Cognitive Science 276 -- Neuroimaging

Homework #1: Tissue Contrast in Spin-Echo and Gradient Echo Sequences

The goal of this homework is: (1) to understand the Bloch equation solutions, (2) to learn enough Matlab (or Mathematica) syntax to allow you plot simple exponential functions and their differences, and (3) to use this skill to understand how varying TR (repetition time) and TE (echo time) can result in contrast between tissues with different T1 (longitudinal) and T2 (transverse) time constants. Turn in answers, graphs, and code.

1. Plot time course (t=0-2500 msec) of *longitudinal* magnetization regrowth, $M_z(t)$, following a single 110 degree RF pulse starting from equilibrium. Use equation below (the solution to the longitudinal relaxation part of the Bloch equation) assuming T1=700 msec, and equilibrium magnetization, M_z^0=1.0. (N.B.: $M_z(0_+)$ means longitudinal magnetization *immediately* after flip).

$$M_z(t) = M_z^0\,(1 - e^{-t/T1}) + M_z(0_+)\,e^{-t/T1}$$

2. Apply the Bloch equations to the following simple spin echo sequence. At the *very beginning* of a spin-echo pulse sequence, *(a)* the equilibrium *longitudinal* magnetization is flipped by a 90 deg RF pulse, *(b)* it recovers (from zero) for time t=TE/2, *(c)* a 180 deg RF pulse is applied at time t=TE/2, *(d)* an echo occurs at time t=TE, *(e)* the *longitudinal* magnetization recovers for t=TR (measured from the first 90 deg RF pulse), and finally, *(f)* the second 90 degree RF pulse occurs. Derive the equation (this means show each step) for the amount of *longitudinal* magnetization present *just **before*** the *second* 90 degree RF pulse by giving equations for the longitudinal magnetization at *each* of the stages *(a)* to *(e)* above (Hint: remember M_z^0 is a *constant* while $M_z(0_+)$ is a variable). No graphs needed.

3. Plot the time course of the decay of *transverse* magnetization after a 90 degree RF pulse for two different tissue types with T2=59 and T2=71 msec, and then plot the *difference* between these two curves to illustrate the time point where their *transverse* magnetizations are the most *different*. Use equation below (assume longitudinal magnetization before the flip, M_z^0, is same for both tissue types, namely 1.0).

$$M_{xy}(t) = M_z^0\,e^{-t/T2}$$

4. Assume the following T1, T2, and proton-density (PD) values for gray matter (GM), white matter (WM), and cerebrospinal fluid (CSF): **T1** (msec): GM=1200, WM=830, CSF=2300; **T2** (msec): GM=87, WM=81, CSF=340; **PD** (water=1.0): GM=0.68, WM=0.59, CSF=0.98. Use this equation for *spin-echo* signal intensity:

$$M_{xy} = M_z^0\,(1 - e^{-TR/T1})\,e^{-TE/T2}$$

(a) In T1-weighted images, WM > GM > CSF (signal intensity, brightness). For a *fixed* TE=7 msec, determine the TR that maximizes the contrast between GM and WM (TR that results in largest value of WM-minus-GM). Do this by plotting $M_{xy}(TR)$ curves for each tissue type (TR=0 to TR=2500). Explain why T1-weighted images have intermediate TR and short TE.

(b) In typical T2-weighted images, CSF > GM > WM. For a fixed TR=3400 ms, determine a TE that maximizes CSF-minus-WM contrast. Plot the curve of $M_{xy}(TE)$ for each tissue (from TE=1 to TE=200 msec). Explain why T2-weighted images have long TR and intermediate TE.

5. Use the following equation for fast *gradient echo* signal intensity (and T1, T2, and PD values from problem 4):

$$M_{xy} = \left[\, [M_z^0 (1 - e^{-TR/T1})] \,/\, [1 - \cos\alpha\, e^{-TR/T1}] \,\right] \sin\alpha\, e^{-TE/T2}$$

Make a *2-D plot* of the dependence of M_{xy} on flip angle (from 0-30 deg) *and* TE (from 0-30 msec). Assume that the TR=21 msec. (a 2-plot means a plot that shows a value of M_{xy} for every (suitably sampled) combination of flip angle and TE from the ranges given above. The *2-D plot* can be a illustrated as a height map, a brightness map, a contour map, (or, if you are really desperate, a grid of numbers :-}).