## Foundations of Neuroimaging

*Homework #1: Tissue Contrast in Spin-Echo and Gradient Echo Sequences*

The goal of this homework is: (1) to understand the Bloch equation solutions, (2) to learn enough Matlab or Mathematica or Octave or GnuPlot syntax to allow you plot simple exponential functions and their differences, and (3) to use this skill to understand how varying TR (repetition time), TE (echo time), and flip angle (alpha) can result in contrast between tissues with different PD (proton density), T1 (longitudinal regrowth) and T2 (transverse decay) constants. Turn in answers, graphs, and code (compact code appreciated).

**1**. Plot time course (t=0-2000 msec) of *longitudinal* magnetization regrowth, $M_z(t)$, following a single 100 degree RF pulse starting from equilibrium. Use equation below (the solution to the longitudinal relaxation part of the Bloch equation) assuming T1=750 msec, and equilibrium magnetization, $M_z^0$=1.0. (N.B.: $M_z(0_+)$ means *longitudinal* magnetization immediately *after* flip) *Hint: don't confuse degrees/radians, or sin/cos!*.

$$M_z(t) = M_z^0\,(1 - e^{-t/T1}) + M_z(0_+)\,e^{-t/T1}$$

**2**. Apply the Bloch equation *longitudinal* magnetization solution (above) to the following simple spin echo sequence. At the *very beginning* of a spin-echo pulse sequence, *(a)* the equilibrium *longitudinal* magnetization, $M_z^0$, is flipped by a 90 deg RF pulse, *(b)* it recovers (from zero) for time t=TE/2, *(c)* a 180 deg RF pulse is applied at time t=TE/2, *(d)* an echo occurs at time t=TE, *(e)* the *longitudinal* magnetization recovers for t=TR (measured from the first 90 deg RF pulse), and finally, *(f)* the second 90 degree RF pulse occurs. Derive the equation (this means show each step) for the amount of *longitudinal* magnetization present *just* ***before*** the *second* 90 degree RF pulse by giving equations for the *longitudinal* magnetization at *each* of the stages *(a)* to *(e)* above. No graphs needed. *Hint: remember $M_z^0$ is a constant while $M_z(0_+)$ is a variable!*

**3**. Plot the time course of the decay of *transverse* magnetization after an $\alpha$ = 70 degree RF pulse for two different tissue types with T2=50 and T2=71 msec, and then plot the *difference* between these two curves to illustrate the time point where their *transverse* magnetizations are the most *different*. Use equation below (assume longitudinal magnetization before the flip, $M_z^0$, is same for both tissue types, namely 1.0).

$$M_{x'y'}(t) = M_{x'y'}(0_+)\,e^{-t/T2}$$

**4**. Assume the following T1, T2, and proton-density (PD [=$M_z^0$]) values for gray matter (GM), white matter (WM), and cerebrospinal fluid (CSF): **T1** (msec): GM=1150, WM=820, CSF=2200; **T2** (msec): GM=86, WM=80, CSF=330; **PD** (water=1.0): GM=0.67, WM=0.58, CSF=0.98. Use this equation for *spin-echo* signal intensity:

$$M_{x'y'} = M_z^0\,(1 - e^{-TR/T1})\,e^{-TE/T2}$$

(a) In T1-weighted images, WM > GM > CSF (signal intensity, brightness). For a *fixed* TE=6 msec, determine the TR that maximizes the contrast between GM and WM (TR that results in largest value of WM-minus-GM). Do this by plotting $M_{x'y'}(TR)$ curves for each tissue type (TR=0 to TR=2500). Briefly explain why T1-weighted images have intermediate TR and short TE.

(b) In typical T2-weighted images, CSF > GM > WM. For a *fixed* TR=3200 msec (vs. fixing the TE above), determine a *TE* that maximizes CSF-minus-WM contrast. Plot the curve of $M_{x'y'}(TE)$ for each tissue (from TE=1 to TE=200 msec). Briefly explain why T2-weighted images have long TR and intermediate TE.

**5**. Use the following equation for fast spoiled *gradient echo* signal intensity (and T1, T2, and PD values from problem 4):

$$M_{x'y'} = \left[ \left[M_z^0 (1 - e^{-TR/T1})\right] / \left[1 - \cos(\alpha)\, e^{-TR/T1}\right] \right] \sin(\alpha)\, e^{-TE/T2}$$

Make a *2D plot* of the dependence of *white matter* $M_{x'y'}$ *minus gray matter* $M_{x'y'}$ (white-gray contrast) on both *flip angle* (from 3-20 deg) and *TR* (from 5-15 msec). Assume that the TE=4 msec. A '2D plot' means, show value of the difference in $M_{x'y'}$'s for regularly sampled combinations of flip angle and TR as a brightness map (contour map) or a height map (surface plot). Which adjustable scan parameter (TE, TR, flip angle) directly affects scan length?