Cognitive Science 276 -- Neuroimaging

Homework #1: Tissue Contrast in Spin-Echo and Gradient Echo Sequences

The goal of this homework is to learn enough Matlab (or Mathematica) syntax to allow you plot simple exponential functions and their differences, and then use this skill to understand how varying TR (repetition time) and TE (echo time) can result in contrast between tissues with different T1 (longitudinal) and T2 (transverse) time constants.

1. Plot the time course of *longitudinal* magnetization regrowth, $M_z(t)$, following one 90 degree RF pulse (from t=0 to t=2000 msec). Use the following equation and assume that T1=760 msec (gray matter), and equilibrium magnetization, M_0=1.0:

$$M_z(t) = M_0 (1 - e^{-t/T1})$$

2. If the pulse is repeated with TR=2000, the longitudinal magnetization will not completely recover before the next pulse happens. Now plot the decay in the *transverse* magnetization as a function of time, $M_{xy}(t)$, that would occur immediately *after* a *second* RF pulse (from t=0 to t=1999 msec after the second pulse) using the following equation (which takes the incomplete longitudinal regrowth into account) and use the parameters, T1=760 msec and T2=77 msec:

$$M_{xy}(t) = M_0 (1 - e^{-TR/T1}) e^{-t/T2}$$

3. Assume the following T1 and T2 values for gray matter (GM), white matter (WM), and cerebrospinal fluid (CSF): **T1**: GM=760, WM=510, CSF=2650; **T2**: GM=77, WM=67, CSF=280; all in msec. Use the equation above for *spin-echo* signal intensity:

$$M_{xy} = M_0 (1 - e^{-TR/T1}) e^{-TE/T2}$$

(a) In typical T1-weighted images, WM > GM > CSF (signal intensity, brightness). For a fixed TE=10 msec, determine the TR that maximizes the contrast between GM and WM (TR that results in largest value of WM-minus-GM). Do this by plotting M_{xy}(TR) curves for each tissue type (TR=0 to TR=3000). Explain why T1-weighted images have short TR and short TE.

(b) In typical T2-weighted images, CSF > GM > WM. For a fixed TR=3000 ms, determine a TE that maximizes CSF-minus-WM contrast. Plot the curve of M_{xy}(TE) for each tissue (from TE=1 to TE=200 msec). Explain why T2-weighted images have long TR and long TE.

4. Use the following equation for *gradient* echo signal intensity (and the T1 and T2 values from the previous problem):

$$M_{xy} = \frac{M_0 (1 - e^{-TR/T1})}{1 - \cos \alpha \, e^{-TR/T1}} \sin \alpha \, e^{-TE/T2}$$

For TR=18 msec and TE=5 msec, find the flip angle, α, that results in the greatest contrast between WM and GM, under the constraint that WM > GM. Plot $M_{xy}(\alpha)$ curves for both types.