CENTRAL TENDENCY: MEAN, MEDIAN, MODE

ASSIGNMENTS

- Pollock, *Essentials*, chs. 2-3
- Begin exploring Pollock, *SPSS Companion,* introduction and chs. 1-2
- Sections in Solís 105

Why Central Tendency?

- Summation of a variable

- Possibilities of comparison:
  - Across time
  - Across categories (e.g. countries, classes)
  - But beware of limitations!

Outline: Measures of Central Tendency

Mean or "average" value:

- ✓ Definition
- ✓ Illustration
- ✓ Special properties (for future reference)

Median or middle value:

- ✓ Definition
- ✓ Comparison with mean

Mode or most frequent value:

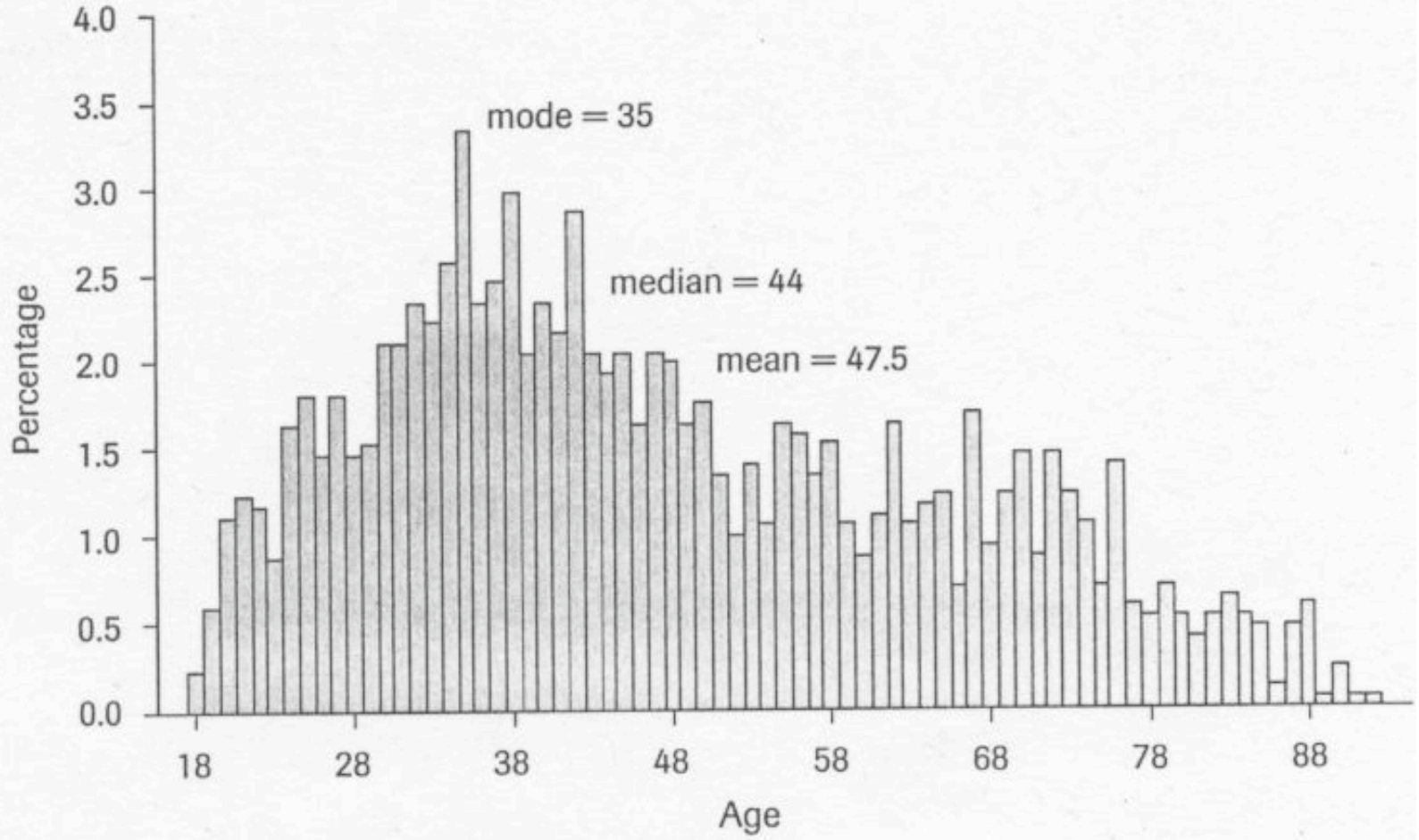
- ✓ Definition
- ✓ Applications

Table 3-3 Age of Respondents (tabular)

Age	Frequency	Percentage	Cumulative percentage	Age	Frequency	Percentage	Cumulative percentage
18	4	0.2	0.2	56	27	1.6	69.8
19	10	0.6	0.8	57	23	1.3	71.1
20	19	1.1	1.9	58	26	1.5	72.7
21	21	1.2	3.2	59	18	1.1	73.7
22	20	1.2	4.3	60	15	0.9	74.6
23	15	0.9	5.2	61	19	1.1	75.7
24	28	1.6	6.8	62	28	1.6	77.3
25	31	1.8	8.6	63	18	1.1	78.4
26	25	1.5	10.1	64	20	1.2	79.6
27	31	1.8	11.9	65	21	1.2	80.8
28	25	1.5	13.4	66	12	0.7	81.5
29	26	1.5	14.9	67	29	1.7	83.2
30	36	2.1	17.0	68	16	0.9	84.1
31	36	2.1	19.1	69	21	1.2	85.3
32	40	2.3	21.4	70	25	1.5	86.8
33	38	2.2	23.7	71	15	0.9	87.7
34	44	2.6	26.2	72	25	1.5	89.1
35	57	3.3	29.6	73	21	1.2	90.4
36	40	2.3	31.9	74	18	1.1	91.4
37	42	2.5	34.3	75	12	0.7	92.1
38	51	3.0	37.3	76	24	1.4	93.5
39	35	2.0	39.4	77	10	0.6	94.1
40	40	2.3	41.7	78	9	0.5	94.6
41	37	2.2	43.9	79	12	0.7	95.3
42	49	2.9	46.7	80	9	0.5	95.9
43	35	2.0	48.8	81	7	0.4	96.3
44	33	1.9	50.7	82	9	0.5	96.8
45	35	2.0	52.7	83	11	0.6	97.4
46	28	1.6	54.4	84	9	0.5	98.0
47	35	2.0	56.4	85	8	0.5	98.4
48	34	2.0	58.4	86	2	0.1	98.5
49	28	1.6	60.0	87	8	0.5	99.0
50	30	1.8	61.8	88	10	0.6	99.6
51	23	1.3	63.1	89	1	0.1	99.6
52	17	1.0	64.1	91	4	0.2	99.9
53	24	1.4	65.5	92	1	0.1	99.9
54	18	1.1	66.6	93	1	0.1	100.0
55	28	1.6	68.2	Total	1,712	100.0	

Source: 1996 National Election Study.

Figure 3-3 Age of Respondents (graphic)



Source: 1996 National Election Study.

Computing the Arithmetic Mean

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

Finding the Arithmetic Mean

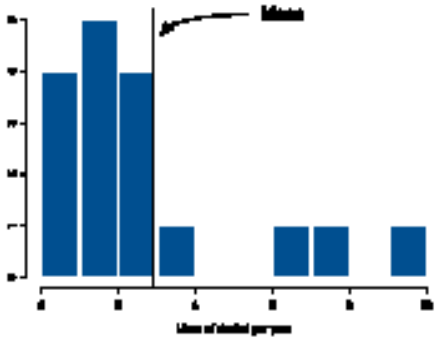
The mean of a variable

Mean

- Average value
- Sum of all values divided by number of cases
- Characteristics of "average" country
- Center of distribution

Country	Alcohol from wine in liters	Heart disease deaths in 100,000
Australia	2.5	211
Austria	3.9	167
Belgium	2.9	131
Denmark	2.9	220
Finland	0.8	297
France	9.1	71
Iceland	0.8	211
Ireland	0.7	300
Italy	7.9	107
Netherlands	1.8	167
New Zealand	1.9	266
Norway	0.8	227
Spain	6.5	86
Sweden	1.6	207
United Kingdom	1.3	285
United States	1.2	199
West Germany	2.7	172
Sum	49.3	3324
	(49.3/17)	(3324/17)
Mean=	2.9	195.6294118

Understanding the mean



Think of mean as physical center of distribution

"Weight" of observations on one side equal to "weight" of observations on other side

Individual observations can be described as "low" or "high" in terms of deviation from mean, indicating distance from mean

Examples from alcohol consumption

- United States below average consumption: 1.2-2.9= -1.7
- West Germany about average consumption: 2.7-2.9= -0.2
- France far above average consumption: 9.1-2.9=6.2

Properties of the Arithmetic Mean

1. SUM OF DEVIATIONS EQUALS ZERO

$$\sum (x_i - \bar{x}) = \text{SUM}$$

$$= \sum x_i - \sum \bar{x}$$

$$= \sum x_i - N\bar{x}$$

$$= \sum x_i - N\left(\frac{\sum x_i}{N}\right)$$

$$= \sum x_i - \sum x_i$$

$$= 0$$

2. SUM OF SQUARED DEVIATIONS MUST BE MINIMAL — "LEAST" SQUARES

WHERE $\bar{x}'$ IS A GUESSED MEAN

$$\Sigma (x_i - \bar{x}') = \Sigma (x_i - \bar{x} + \bar{x} - \bar{x}')$$

$$= \Sigma (x_i - \bar{x}) + \Sigma (\bar{x} - \bar{x}')$$

SQUARING →

$$\Sigma (x_i - \bar{x}')^2 = \Sigma (x_i - \bar{x})^2 + 2\Sigma (x_i - \bar{x})(\bar{x} - \bar{x}') + \Sigma (\bar{x} - \bar{x}')^2$$

AND

$$\Sigma (x_i - \bar{x}')^2 = \Sigma (x_i - \bar{x})^2 + \Sigma (\bar{x} - \bar{x}')^2$$

SO, MINIMAL WHEN $\bar{x}' = \bar{x}$

Illustration: Properties of the Arithmetic Mean

- Data: 3, 4, 5, 6, 7
- Mean = 25/5 = 5

Differences from mean:	Squared differences:
3 – 5 = -2	4
4 – 5 = -1	1
5 – 5 = 0	0
6 – 5 = +1	1
7 – 5 = +2	4
Sum (Σ) = 0	Sum (Σ) = 10

Treating Other Numbers (e.g., 3) as Mean:

Differences:	Squared differences:
3 – 3 = 0	0
4 – 3 = +1	1
5 – 3 = +2	4
6 – 3 = +3	9
7 – 3 = +4	16
Sum (Σ) = 10 [not zero]	Sum = 30

Sum of squared differences for 4 = 15
Sum of squared differences for 6 = 15
Sum of squared differences for 7 = 30

True Mean (5) has **least** sum^2 = 10

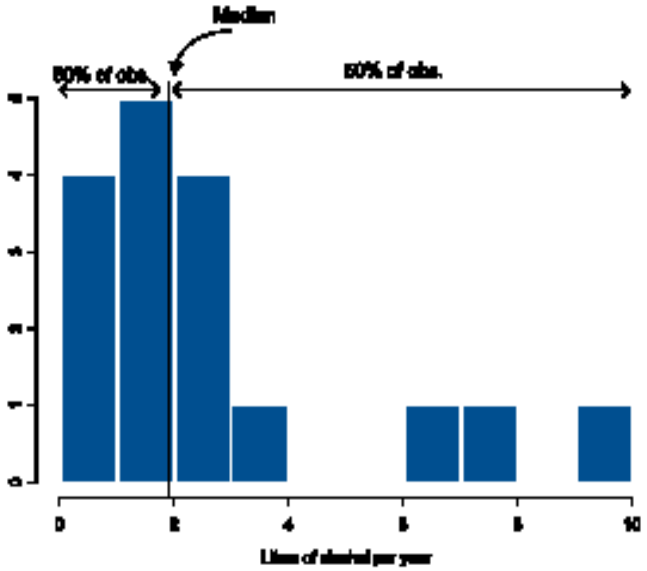
The median of a variable

- Order variable from high to low
- Find minimum and maximum values
- Median is middlemost value of variable
  - (Even number: mean of the middle two)

	Deaths	
France	71	Minimum is 71
Spain	86	
Italy	107	
Belgium	131	
Austria	167	
Netherlands	167	
West Germany	172	
United States	199	Since we have 17 observations (which is an o
Sweden	207	the median is the 9th highest observations (20
Australia	211	
Iceland	211	Quartiles are Q1=149, Q2/median=207, Q3=2
Denmark	220	
Norway	227	
New Zealand	266	
United	285	
Finland	297	
Ireland	300	Maximum is 300

Understanding the median

- Median is the middlemost observation
- At median, 50% of obs. above, 50% below
- Mean "weighted", based on actual value of observations
- Median only based on number of observations



Interpreting the Mode

- Definition: The **most frequent value**
- Especially useful for **categorical** variables
- Focus upon the **distribution** of values
- Applicable to quantitative data through **shape** of distribution
- Illustrations: heart disease, alcohol consumption, GDP per capita

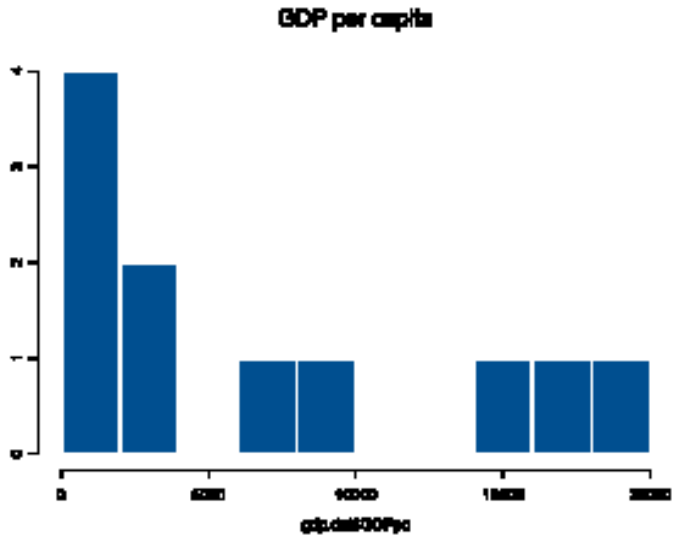
Cautionary Tales

- Mean is the most common measure of central tendency

- It is misleading for variables with skewed distributions (e.g., power, wealth, education, income)

- Mean, median, and mode **converge** with normal distributions, and **diverge** in cases of skewed distributions.

The mean of a skewed distribution



GDP per capita (ordered)

ID	Country	GDPpc
2	Afghanistan	591
6	Angola	886
3	Albania	1164
11	Azerbaijan	1281
8	Armenia	2109
4	Algeria	3076
7	Argentina	7044
1	Antigua & Barbuda	9078
5	Andorra	14748
10	Austria	16989
9	Australia	18500

Mean (6860.5) not characteristic of many states

Median 3076 (Algeria)