0,0] 0.79999995
PP[0,0] -> P[0,0] NP[0,0] 1.0
P[0,0] -> on[0,0] 1.0
NP[0,1] -> NP[0,0] PP[0,1] 0.0044827187
NP[0,1] -> NP[0,1] PP[1,1] 0.19999103
NP[0,1] -> NP[0,2] PP[2,1] 0.014011657
NP[0,1] -> NP[0,3] PP[3,1] 0.52381814
NP[0,1] -> N[0,1] N[1,1] 0.018886833
NP[0,1] -> N[0,2] N[2,1] 0.018886833
NP[0,1] -> N[0,3] N[3,1] 0.1875152
NP[0,1] -> N[0,1] 0.031478055
NP[0,2] -> NP[0,0] PP[0,2] 0.0029028896
NP[0,2] -> NP[0,3] PP[3,2] 0.07255861
NP[0,2] -> N[0,0] N[0,2] 0.0039330553
NP[0,2] -> N[0,2] N[2,2] 0.0039330553
NP[0,2] -> N[0,3] N[3,2] 0.78431565
NP[0,2] -> N[0,2] 0.13166262
NP[0,3] -> N[0,0] N[0,3] 0.028147077
NP[0,3] -> N[0,3] N[3,3] 0.028147077
NP[0,3] -> N[0,3] 0.94224906
N[0,1] -> fox[0,1] 0.08422382
N[0,1] -> box[0,1] 0.9157763
PP[0,1] -> P[0,0] NP[0,1] 0.15490696
PP[0,1] -> P[0,1] NP[1,1] 0.0111635
PP[0,1] -> P[0,2] NP[2,1] 0.5433943

PP[0,1] -> P[0,3] NP[3,1] 0.29053518
PP[1,1] -> P[1,1] NP[1,1] 1.0000267
PP[2,1] -> P[2,1] NP[1,1] 0.5286748
PP[2,1] -> P[2,2] NP[2,1] 0.4713301
PP[3,1] -> P[3,1] NP[1,1] 0.019618757
PP[3,1] -> P[3,2] NP[2,1] 0.954962
PP[3,1] -> P[3,3] NP[3,1] 0.025420623
N[1,1] -> fox[1,1] 0.19999999
N[1,1] -> box[1,1] 0.8
N[0,2] -> fox[0,2] 0.19999999
N[0,2] -> box[0,2] 0.79999995
N[2,1] -> fox[2,1] 0.08422382
N[2,1] -> box[2,1] 0.9157762
N[0,3] -> fox[0,3] 0.40460965
N[0,3] -> box[0,3] 0.5953903
N[3,1] -> fox[3,1] 0.0842238
N[3,1] -> box[3,1] 0.91577613
P[0,1] -> on[0,1] 1.0
NP[1,1] -> NP[1,1] PP[1,1] 0.19999105
NP[1,1] -> N[1,1] N[1,1] 0.3000053
NP[1,1] -> N[1,1] 0.50000894
P[0,2] -> on[0,2] 1.0
NP[2,1] -> NP[2,1] PP[1,1] 0.19999108
NP[2,1] -> NP[2,2] PP[2,1] 0.0014732869
NP[2,1] -> N[2,1] N[1,1] 0.29396322
NP[2,1] -> N[2,2] N[2,1] 0.01463557
NP[2,1] -> N[2,1] 0.48993886
P[0,3] -> on[0,3] 1.0
NP[3,1] -> NP[3,1] PP[1,1] 0.19999103
NP[3,1] -> NP[3,2] PP[2,1] 0.02100433
NP[3,1] -> NP[3,3] PP[3,1] 0.027316585
NP[3,1] -> N[3,1] N[1,1] 0.20226231

NP[3,1] -> N[3,2] N[2,1] 0.20226222
NP[3,1] -> N[3,3] N[3,1] 0.010070053
NP[3,1] -> N[3,1] 0.33710387
PP[0,2] -> P[0,0] NP[0,2] 0.23921143
PP[0,2] -> P[0,2] NP[2,2] 0.088231556
PP[0,2] -> P[0,3] NP[3,2] 0.672557
PP[2,2] -> P[2,2] NP[2,2] 1.0
PP[3,2] -> P[3,2] NP[2,2] 0.72489583
PP[3,2] -> P[3,3] NP[3,2] 0.27510414
N[2,2] -> fox[2,2] 0.19999999
N[2,2] -> box[2,2] 0.79999995
N[3,2] -> fox[3,2] 0.20000005
N[3,2] -> box[3,2] 0.79999995
NP[2,2] -> N[2,2] N[2,2] 0.02898563
NP[2,2] -> N[2,2] 0.9703199
NP[3,2] -> NP[3,3] PP[3,2] 0.002524153
NP[3,2] -> N[3,2] N[2,2] 0.028097434
NP[3,2] -> N[3,3] N[3,2] 0.028097434
NP[3,2] -> N[3,2] 0.9405865
PP[0,3] -> P[0,0] NP[0,3] 0.91090643
PP[0,3] -> P[0,3] NP[3,3] 0.08909376
PP[3,3] -> P[3,3] NP[3,3] 1.0
N[3,3] -> fox[3,3] 0.19999999
N[3,3] -> box[3,3] 0.79999995
NP[3,3] -> N[3,3] N[3,3] 0.02898563
NP[3,3] -> N[3,3] 0.9703199
P[1,1] -> on[1,1] 1.0
P[2,1] -> on[2,1] 1.0
P[2,2] -> on[2,2] 1.0
P[3,1] -> on[3,1] 1.0
P[3,2] -> on[3,2] 1.0
P[3,3] -> on[3,3] 1.0
```

Intersecting weighted FSAs and CFGs

- Really cool: we can *sample* from this grammar just like we can from any PCFG

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