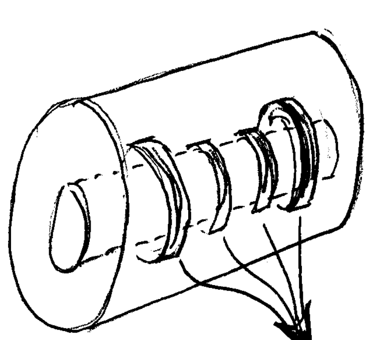
Hardware

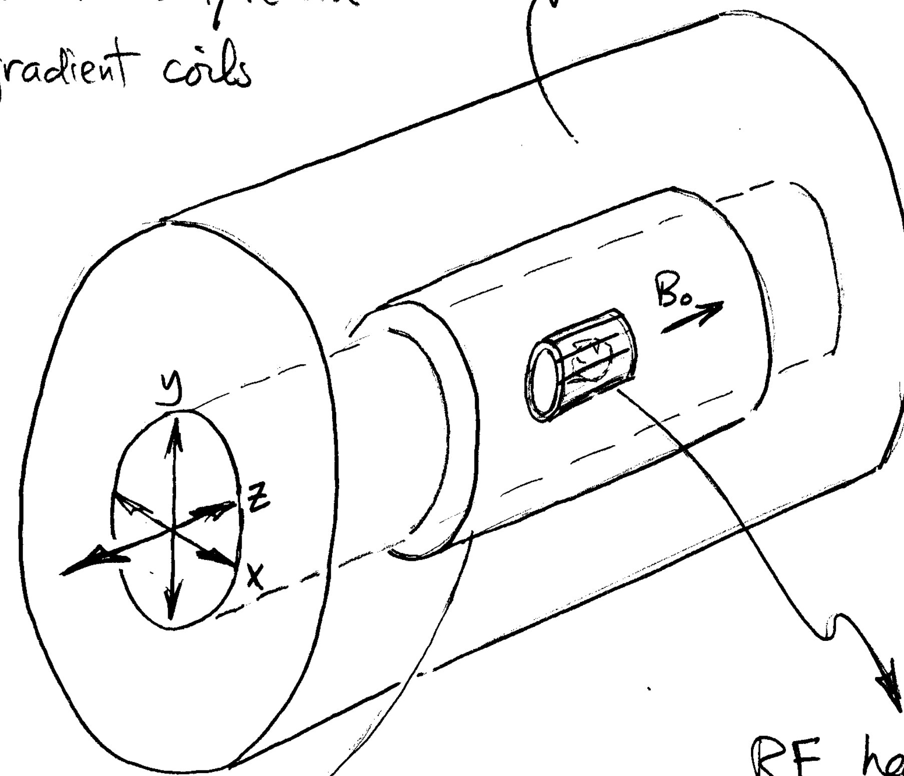
MAGNET HARDWARE

- B_0 field from superconducting magnet
- RF transmit/receive
- gradient coils



superconducting coils in liquid helium (no power required after current injected to bring up field using induction)

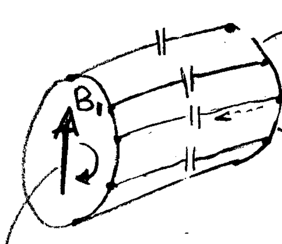
B_0 field



$1T = 10{,}000$ Gauss

$\frac{\gamma}{2\pi} = 42.6\ MHz/T$

RF head coil



RF transmitter (10 kW)

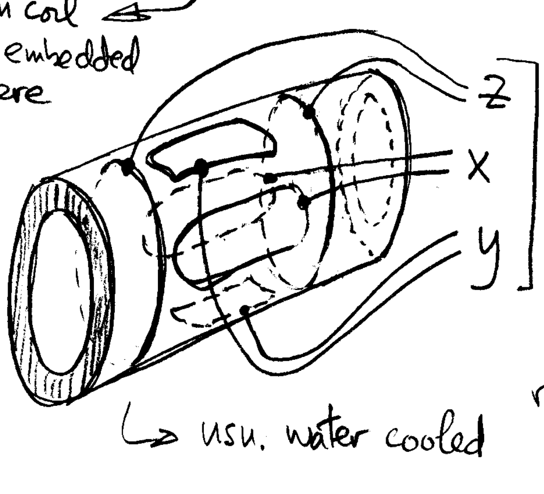
RF receiver

circularly polarized B1 field rotating $\perp$ to B0 at Larmor freq.

(B1 is several orders of magnitude smaller than B0)

body gradient coils

shim coil also embedded in here (not shown)



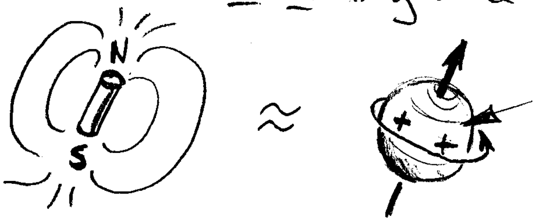
three 0.5 million watt amplifiers to add ramps to B_0^z field

usu. water cooled

SPIN & PRECESSION

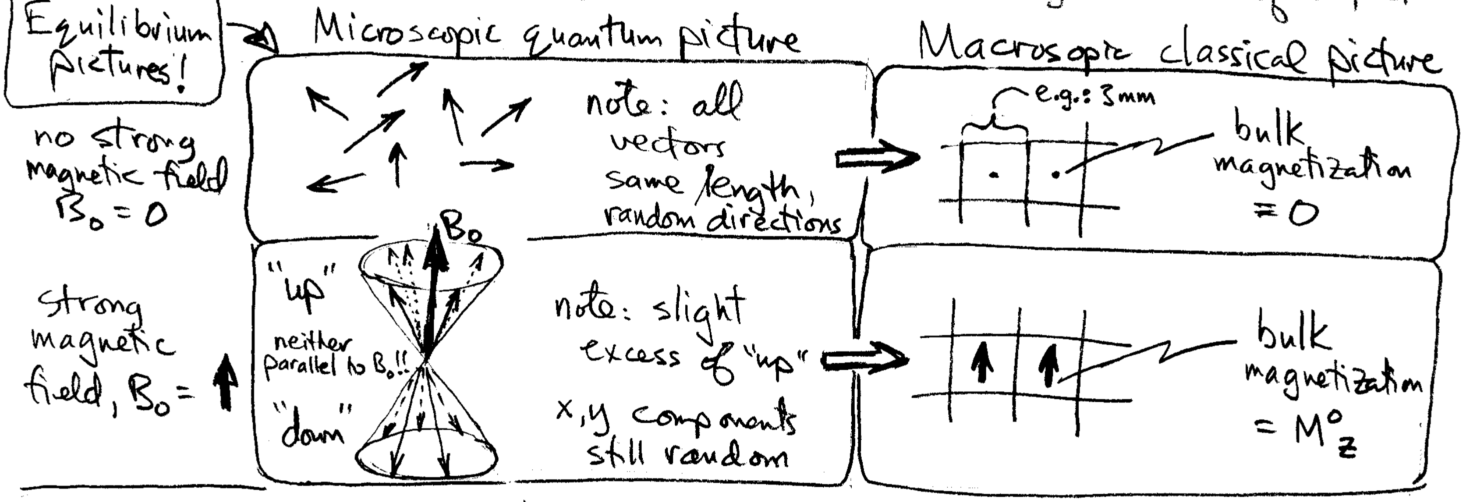
N.B.: compared to top + gravity magnetic relaxation is:
- frictionless spin
- signed gravity (can change direction of precession)
- neighbor bumping causes decay

- nuclei act like spinning charges
  - → nuclei w/ odd atomic weight or odd proton numbers (e.g., hydrogen)
- moving charges create a magnetic field
- protons act as if they were spinning (tho classically, this would cause energy to be lost, resulting in spin-down)



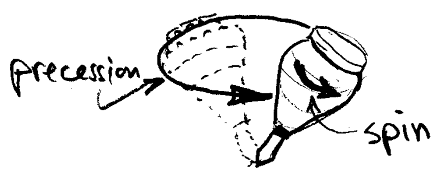
classical picture
charges on surface of spinning sphere make current loops, creating magnetic dipole field

- strong magnetic field required to activate bulk magnetization of object



Precession
- add to equilibrium pictures above!! (distinguish precession from spin)
- treat classically, like top

$$2\pi \underbrace{f_0}_{\text{Larmor freq (63 MHz)}} = \underbrace{\omega_0}_{\text{same in radians/sec}} = \underbrace{\gamma}_{\text{gyro-magnetic ratio}} \underbrace{B_0}_{\text{static field (1.5 T)}}$$



like top: precession faster w/ more gravity

- bulk magnetization: parallel to B_0

two non-constants

$$M_z^0 = |\vec{M}| = \frac{\gamma^2 h^2 B_0 N_s}{4 K T_s}$$

- γ – gyromagnetic ratio (const.)
- h – Planck's constant
- B_0 – M_z^0 proportional to mag strength
- N_s – " " to number spins
- K – Boltzmann constant
- T_s – abs. temp sample

BLOCH EQUATION

vector sum of all microscopic magnetization vectors

- time-dependent behavior of $\vec{M}$ in the presence of an applied magnetic field (excitation & relaxation)

precession: $B = B_0$
excitation: $B = B_0 + B_1$

"transverse" "spin-spin"

"longitudinal" "spin-lattice"

equilibrium $\vec{M}$ value in only the B_0 field

(laboratory frame)

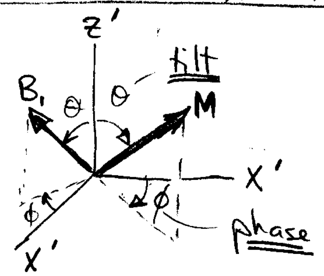
$$\underbrace{\frac{d\vec{M}}{dt}}_{\text{change in mag vector}} = \underbrace{\vec{M}}_{\text{existing mag vector sum}} \times \underbrace{\gamma\vec{B}}_{\text{applied field (typically rotating)}} \underbrace{- \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{(M_z - M_z^0)\vec{k}}{T_1}}$$

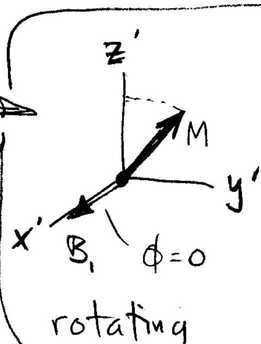
$\vec{k}$ ~ unit vector in z-direction

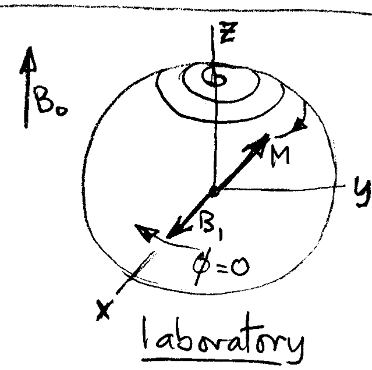
↳ these can be ignored during short excitation, so magnetic vector stays same length as it spirals down (vs. relaxation, where it shrinks, then grows)

- in the Larmor-rotating coordinate system, a tilt w/o a phase shift for a standard B_1 excitation is rotation around **x**-axis

general case rotating (distinguish B_1 & M!)







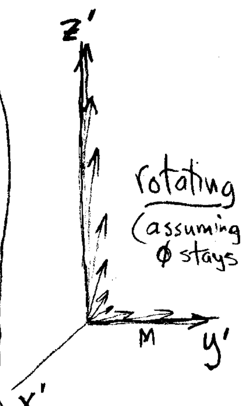
- longitudinal and transverse relaxations ⇗drop $\vec{i},\vec{j},\vec{k}$

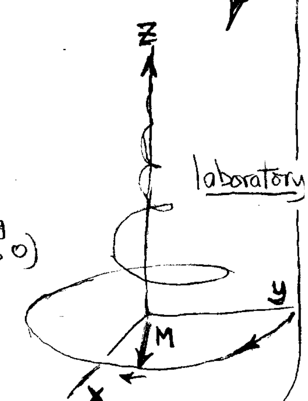
(rotating frame)

M_L: $$\frac{dM_{z'}}{dt} = -\frac{M_{z'} - M_z^0}{T_1}$$

M_T: $$\frac{dM_{x'y'}}{dt} = -\frac{M_{x'y'}}{T_2}$$

from Bloch equation after dropping applied field term





- solution to equations above: time-dependent free precession eq's

lab frame: same!

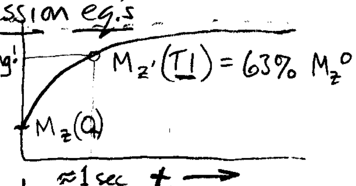
(rotating frame)

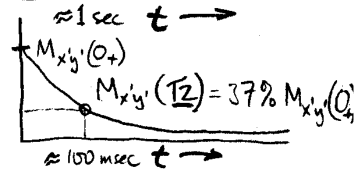
M_L: $$M_{z'}(t) = \overbrace{\underbrace{M_z^0}_{\text{initial mag}}\left(1 - e^{-t/T_1}\right)}^{\text{re-growing from 0}} + \overbrace{M_{z'}(0_+)e^{-t/T_1}}^{\text{leftover after pulse – decaying!}}$$

M_T: $$M_{x'y'}(t) = \underbrace{M_{x'y'}(0_+)}e^{-t/T_2}$$

M_L, M_T at time immediately after pulse

lab frame: times $e^{-i\omega_0 t}$

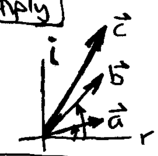




BLOCH EQ. – MATRIX VERSION

Complex multiply (3rd kind!)

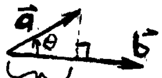
$$\vec{c} = \vec{a}\cdot\vec{b} = \begin{bmatrix} b_x & -b_y \\ b_y & b_x \end{bmatrix}\begin{bmatrix} a_x \\ a_y \end{bmatrix} = [a_x b_x - a_y b_y \quad a_x b_y + a_y b_x] \quad \text{(vector)}$$



- angles add
- magnitudes multiply

inner product (= dot product) (= scaled projection onto)

$$c = \vec{a}\cdot\vec{b} = [a_x \; a_y \; a_z]\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = a_x b_x + a_y b_y + a_z b_z \quad \text{(scalar)}$$



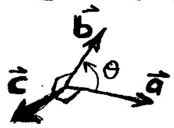
$p = |\mathbf{a}|\cos\theta$

$c = p\,|\mathbf{b}| \rightarrow c = p$ if $|\mathbf{b}| = 1$

$$= \underbrace{|\mathbf{a}|}_{\text{length of } \mathbf{a}}\,|\mathbf{b}|\cos\theta$$

outer product (= cross product)

$$\vec{c} = \vec{a}\times\vec{b} = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = [a_y b_z - a_z b_y \quad a_z b_x - a_x b_z \quad a_x b_y - a_y b_x] \quad \text{(vector)}$$

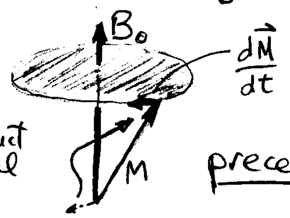


$$= |\mathbf{a}|\,|\mathbf{b}|\sin\theta$$

Just Precession

$$\frac{d\vec{M}}{dt} = \vec{M}\times\gamma\vec{B}_0$$

(assume only z component of B_0 present)



cross product is orthogonal to B & M

precession

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \gamma B_0 & 0 \\ -\gamma B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Solution: (M_x, M_y component coupling) $\omega_0 = \gamma B_0 \Longrightarrow$ precession around z

$$\vec{M}(t) = \begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} M_x^\circ \\ M_y^\circ \\ M_z^\circ \end{bmatrix} = \overset{\square}{\mathbf{R}}_z(\omega_0 t)\,\vec{\mathbf{M}}^\circ$$

Including Relaxation

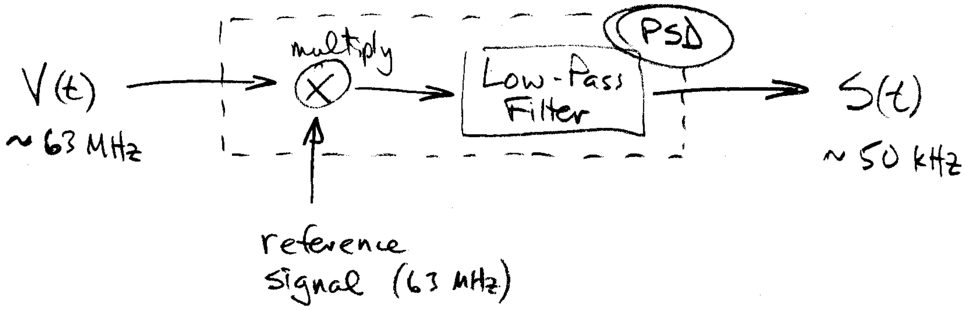
$$\frac{d\vec{M}}{dt} = \begin{bmatrix} -1/T_2 & \gamma B_0 & 0 \\ -\gamma B_0 & -1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{bmatrix}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Solution:

$$\vec{M}(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix}\overset{\square}{\mathbf{R}}_z(\omega_0 t)\,\vec{\mathbf{M}}^\circ + \begin{bmatrix} 0 \\ 0 \\ M_0(1-e^{-t/T_1}) \end{bmatrix}$$

PHASE - SENSITIVE DETECTION

how we get rotating frame



- method for moving very high frequency Larmor oscillations down to more tractable frequency range

demodulated signal $\propto$ reference $\cdot$ RF coil signal

$$\propto \sin(\omega_0 + \delta\omega)t \cdot \sin \omega_0 t$$

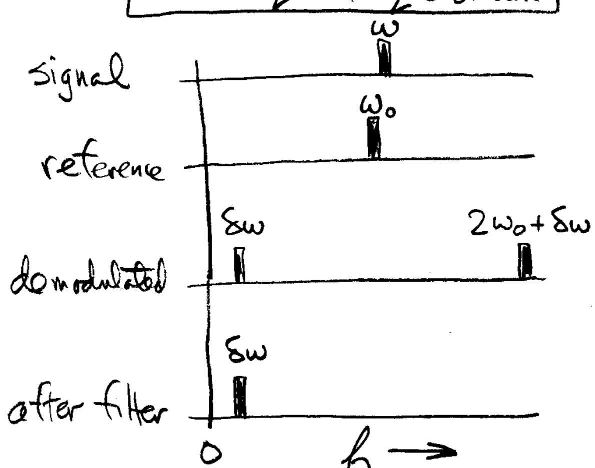
$$\sin a \sin b = \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\propto \tfrac{1}{2}\Big[\underbrace{\cos \delta\omega t}_{\text{diff between RF input \& ref}} - \underbrace{\cos(2\omega_0 + \delta\omega)t}_{\text{filter this one out w/ low pass filter}}\Big]$$

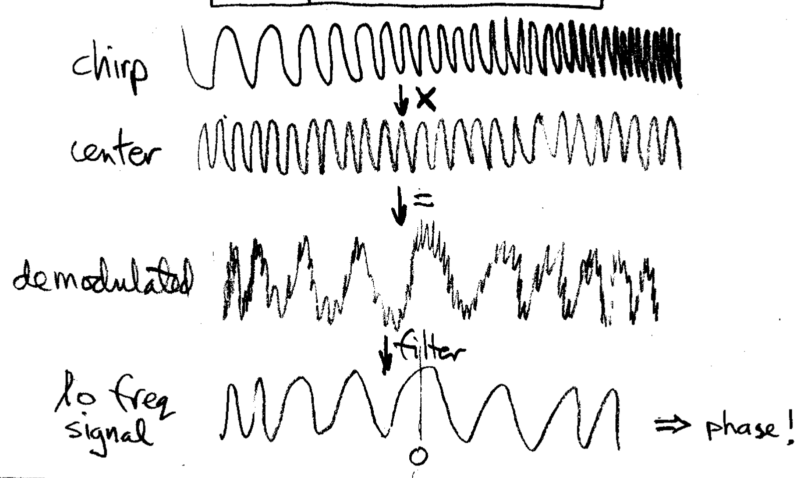
this signal is digitized

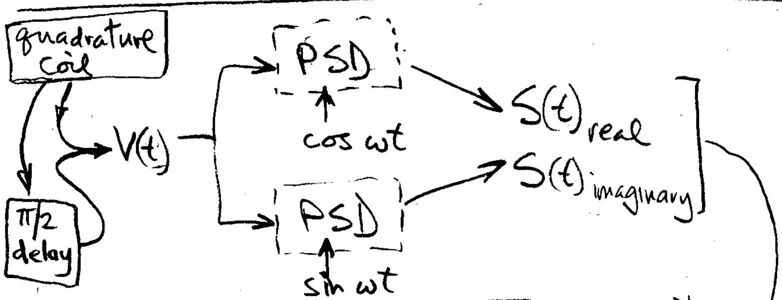
---

one freq — freq domain



chirp — time domain





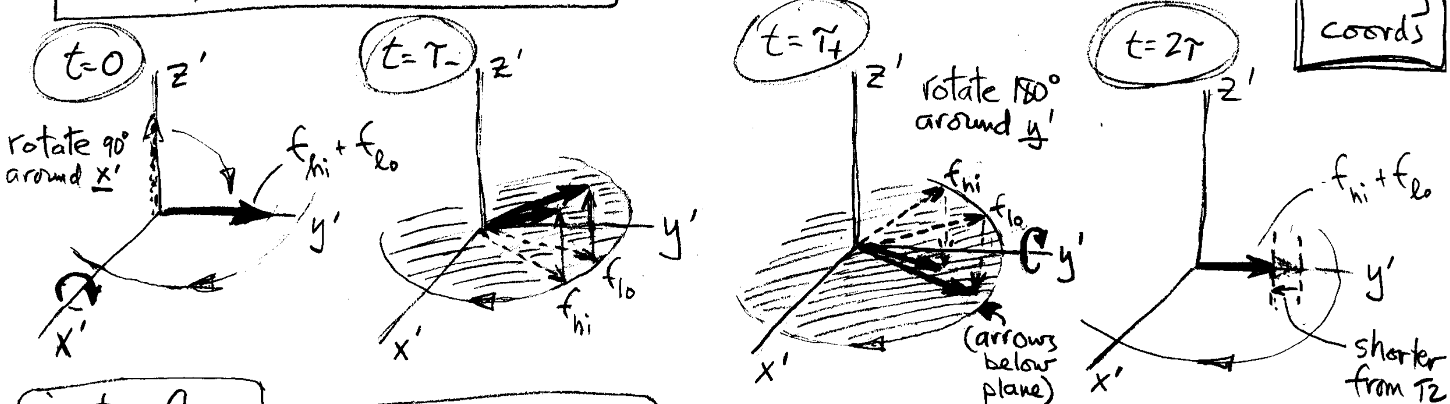
$$\boxed{S(t)_{complex} = e^{i\,\delta\omega t}}$$

- two signals are made from a single receiving RF coil
- a quadrature coil can be treated the same way (OK to combine after adding $\pi/2$ phase, then PSD)
- quadrature coil has better S/N since noise in each part is uncorrelated ($\sqrt{2}$ better)

ECHOES — spin echo

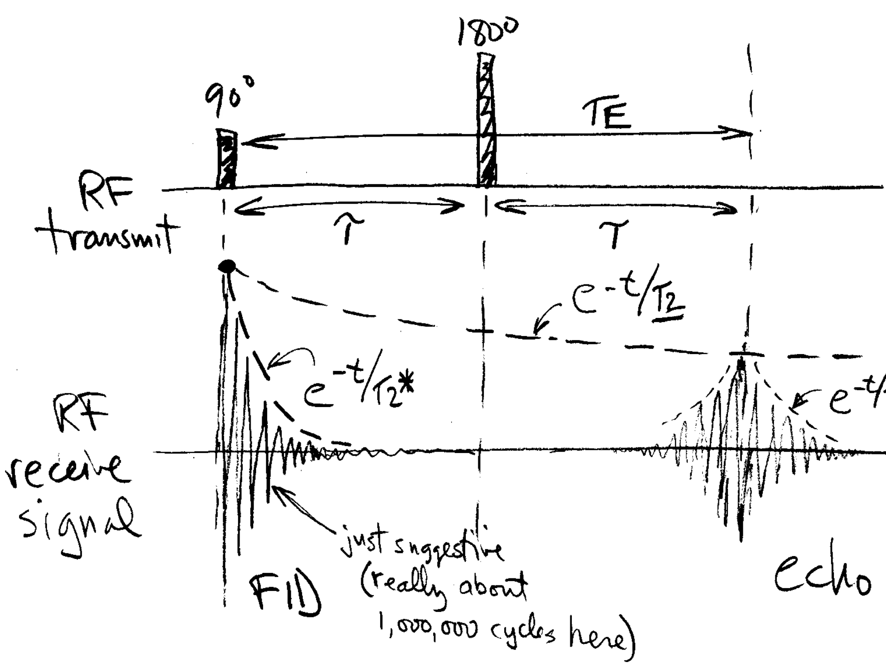
$90° - \tau - 180°$, T2/T2* & echo

rotating coords



just after 90° x' pulse $f_{lo} + f_{hi}$ have <u>same phase</u>	relaxation + phase dispersion of $f_{lo} + f_{hi}$ (both from $B > B_0$)	just after 180° y' pulse (y' pulse like x' pulse but RF has +90° phase)	echo caused by re-phasing of $f_{lo} + f_{hi}$ (w/ decay due to T2)

- remember [RF just tips vector(s) while retaining length; relaxation includes tips <u>and</u> shrinks (<u>and</u> grows for echo)
- 180° <u>x'</u> pulse works, too, but echo will be $+\pi$ phase (<u>left</u> side in figs above)
- echos generated even if second pulse <u>not</u> 180° (see next)



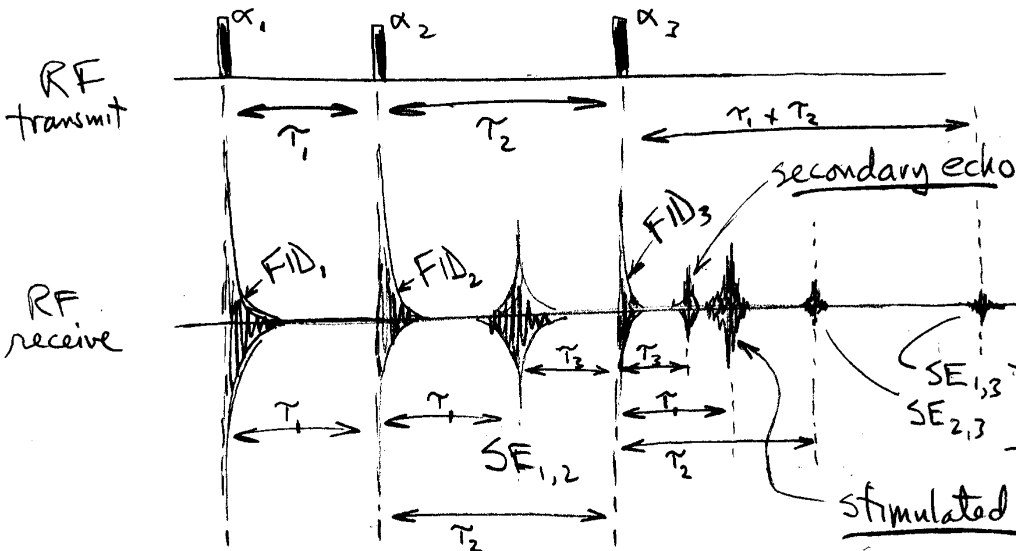
- FID decay (<u>and</u> <u>echo</u> growth/decay) described by T2*, <u>from</u> <u>inhomogeneity</u>
- reduction in <u>height</u> of echo compared to initial described by T2, <u>echo fixes the "star"</u>

ECHO TRAINS — spin-echo trains

$$E_n = \frac{3^{(n-1)} - 1}{2}$$

echos after <u>end</u> of n^{th} pulse
3 RF's → 4 echos (here)
6 RF's → 121 echos (!!)

- it's (too) easy to make echos...



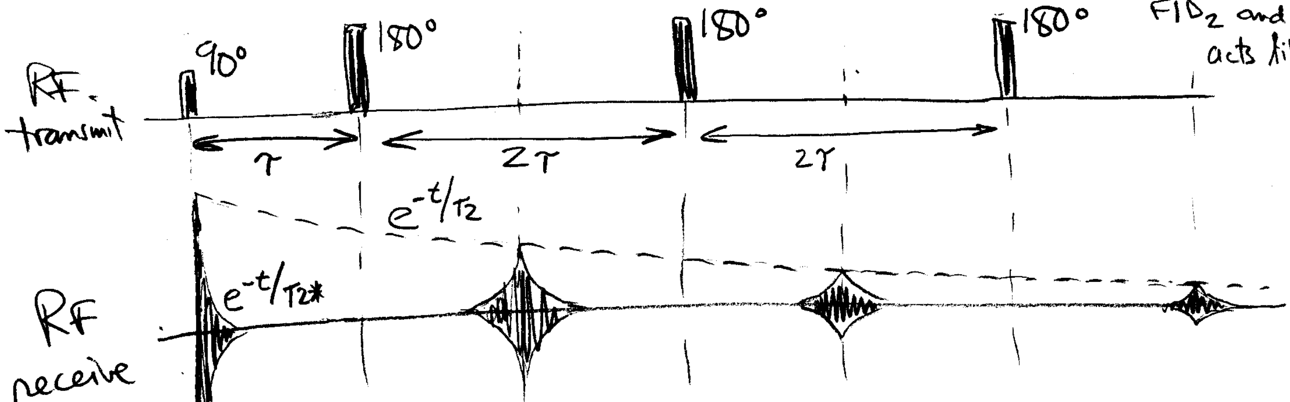
<u>secondary echo</u>: [$SE_{1,2}$ acts like RF pulse; α_3 makes an echo from it]

$SE_{1,3}$, $SE_{2,3}$ — two more conventional two pulse spin echoes

<u>stimulated echo</u>: combined effect of 3

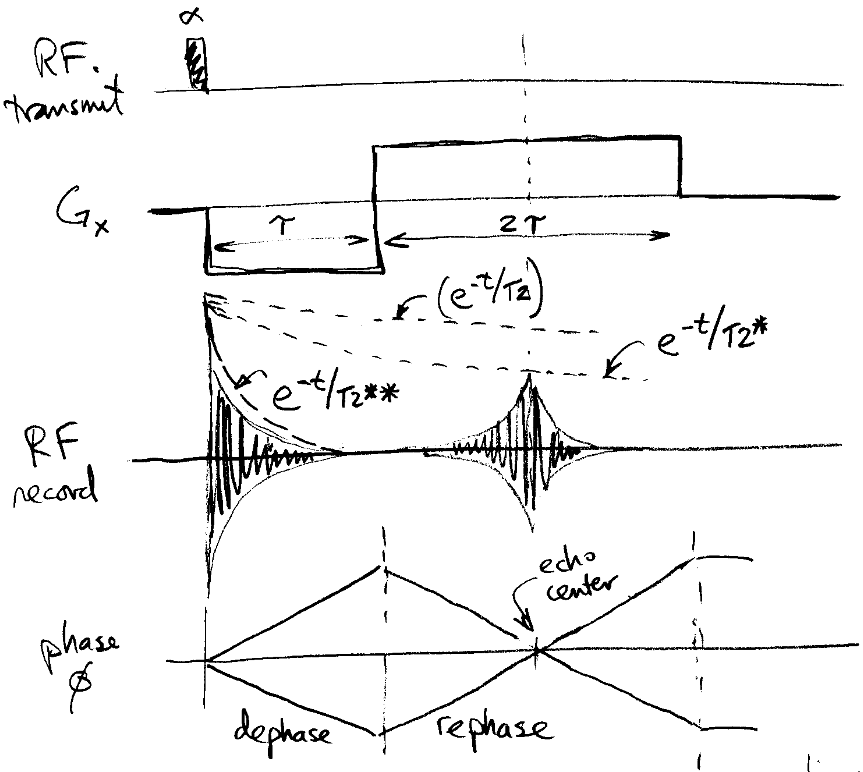
- α_1: $M_L \rightarrow M_T$
- α_2: leftover $\underline{\underline{M_T}}$ flipped to $\underline{\underline{M_L}}$ ("saved")
- α_3: flip saved $M_L \rightarrow M_T$ which can then begin to cancel delays (after being held "in limbo" between FID_2 and FID_3); acts like 2-pulse echo

- a useful multi-echo sequence (CPMG) is a 90° followed by 180° at 2τ spacing



- typically, 90° and 180° applied in different axes (x', then $y', y', y', \ldots$) which reduces phase errors due to imperfect 180° pulses (since slightly-off rotation around y' affects phase less)

GRADIENT ECHOES — T2*, GE chains

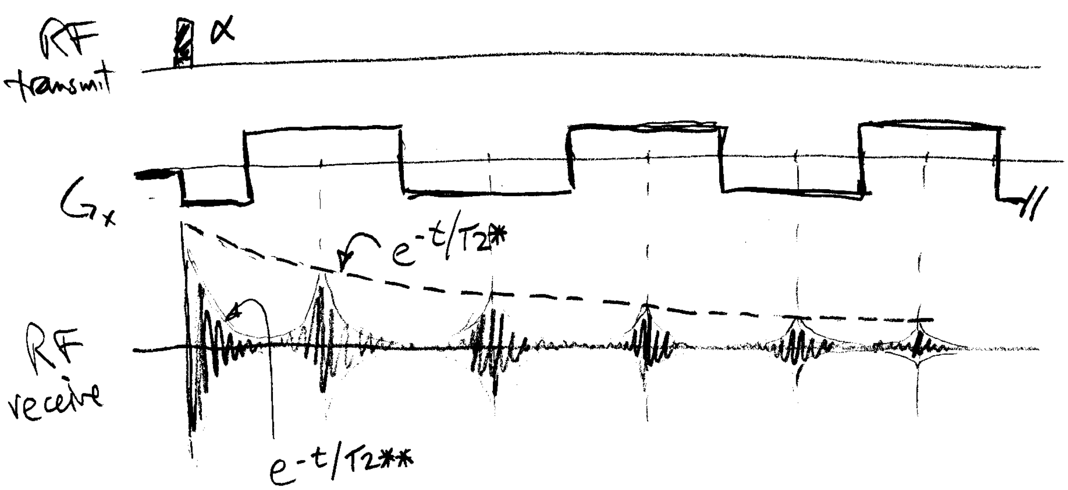


- initial negative gradient <u>de</u>phases spins
- after $t = \tau$ of positive gradient, spins rephase
- does <u>not</u> correct for T2* inhomogeneities so <u>echo amplitude</u> is $\boxed{A_E = e^{-t/T2^*}}$
- the initial "FID" is not "free" since it is being actively de-phased by gradient, so FID decay

$\frac{1}{T2} < \frac{1}{T2^*} < \frac{1}{T2^{**}} \leftarrow \boxed{A_E = e^{-t/T2^{**}}}$

- key difference between spin-echo (SE) and gradient echo (GE) is that B_0 inhomogeneities <u>not</u> corrected
  ↳ hence, echoes are T2*-weighted, <u>not</u> T2-weighted ⟹ more susceptible to inhomogeneities

- echo trains possible w/ gradient echo (CPMG-like)

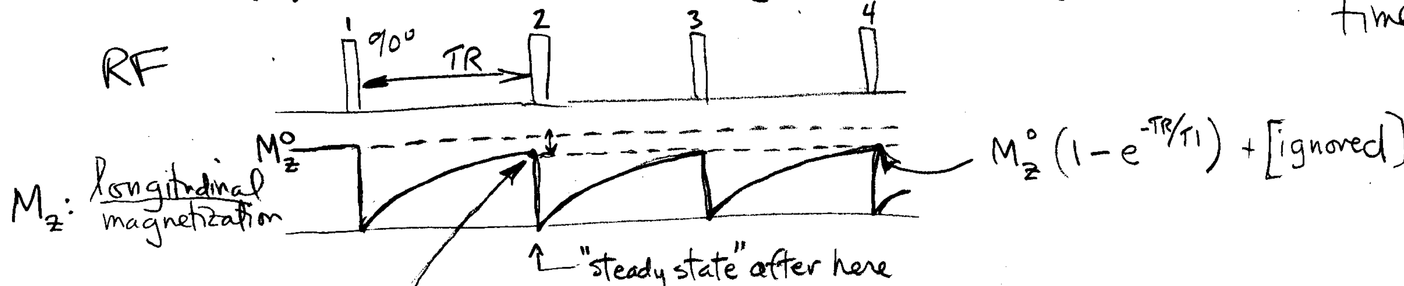


- the faster the gradients are switched, the more echoes you get
- EPI hardware ⟹ 64 echoes

IMAGE CONTRAST

T1 saturation-recovery (no echo, just FID)

- contrast (PD, T1, T2, T2*) depends on magnetization <u>not</u> getting back to equilibrium, and then <u>differences</u> in how far away each tissue type is at measurement time

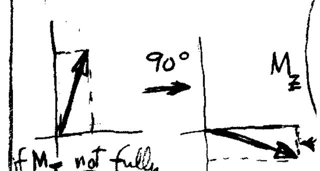


- simple saturation/recovery w/ <u>no</u> echo
- initial conditions: [M_z before first pulse = M_z°; $M_z = 0$ immed. after first pulse (i.e., 90° pulse)]
- from Bloch eq, M_z just before second pulse:

$$\underbrace{M_{z'}^{(n)}(0_-)}_{M_L \text{ before current pulse}} = \underbrace{M_z^{(0)}\left(1-e^{-TR/T1}\right)}_{M_L \text{ "regrowth-from-zero" term}} + \underbrace{M_z^{(n-1)}(0_+)e^{-TR/T1}}_{M_L \text{ "left-immed.-after-pulse" term (N.B. decaying)}}$$

2, 1, t, t

assume this is 0 because we assume that M_{xy} (<u>transverse</u>) completely decayed so that a 90° pulse doesn't generate any <u>initial</u> <u>longitudinal</u>



- given [(1) 90° pulse; (2) no M_{xy} left] → pure tip: $M_{xy} = M_z$

- tip existing mag

$$* \quad \underbrace{M_{z'}^{(n)}(0_-)}_{\text{longitudinal mag just \underline{before} pulse}} = \underbrace{M_{x'y'}(0_+)}_{\text{transverse we can record \underline{after} pulse}} = \underbrace{M_z^\circ\left(1-e^{-TR/T1}\right)}_{\text{transverse mag depends on \underline{T1}!}}$$

- that is, the <u>not-completely-regrown</u> longitudinal magnetization, which depends on T1, but which we <u>cannot</u> record, is completely converted to recordable transverse magnetization

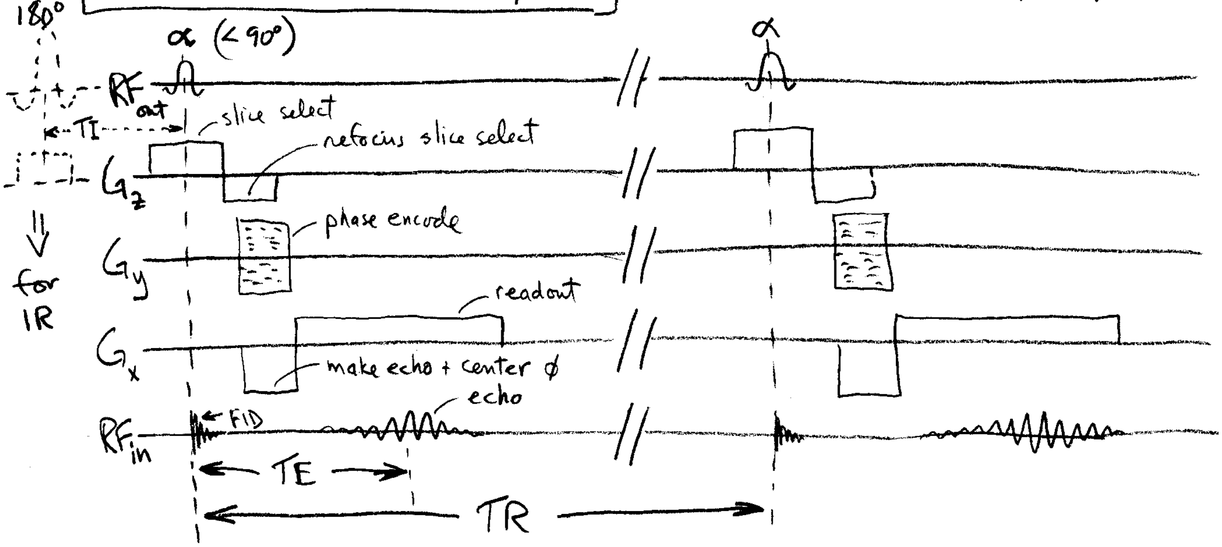
assume immediate recording of signal

$$I(\mathbf{r}) = \underbrace{C}_{\text{recon const.}} \underbrace{\rho(\mathbf{r})}_{\text{spectral dens}} \left(1 - e^{-TR/T1(\mathbf{r})}\right)$$

spectral dens $\rho(\mathbf{r})$ ≈ p. density: underlies equilib. M_z°

IMAGE CONTRAST

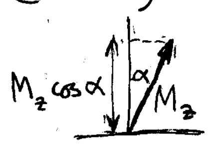
GRE w/ small tip angle



- use basic longitudinal relaxation from Bloch eq. again
  ⟶ & assume $M_{x'y'}^{(n)}(0_-) = 0$ ⟶ transverse dephased before next pulse

$$M_{z'}^{(n)}(0_-) = M_z^0\left(1 - e^{-TR/T2}\right) + M_{z'}^{(n-1)}(0_+)\, e^{-TR/T1}$$

- assume we have a small tip angle:



$$\Rightarrow \quad M_{z'}^{(n)}(0_+) = M_z^{(n)}(0_-)\cos\alpha$$

$$M_{z'}^{(n)}(0_-) = M_z^0\left(1 - e^{-TR/T1}\right) + M_{z'}^{(n-1)}(0_-)\cos\alpha\, e^{-TR/T1}$$

- assume we are in dynamic equilibrium:

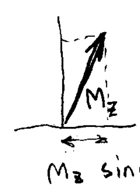
$$M_{z'}^{(n)}(0_-) = M_{z'}^{(n-1)}(0_-) = \underbrace{M_{z'}^{ss}(0_-)}_{\text{steady state}}$$

prepulse steady state <u>longitudinal</u>

$$M_{z'}^{ss}(0_-) = \frac{M_z^0\left(1 - e^{-TR/T1}\right)}{1 - \cos\alpha\, e^{-TR/T1}}$$

post-pulse <u>transverse</u> magnetization

$$M_{x'y'}^{ss}(t) = \frac{M_z^0\left(1 - e^{-TR/T1}\right)}{1 - \cos\alpha\, e^{-TR/T1}} \cdot \sin\alpha\, e^{-t/T2^*}$$



gradient echo amplitude

$$* \quad \boxed{A_E = \frac{M_z^0\left(1 - e^{-TR/T1}\right)}{\underbrace{1 - \cos\alpha\, e^{-TR/T1}}_{\text{T1 contrast mainly depends on flip angle, not TR}}} \sin\alpha\, e^{-TE/\underbrace{T2^*}_{\text{from gradient echo}}}}$$

T1 contrast mainly depends on flip angle, not TR ⟶ cos 90° = 1 ⟶ eliminates T1 weight since denom = numer

SIGNAL-TO-NOISE S/N

- generalized dependence of SNR on 3D imaging parameters

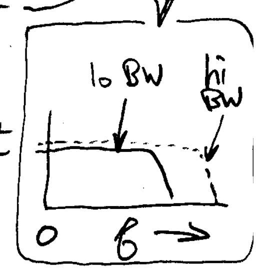
$$\text{SNR/voxel} \propto \underbrace{\Delta x \Delta y \Delta z}_{\substack{\text{voxel size} \\ \text{(of same number)}}} \sqrt{\underbrace{N_{acq}}_{\substack{\text{num} \\ \text{repeats}}} \underbrace{N_x N_y N_z}_{\substack{\text{number of} \\ \text{voxels (of} \\ \text{same size)}}} \underbrace{\Delta t}_{\substack{\text{read} \\ \text{timestep}}}}$$

- size (volume) of voxels (with the <u>number</u> of voxels held constant), <u>linear</u> effect on S/N
  - ↳ e.g., $\boxed{3\times3\times3 \text{ mm} \rightarrow 4\times4\times4 \text{ mm}} \rightarrow \frac{64}{27} = \underline{2.37 \text{ times}}$ better S/N
- more voxels (with size of voxels, Δt per readstep constant) $\sqrt{n}$ effect on S/N
  - ↳ e.g., $\boxed{64\times64 \rightarrow 128\times128} \rightarrow \frac{\sqrt{128\times128}}{\sqrt{64\times64}} = \underline{2 \text{ times}}$ better S/N
- # acquisitions $\sqrt{n}$ better S/N
  - ↳ e.g. $\boxed{1 \text{ acq} \rightarrow 2 \text{ acq}} \rightarrow \frac{\sqrt{2}}{\sqrt{1}} = \underline{1.41 \text{ times}}$ better S/N

- longer timestep during readout, $\sqrt{\Delta t}$ better S/N

$\Delta t = \frac{1}{BW_{read}}$, digitization timestep during echo acquisition

- BW_{read} determined by cutoff freq, analog lowpass filter
- Δt controls BW because low-pass cutoff has to be set higher for smaller (higher freq-detecting) Δt
- must filter out freq's $> f_{max} = \frac{1}{2\Delta t}$ because they <u>alias</u>



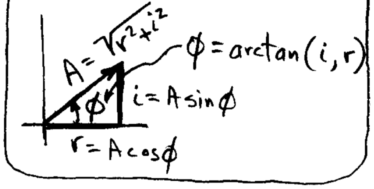
COMPLEX ALGEBRA

- don't confuse freq, angle

$e^{-i\omega t}$

makes linear var into circular — freq is linear × current time — ↳ angle is linear!

- how to convert: $(r, i) \rightarrow (A, \phi)$



real/imaginary

add: $(r_1, i_1) + (r_2, i_2) = (r_1 + r_2,\ i_1 + i_2)$

mult: $(r_1, i_1) \times (r_2, i_2) = (r_1 r_2 - i_1 i_2,\ r_1 i_2 + i_1 r_2)$

ampl./phase

add: $(A_1, \phi_1) + (A_2, \phi_2) = (A_1 \cos\phi_1 + A_2 \cos\phi_2,\ A_1 \sin\phi_1 + A_2 \sin\phi_2)$

mult: $(A_1, \phi_1) \times (A_2, \phi_2) = (A_1 A_2,\ \phi_1 + \phi_2) \longrightarrow$ N.B.: 3rd kind of vector mult. different than dot product and cross product

complex to real power: $(A, \phi)^n = (A^n, n\phi)$

---

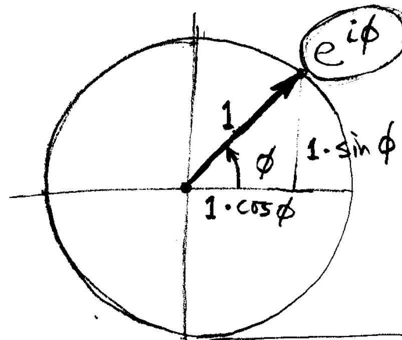
real to complex power

$e^{i\phi} =$ [expand as series, recognize cos, sin series

$= \cos\phi + i \sin\phi$

$= \cos\phi, \sin\phi$

$=$ vector on unit circle



shorthand for a unit vector (2D) pointing in the direction of ϕ

- for arbitrary amplitude, multiply $A e^{i\phi}$

$e^{i\phi n} = (\cos\phi + i \sin\phi)^n$

$= \cos n\phi + i \sin n\phi$

- change in phase is angle $\dfrac{d\phi}{dt} = \omega$ ↔ - phase is integral of angle variable $\phi = \int \omega \, dt$

Fourier transform

$H(f) = \int h(t) e^{-i 2\pi f t} dt$

$H(-f) = \int h(t) e^{i 2\pi f t} dt$

Convolution

$f(x) = g(x) * h(x) = \int_{-\infty}^{\infty} g(z) \cdot h(x - z) dz$

Cross-correlation

$f(x) = g(x) \otimes h(x) = \int_{-\infty}^{\infty} g(z) \cdot h(x + z) dz$

Convolution Theorem

$F[g(x) \cdot h(x)] = G(k) * H(k) \longrightarrow$ [the Fourier transform of two functions multiplied by each other equals the convolution of the Fourier transform of each funct

⇓

because of FFT, faster than convolution for small kernel

Fourier transform (1)

ft is a phase angle

$f \cdot t = ft$

cyc/sec $\cdot$ sec = cyc

for one f

$$\underbrace{\overset{\text{real}}{H(f)}}_{\text{complex freq. domain}} = \int_{t=-\infty}^{t=\infty} \underbrace{\overset{\text{real}}{h(t)}}_{\text{complex time domain}} \overset{\text{complex multiply}}{\cdot} \underbrace{e^{-i\,\overbrace{2\pi f}^{\substack{\omega \\ \text{ang. freq (rad/sec)}}}\,t}}_{\text{complex periodic}} \underbrace{dt}_{\text{real}}$$

lin freq (cyc/sec)

sum across t (complex add)

Fourier integral terms written out

$h_r(t)\cos(2\pi ft) + h_i(t)\sin(2\pi ft)$,

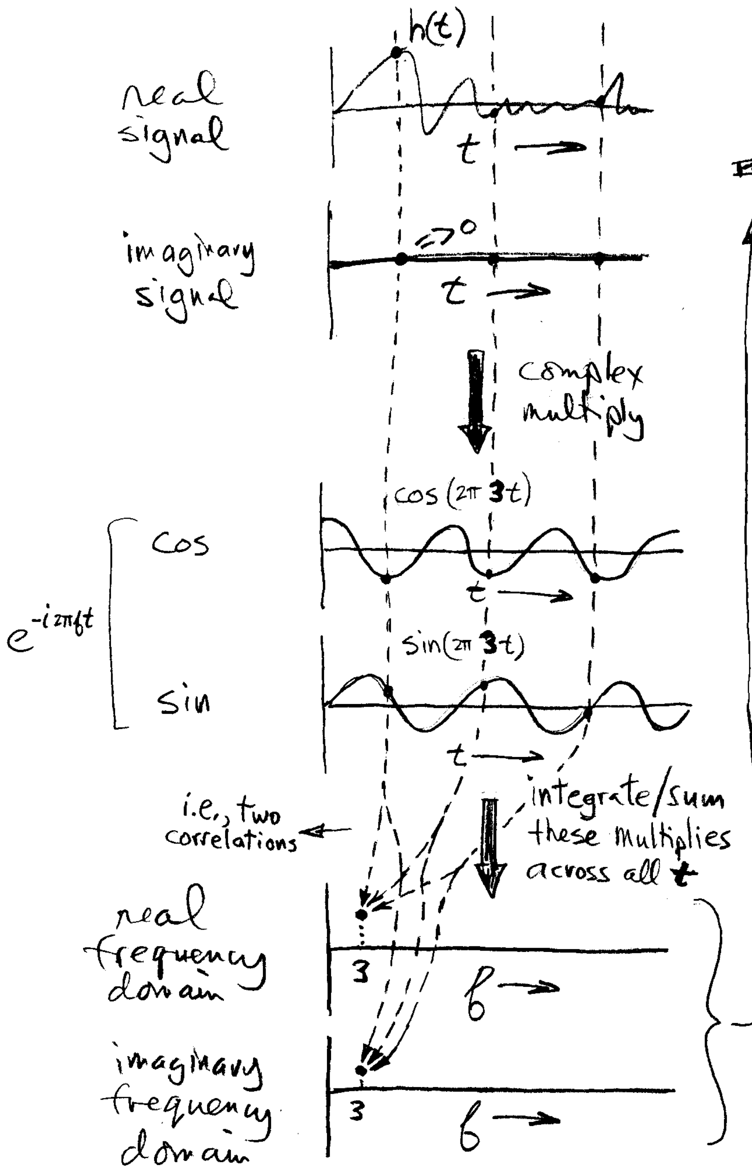
$-h_r(t)\sin(2\pi ft) + h_i(t)\cos(2\pi ft)$

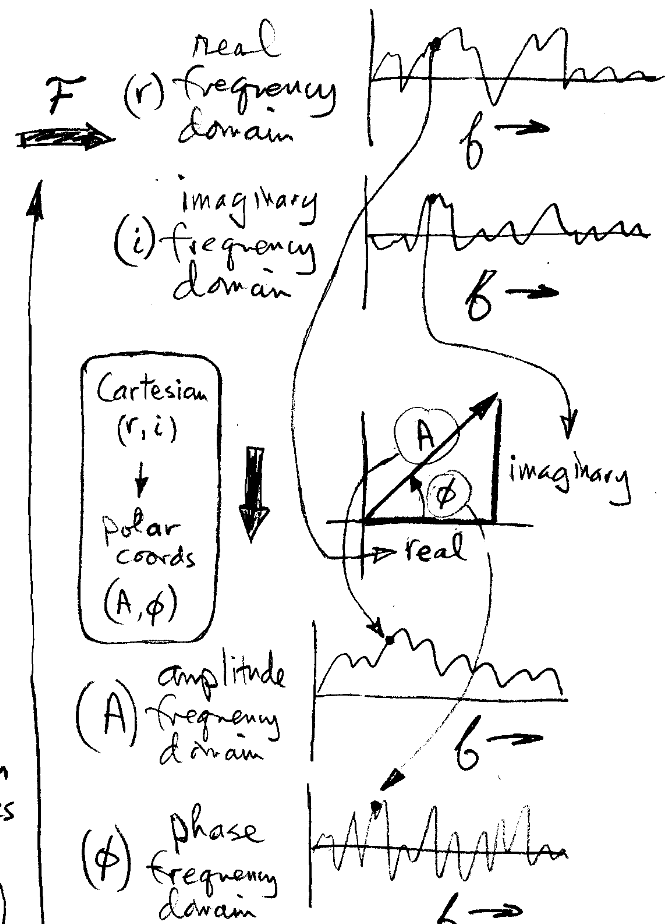
Fourier integral w/ real signal

$h_r(t)\cos(2\pi ft)$,

$-h_r(t)\sin(2\pi ft)$

- How to calculate $H(f)$ for one f ($f=3$):

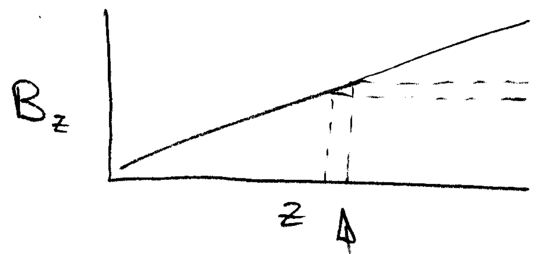




like correlating with sin and cos (at each freq.) so we get phase (at each freq.)

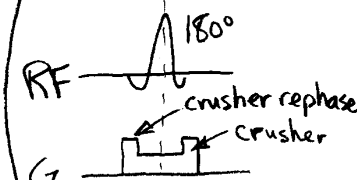
SLICE SELECTION (G_z)

- slice select gradient on <u>during</u> RF stim



$$f = \overset{\frac{\gamma}{2\pi}}{\gamma}\left(B_0 + \overset{G_z z}{B_{G,z}}\right)$$

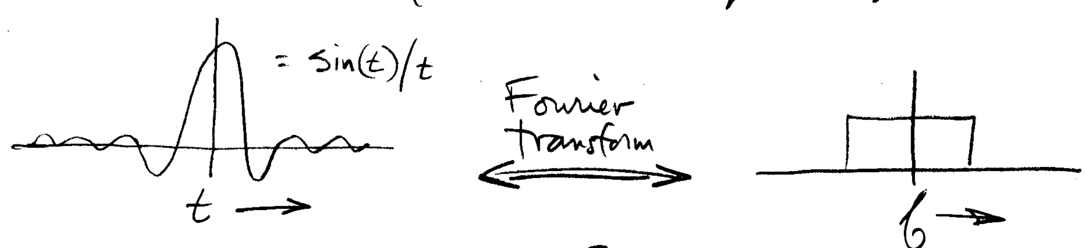
- protons here can only be excited by a narrow band of radio frequencies



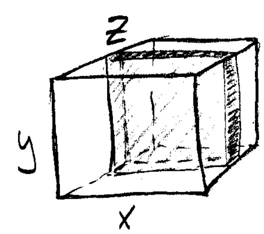
- 180° pulses don't need rephasing (self-rephasing)
- crusher lobes added to kill spurious transvese

- to apply a pulse containing a narrow band of frequencies, we use a <u>sinc</u> pulse envelope (Fourier transform of a narrow freq. band)

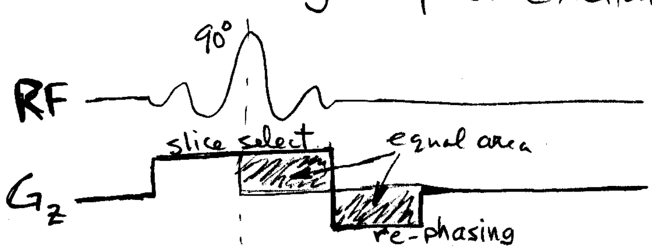
in practice, Gaussian pulse envelope good, too



- this excites protons in a narrow slab



- since the slice-selection gradient introduces (space-dependent) phase shifts (see freq encode) these have to be removed by a post-excitation rephasing z-gradient

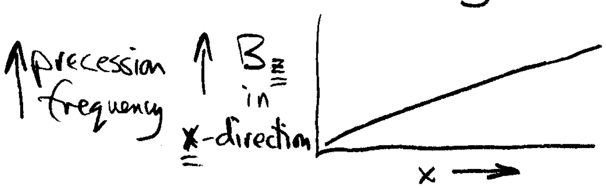


- approximation from assuming tip occurs instantaneously in middle
- valid for small tip: 90° → 52%
- in practice: adjust to max, use crusher to kill spurious transverse <u>on 180°</u>

FREQUENCY ENCODING (1)

<u>avoid</u> this mistaken intuition!

- frequency encode gradient (G_x) causes precession rates to vary linearly in x-direction



↳ <u>correct</u> (remember that strength of G_x causes variation of slope of B_z in x-direction)

- different frequency signals are mixed together and recorded as a 1-D signal over time

  ↳ <u>correct</u>, but remember, we are recording the summed <u>phase angle</u> after de-modulation

- a Fourier transform, which converts back and forth between <u>x-position</u> (cf. time) and spatial frequency (cf. temporal freq) is done on signal

  ↳ <u>correct</u>

- <u>spatial</u> frequencies get confused/conflated with <u>precession</u> frequencies

  ↳ <u>wrong</u> !!

- therefore, the Fourier transform is used to convert position-dependent <u>precession</u> frequencies into <u>spatial</u> position

  ↳ <u>wrong</u>!! → it actually converts <u>spatial</u> <u>frequencies</u> to <u>spatial</u> <u>position</u>

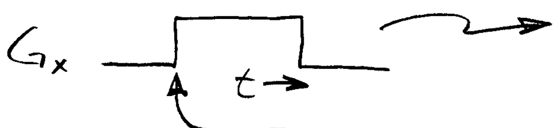
  → the spatial frequency <u>increases</u> for each time point in the readout

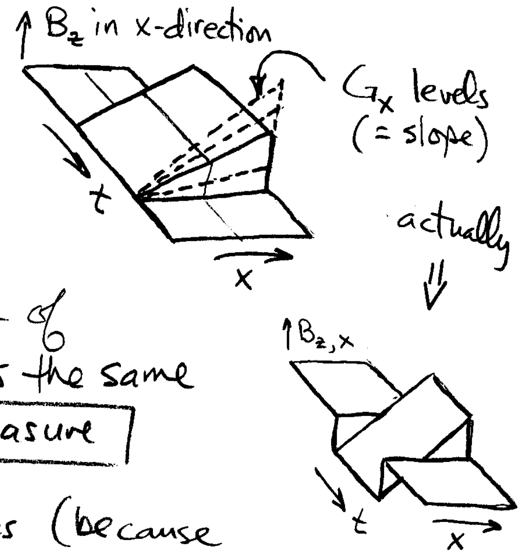
  → the precession freq ramp is <u>constant</u> each timestep

FREQUENCY ENCODING (2)

correct intuition — why phase critical

- "frequency"-encode gradient (G_x) turned on during during echo causes precession rates to immediately vary with x-position

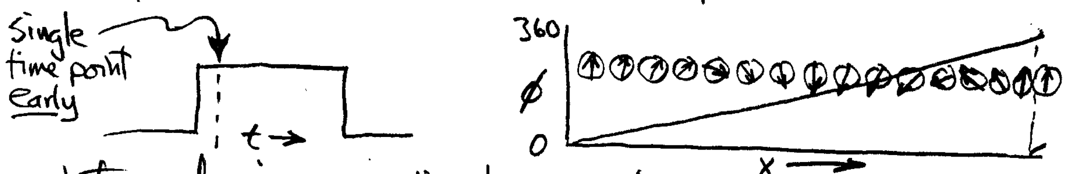




- at beginning of gradient on, the phase of signal coming from each x-position is the same

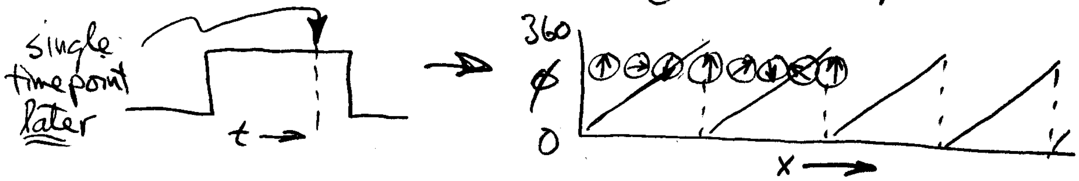
$\boxed{\text{summed phase angle is what we measure}}$

- early after gradient on, phase advances (because of faster precession frequency) arise with greatest phase advance at largest x-position



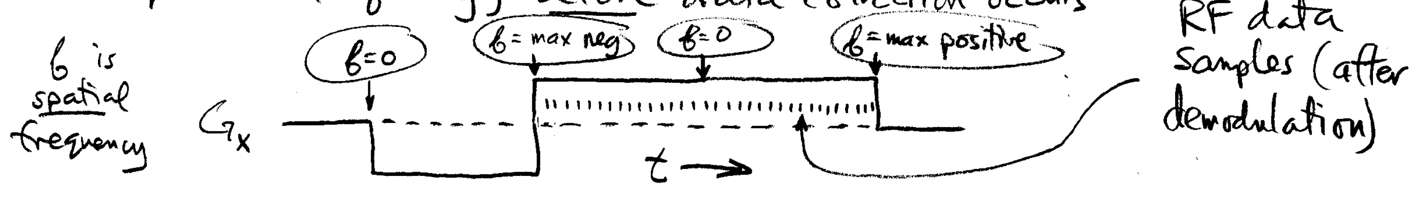
⇒ one cycle of spatial frequency of phase angle (= low spatial freq)

- later during gradient on, phase advances cause multiple wraparounds of phase angle across space



⇒ multiple cycles of spatial frequency of phase angle (= hi spatial freq)

- in practice, the lowest spatial frequency (= 0) occurs in the middle of the gradient on time because the phase is "wound" negatively by a preparatory gradient (to the highest negative spatial frequency) before data collection occurs



$\boxed{\text{FREQUENCY ENCODING (3)}}$ why each datapoint is 1 spatial freq

Standard Fourier transform : (Temporal freq $\longleftrightarrow$ time)

$$\underbrace{H(f)}_{\text{freq domain}} = \int_{t=-\infty}^{t=\infty} \underbrace{h(t)}_{\text{time domain}} \cdot \underbrace{e^{-i\,2\pi f t}}\, \underbrace{dt}$$

time domain → r, i=0

→ [cos $2\pi f t$, −sin $2\pi f t$]

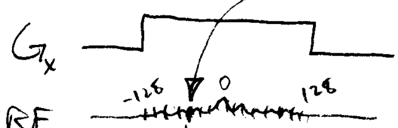
sum across all time

"k" is often used instead of "f" for the frequency variable

Imaging equation : (spatial freq. $\longleftrightarrow$ space)

$$\underbrace{S(f)} = \int_{x=-\infty}^{x=\infty} \underbrace{I(x)} \cdot \underbrace{e^{-i\,2\pi f x}}\, \underbrace{dx}$$

one data point (i.e., one spatial freq) during readout (2 components)



get this single readout point by summing signal across x-position (RF coil records sum)

even though variable is f, it represents one time point during readout

signal strength at one x-position (brightness of image point)

spin density (spectral dens.)

to make image, do inverse Fourier transform of recorded signal $S(f)$

cos $(2\pi f x)$, −sin $(2\pi f x)$

oscillations come from readout phase wrapping, where f is single spatial freq (e.g. 5) and x goes across object

[FID: $G_x t$
SE: $G_x(t-TE)$]

$f = G_x t$, that is, spatial freq depends on amount of time gradient was on (this f increases with time!)

don't confuse with instantaneously changed precession freqs which are constant across entire readout time (for each x position)

Sum across x of object

this is done by RF coil recording sum

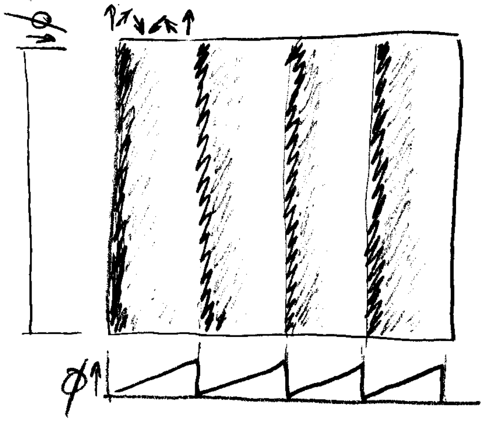
PHASE & FREQ, 2D & 3D

pictures of phase in <u>image</u> space

N.B.: stripes have sharp edges from phase wrap (<u>not</u> sinusoid since ϕ from 2-comp quadrature!)

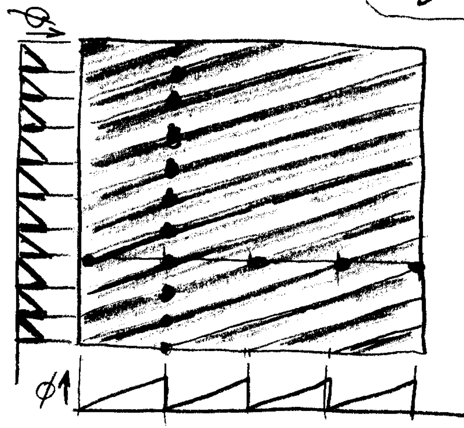
- since the phase-encode gradient and the freq encode gradient both affect <u>phase</u> the result is a <u>rotation</u> of phase "stripes" when the two add

phase encode $f=0$



freq encode $f=4$

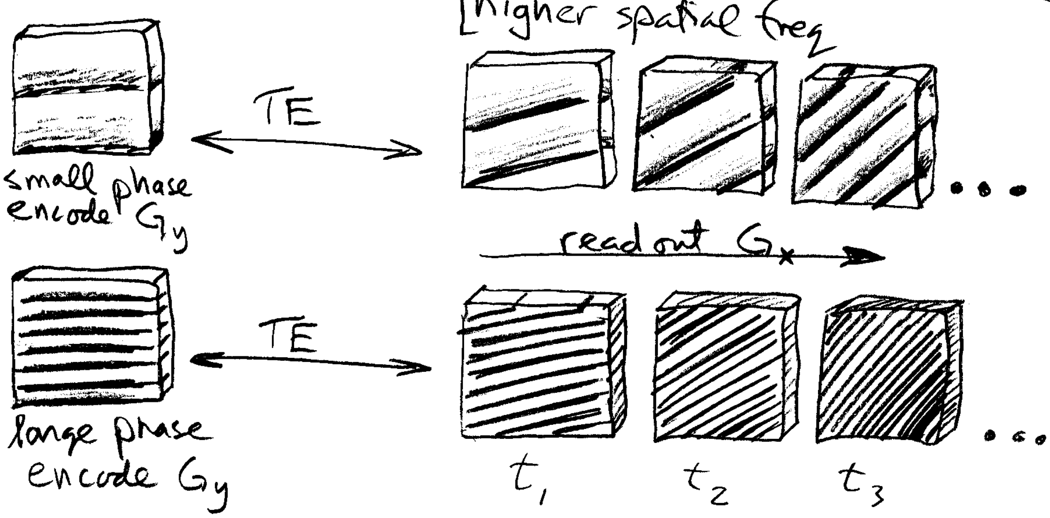
phase encode $f=10$



freq encode $f=4$
(unchanged in x-axis)

stripes here represent complex value
↓
phase of whole image summed to one (complex) number by RF coils

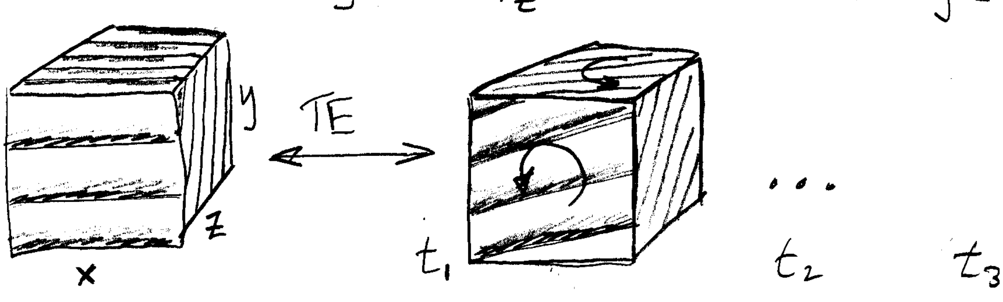
- successive read out steps: [more rotation
[higher spatial freq



e.g., after **x**-gradient, spins at a point might be 3 cyc ahead while after **y**-gradient spins at <u>same</u> pnt 7 cyc ahead: <u>but</u> counting <u>wraps</u> in X-direction, still only 3 ahead

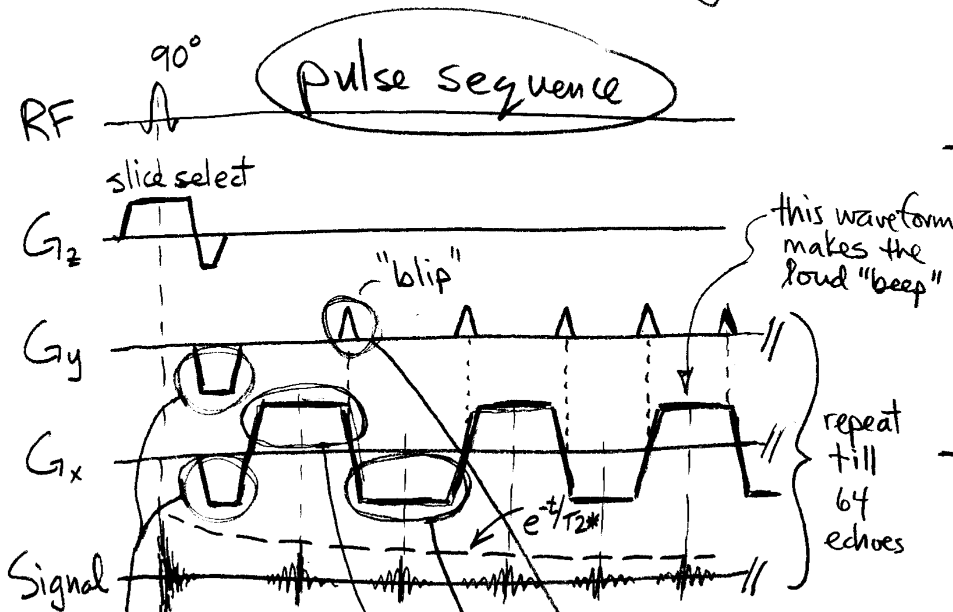
- 3D phase encode w/ G_y <u>and</u> G_z starts rotated in y-z plane

large phase encode G_z
+
small phase encode G_y



ECHO PLANAR IMAGING EPI (another fast gradient echo)

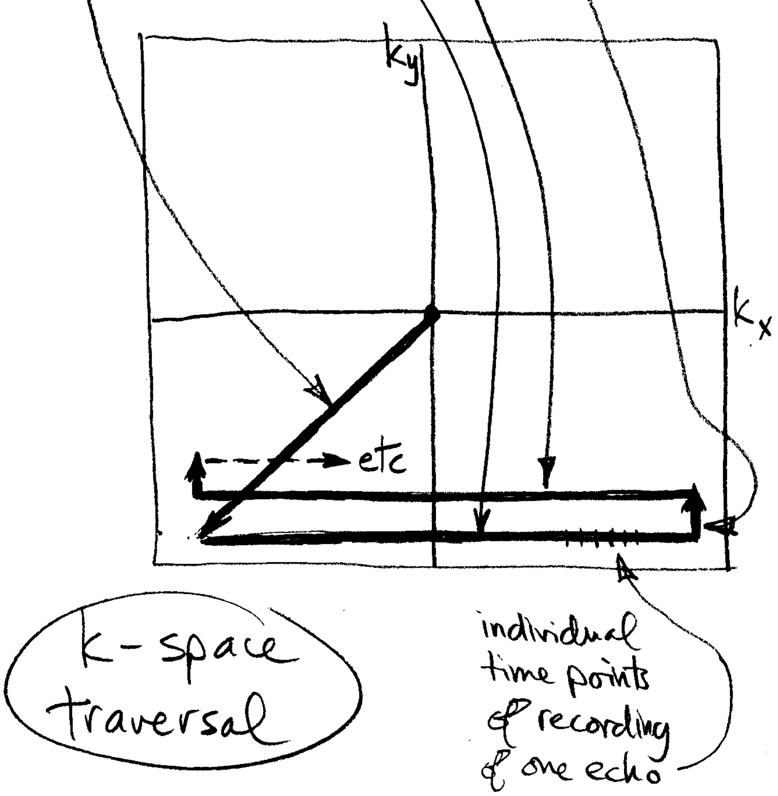
– single shot EPI collects <u>all</u> k-space lines (e.g. 64) after a 90° RF pulse using a train of <u>gradient</u> echoes



– since there is only one RF pulse per slice, spins never get reset to all-the-same (= zero freq, center of k-space)

– therefore, the recording point (Δt) in k-space (= spin phase stripe pattern) stays wherever the x and y gradients last left it

– that explains why successive y phase-encode steps are achieved without changing the size of the G_y "blips"



– echoes are T2*-weighted (gradient echo)

– contrast mainly determined by echoes near center of k-space, which are only recorded after about 32 echoes