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3 **Imprecise evidence without imprecise credences**

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7 **Abstract** Does rationality require imprecise credences? Many hold that it does:
8 imprecise evidence requires correspondingly imprecise credences. I argue that this
9 is false. The imprecise view faces the same arbitrariness worries that were meant to
10 motivate it in the first place. It faces these worries because it incorporates a certain
11 idealization. But doing away with this idealization effectively collapses the
12 imprecise view into a particular kind of precise view. On this alternative, our
13 attitudes should reflect a kind of normative uncertainty: uncertainty about what to
14 believe. This view refutes the claim that precise credences are inappropriately
15 informative or committal. Some argue that indeterminate evidential support requires
16 imprecise credences; but I argue that indeterminate evidential support instead places
17 indeterminate requirements on credences, and is compatible with the claim that
18 **AQ1** rational credences may always be precise.

19
20 **Keywords** Epistemology · Bayesianism · Imprecise credences · Imprecise
21 probabilities

22
23 A traditional theory of uncertainty says that beliefs come in degrees. Degrees of
24 belief (“credences”) have real number values between 0 and 1, where 1
25 conventionally represents certain belief, 0 represents certain disbelief, and values
26 in between represent degrees of uncertainty. We have elegant, well-understood
27 normative theories for credences: norms for how credences should hang together at
28 a time, how they should change in response to new evidence, and how they should
29 influence our preferences.

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30 Many have argued, against the traditional theory, that rational subjects have
 31 imprecise credences. Instead of sharp, real number values like .4, rational agents
 32 have credences that are spread out over multiple real numbers, e.g. intervals like [.2,
 33 .8]. There are descriptive and normative versions of the view.

34 The descriptive (psychological) version argues that humans' cognitive capacities
 35 don't allow for infinitely sharp credences. In creatures like us, there's typically no
 36 genuine psychological difference between having credence .8 and credence
 37 .8000000000000001. So we are better represented with imprecise credences.¹

38 The normative (epistemic) version argues that even idealized agents without our
 39 cognitive limitations shouldn't have sharp credences. Epistemic rationality some-
 40 times demands imprecise credences.²

41 This paper concerns the latter view. The former view may be correct as a matter
 42 of psychological fact. Creatures like us may be better represented by some
 43 imprecise credence model or other. But this doesn't tell us much about
 44 epistemology. Epistemic theories of imprecise credences say that, for evidential
 45 reasons, it can be irrational to have precise credences. But these theories typically
 46 represent ideal rationality in ways that go far beyond human cognitive capacities.

47 This paper concerns, instead, the contents of epistemic norms.³ Does rationality
 48 require imprecise credences? Many hold that it does: imprecise evidence requires
 49 correspondingly imprecise credences. I'll argue that this is false. Imprecise
 50 evidence, if such a thing exists, at most requires uncertainty about what credences to
 51 have. It doesn't require credences that are themselves imprecise.

52 In Sects. 1 and 2, I summarize motivations for the imprecise view and put
 53 forward a challenge for it. Briefly, the view faces the same arbitrariness worries that
 54 were meant to motivate it in the first place. The imprecise view faces these
 55 challenges because it incorporates a certain idealization. But doing away with that
 56 idealization effectively collapses the imprecise view into a particular kind of precise
 57 view. On this alternative, our attitudes should reflect a kind of normative
 58 uncertainty: uncertainty about what to believe. I defend this proposal in Sect. 3.

59 In Sects. 4 and 5, we reach the showdown. Section 4 argues against the claim that
 60 precise credences are inappropriately informative or committal. Section 5 argues
 61 against the claim that indeterminate evidential support requires imprecise credences.
 62 Are there any reasons to go imprecise that don't equally support going precise with
 63 normative uncertainty? The answer, I argue, is no. Anything mushy can do, sharp
 64 can do better.

¹ See e.g. Jeffrey (1983) and van Fraassen (1990).

² See e.g. Levi (1974, 1980, 1985), Walley (1991), Joyce (2005, 2010), Weatherson (2008), Sturgeon (2008), Hájek and Smithson (2012), and Moss (2014).

³ This might be interpreted as a question about epistemic norms for cognitively idealized agents. I'd rather interpret it as a question about what evidentialist epistemology requires.

65 **1 Imprecise credences**66 **1.1 What are imprecise credences?**

67 Imprecise credences, whatever they are, are doxastic states that are more coarse-
 68 grained than precise credences. The psychological nature of imprecise credences
 69 widely contested. How are they behaviorally manifested? How are they functionally
 70 distinct from precise credences?⁴ Are nonideal agents like us capable of having
 71 imprecise credences—for example, interval or set-valued credences? No orthodox
 72 answer to these questions has emerged.

73 How should imprecise credences be modeled? Some have suggested representing
 74 imprecise credences with a comparative confidence ranking.⁵ Some suggest using
 75 imprecise credence functions, from propositions (sentences, events, ...) to lower and
 76 upper bounds, or to intervals within $[0, 1]$.⁶ I'll focus on a model, defended at length
 77 by Joyce (2010), that represents agents' doxastic states with sets of precise
 78 probability functions.

79 I'll use the following notation: C is the set of probability functions c that
 80 characterize an agent's belief state; call C an agent's "representor." For
 81 representing imprecise credences toward propositions, we can say $C(A)$ is the set
 82 of probabilities assigned to A by some probability function in an agent's representor.

83 A representor can be thought of as a committee of probability functions. The
 84 committee decides by unanimous vote. If the committee unanimously votes to
 85 assign A credence greater than .5, then the agent is more confident than not in A . If
 86 the committee unanimously votes that the agent should perform some action, then
 87 the agent is rationally required to perform that action.⁷ And so on.

88 **1.2 The epistemic argument for imprecise credences**

89 Many proponents of imprecise credences claim that in the face of some bodies of
 90 evidence, it is simply irrational to have precise credences. These bodies of evidence
 91 are somehow ambiguous (they point in conflicting directions) or unspecific (they
 92 don't point in any direction).⁸ It's an open question how widespread this kind of
 93 evidence is. Often it's implicitly assumed that we can only have precise credences

⁴ Some have associated certain patterns of preferences or behaviors with imprecise credences: for example, having distinct buying and selling prices for gambles (Walley 1991), or being willing to forego sure gains in particular diachronic betting contexts (Elga 2010). But treating these as symptomatic of imprecise credences, rather than precise credences, depends on specific assumptions about how precise credences must be manifested in behavior: for example, that agents with precise credences are expected utility maximizers.

⁵ E.g. Fine (1973).

⁶ E.g. Kyburg (1983).

⁷ Beyond this sufficient condition, there's some controversy among proponents of imprecise credences about what practical rationality requires of agents with imprecise credences. See e.g. Seidenfeld (2004), Weatherson (2008), Joyce (2010), Williams (2014), and Moss (2014).

⁸ "Unspecific bodies of evidence" may include empty bodies of evidence.



94 where we have knowledge of objective chances. Any evidence that doesn't entail
95 chances is imprecise, and requires imprecise credences.

96 Note that the dialectic here is somewhat different from more common debates
97 about imprecise credences. In other contexts, fans of imprecise credences have
98 faced the challenge of showing that imprecise credences are rationally permissible:
99 for example, in debates over whether imprecise credences can serve as a guide to
100 rational action⁹ or allow for inductive learning.¹⁰

101 This paper concerns the evidentialist argument for imprecise credences. The
102 argument is thought to show that imprecise credences aren't merely permissible: in
103 the face of imprecise evidence, they're rationally required. Since the phrase
104 "proponent of imprecise credences" doesn't distinguish between these two views,
105 we have to introduce new terminology. Proponents of rationally required imprecise
106 credences will be called "mushers."¹¹ I'll defend the "sharper" view: that precise
107 credences are always rationally permissible.

108 We'll consider two examples that are commonly used to elicit the musher
109 intuition:

110 **Jellyfish.** "A stranger approaches you on the street and starts pulling out
111 objects from a bag. The first three objects he pulls out are a regular-sized tube
112 of toothpaste, a live jellyfish, and a travel-sized tube of toothpaste. To what
113 degree should you believe that the next object he pulls out will be another tube
114 of toothpaste?" (Elga 2010, 1)

115 If there's any such thing as imprecise evidence, this looks like a good candidate.
116 Unless you have peculiar background beliefs, the evidence you've received will feel
117 too unspecific or ambiguous to support any particular assignment of probability. It
118 doesn't obviously seem to favor a credence like .44 over a credence like .78 or .21.
119 According to the musher, it would be epistemically inappropriate to assign a precise
120 credence. Instead, you should take on a state that equally encompasses all of the
121 probability functions that could be compatible with the evidence.

122 A second case:

123 **Mystery Coin.** You have a coin that was made at a factory where they can
124 make coins of any bias. You have no idea whatsoever what bias your coin has.
125 What should your credence be that the next time you toss the coin, it'll land
126 heads? (see e.g. Joyce 2010.)

127 This case is more theoretically loaded. There is a sharp credence that stands out as a
128 natural candidate: .5. After all, you have no more reason to favor heads than to favor
129 tails; your evidence is symmetric. Credence .5 in both heads and tails seems *prima*
130 *facie* to preserve this symmetry.

131 But Joyce (2010) and others have argued that the reasoning that lands us at this
132 answer is faulty. The reasoning presumably relies on the principle of indifference,

⁹ E.g. in Elga (2010).

¹⁰ White (2009), see Pedersen and Wheeler (2014) for discussion.

¹¹ Imprecise credences are often called "mushy" credences.

133 which says that if there is a finite set of mutually exclusive possibilities and you
 134 have no reason to believe any one more than any other, then you should distribute
 135 your credence equally among them. But the principle of indifference faces serious
 136 (though perhaps not decisive) objections.¹² Without something like it, what
 137 motivates the .5 answer?

138 According to Joyce, nothing does. There's no more reason to settle on a credence
 139 like .5 than a credence like .8 or .421. Joyce sees this as a case of imprecise
 140 evidence. If you knew the objective chance of the coin's landing heads, then you
 141 should adopt a precise credence. But if you have no information about the chance,
 142 you should have an imprecise credence that includes all of the probabilities that
 143 could be equal to the objective chance of the coin's landing heads, given your
 144 evidence. In this case, that may be the full unit interval $[0, 1]$.¹³

145 The musher's assessment of these cases: any precise credence function would be an
 146 inappropriate response to the evidence. It would amount to taking a definite stance
 147 when the evidence doesn't justify a definite stance. It would mean adopting an attitude
 148 that is in some way more informative than what the evidence warrants. It would mean
 149 failing to fully withhold judgment where judgment ought to be fully withheld.

150 Mushers in their own words:

151 [E]ven if men have, at least to a good degree of approximation, the abilities
 152 bayesians attribute to them, there are many situations where, in my opinion,
 153 rational men *ought not* to have precise utility functions and precise probability
 154 judgments. (Levi 1974, 394–395)

155 If there is little evidence concerning Ω then beliefs about Ω should be
 156 indeterminate, and probability models imprecise, to reflect the lack of
 157 information. (Walley 1991)

158 Precise credences ...always commit a believer to extremely definite beliefs
 159 about repeated events and very specific inductive policies, even when the
 160 evidence comes nowhere close to warranting such beliefs and policies. (Joyce
 161 2010, 285)

162 [In Elga's Jellyfish case,] you may rest assured that your reluctance to have a
 163 settled opinion is appropriate. At best, having some exact real number
 164 assessment of the likelihood of more toothpaste would be a foolhardy response
 165 to your unspecific evidence. (Moss 2014, 2)

166 Hillary Clinton will win the 2016 U.S. presidential election. Tesla Motors'
 167 stock price will be over \$310 on January 1, 2016. Dinosaurs were wiped out by

¹² In particular, Bertrand's (1889) paradoxes; see e.g. the hollow cube example in Seidenfeld (1978) and van Fraassen (1989). For a defense of the principle of indifference, see White (2009).

¹³ There are other kinds of motivation for rationally permissible imprecise credences. One is the view that credences intuitively needn't obey Trichotomy, the claim that for all propositions A and B , $c(A)$ is either greater than, less than, or equal to $c(B)$ (see e.g. (Schoenfield 2012). Moss (2014) argues that imprecise credences provide a good way to model rational changes of heart (in a distinctly epistemic sense). Hájek and Smithson (2012) suggest imprecise credences as a way of representing rational attitudes towards events with undefined expected value. Finally, there's the empirical possibility of indeterminate chances, also discussed in Hájek and Smithson (2012): if there are set-valued chances, the Principal Principle seems to demand set-valued credences. Only the last of these suggests that imprecise credences are rationally required; I'll return to it in Sect. 4.

168 a giant asteroid, rather than gradual climatic change. If you have a *perfectly*
 169 *precise* credence on any of these matters, you *might* be a little off your rocker.
 170 Having a precise credence for a proposition X means having opinions that are
 171 are so rich and specific that they pin down a *single* estimate $c(X)$ of the truth-
 172 value of XIf your evidence is anything like mine, it is too incomplete and
 173 ambiguous to justify such a rich and specific range of opinions. (Konek,
 174 forthcoming, 1)

175 The musher's position in slogan form: *imprecise evidence requires imprecise*
 176 *credences*.¹⁴

177 There are two strands of thought amongst mushers about the nature of imprecise
 178 evidence:

- 179 1. The most common assumption is that any evidence that doesn't entail a precise
 180 objective chance for a proposition A is imprecise with respect to A .¹⁵ The notion
 181 of imprecise evidence is often illustrated by contrasting cases like **Mystery**
 182 **Coin** with fair coin tosses; the form of evidence that appears in the Ellsberg
 183 paradox is another example.¹⁶ In practice, this arguably means that unless
 184 evidence entails either A or its negation, it is imprecise with respect to A . There
 185 are arguably no realistic cases where it's rational to be certain of precise, non-
 186 extremal objective chances.
- 187 2. Some mushers¹⁷ seem to treat imprecise evidence as less ubiquitous, arising in
 188 cases where individual items of evidence pull hard in opposing directions: court
 189 cases where both the prosecution and defense have mounted compelling
 190 arguments with complex and detailed evidence; cases of entrenched disagreement
 191 among experts on the basis of shared evidence; and so on. These mushers
 192 generally allow that highly incomplete evidence is also imprecise. (For
 193 example, one's attitude about the defendant's guilt upon receiving a jury
 194 summons, before learning anything about the case; for another, the Jellyfish
 195 example above). For this sort of musher, the notion of imprecise evidence left
 196 both intuitive and vague: there may be no bright dividing line between precise
 197 and imprecise evidence, as there is for the first sort of musher.

198 I intend to cast a broad net, and my use of "imprecise evidence" is intended to be
 199 compatible with either interpretation. It's the mushers' contention that there are some

¹⁴ See e.g. Joyce (2010): "Since the data we receive is often incomplete, imprecise or equivocal, the epistemically right response is often to have opinions that are similarly incomplete, imprecise or equivocal."

¹⁵ More cautiously, we might distinguish between first- and higher-order objective chances. Suppose a coin has been chosen at random from an urn containing 50 coins biased 3/4 toward heads and 50 coins biased 3/4 toward tails. The first-order objective chance that the chosen coin will land heads on the next toss is either .75 or .25, but the second-order objective chance of heads is .5. Musers in the first category might allow for precise credences where only higher-order objective chance are known. This seems to be the position of Joyce (2010).

¹⁶ Ellsberg (1961).

¹⁷ Schoenfield (2012) and Konek (forthcoming) seem to fall into this category, given their choice of motivating examples.

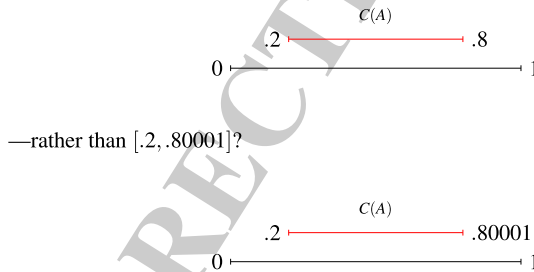
200 bodies of evidence that *intuitively* place special demands on our credences: they
 201 require a kind of uncertainty or suspension of judgment beyond merely having
 202 middling credence in the relevant propositions. The musher holds that the best way of
 203 understanding the required form of uncertainty uses the notion of imprecise credences.
 204 My contention is that if any form of evidence does place these special demands, the
 205 special form of uncertainty they demand is better represented in other ways.

206 2 A challenge for imprecise credences

207 2.1 The arbitrariness challenge

208 In cases like the Jellyfish case, the musher claims that the evidence is too unspecific
 209 or ambiguous to support any precise credence. If you were to adopt a precise
 210 credence that the next item pulled from the bag would be toothpaste, what evidential
 211 considerations could possibly justify it over a nearby alternative?

212 But the musher faces a similar challenge. What imprecise credence is rationally
 213 permissible in the Jellyfish case? The musher is committed to there being one.
 214 Suppose their answer is that, given your total evidence, the rationally obligatory
 215 credence is the interval $[.2, .8]$. Then the musher faces the same challenge as the
 216 sharper: why is it rationally required to have credence $[.2, .8]$ —
 217



(Figures are drawn to scale.)

219 What evidential considerations could possibly justify that particular imprecise
 220 credence over nearby alternatives? How could the evidence be so unambiguous and
 221 specific as to isolate out any one particular interval? This objection is discussed in
 222 Good (1962), Walley (1991), and Williamson (2014).¹⁸

223 Three moves the musher could make:

¹⁸ Walley more or less concedes this point (104–105). He distinguishes “incomplete” versus “exhaustive” interpretations of imprecise credences, similar to the “imprecise” versus “indeterminate” interpretations discussed in Levi (1985). On the incomplete interpretation, which he generally uses, the degree of imprecision can be partly determined by the incomplete *elicitation* of an agent’s belief state. On the exhaustive interpretation, by contrast, imprecision is determined solely by indeterminacy in the agent’s belief state. The latter interpretation, Walley acknowledges, requires “the same kind of precise discriminations as the Bayesian theory” (105). The musher position, as I’ve defined it, is concerned with the exhaustive interpretation: there are no epistemic obligations to be such that someone else has

224 First, the musher might go permissivist. They might say [.2, .8] and [.2, .80001]
 225 are both rationally permissible, given the relevant total evidence. We don't need to
 226 justify adopting one over the other.

227 The problem: going permissivist undermines the most basic motivations for the
 228 musher position. The permissivist sharper can equally say: when the evidence is
 229 murky, more than one precise credence is rationally permitted. The musher was
 230 right, the permissivist sharper allows, that imprecise evidence generated an
 231 epistemic situation where any precise credence would be arbitrary. But any
 232 imprecise credence would also be arbitrary. So, thus far we have no argument that
 233 imprecise credences are epistemically better. (This may be why, as a sociological
 234 fact, mushers tend not to be permissivists).

235 Moreover, the problem still re-emerges for the imprecise permissivist: which
 236 credence functions are rationally permissible, which are rationally impermissible,
 237 and what could possibly ground a sharp cutoff between the two? Suppose the
 238 greatest permissible upper bound for an imprecise credence in the toothpaste case is
 239 .8. What distinguishes permissible imprecise credences with an upper bound of .8
 240 from impermissible imprecise credences with an upper bound of .80001? (You
 241 might say: go radically permissive; permit all possible sets of credences. But this is
 242 to abandon the musher view, since precise credences will be rationally permissible.
 243 Otherwise, what justifies permitting set-valued credences of cardinality 2 while
 244 forbidding those with cardinality 1?)

245 Second, the musher might say that in cases like the Jellyfish case, there's only
 246 one rationally permissible credence, and it's [0, 1]. The only way it would be
 247 permissible to form any narrower credence would be if you knew the objective
 248 chances.

249 Problem: given how rarely we ever have information about objective chances of
 250 unknown events—personally, I've never seen a perfectly fair coin—virtually all of
 251 our uncertain credences would be [0, 1]. Surely rational thinkers can be more
 252 confident of some hypotheses over others without knowing anything about chances.

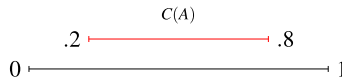
253 One might object: even with ordinary coins, you do know that the chance of
 254 heads is not .9. So you needn't have maximally imprecise credences. Reply: there's
 255 no determinate cutoff between chances of heads I can rule out and those I can't. So
 256 the musher still must explain how a unique imprecise credence could be rationally
 257 required, or how a unique set of imprecise credences could be rationally
 258 permissible. Musers would still rely unjustifiably on sharp cutoff points in their
 259 models—cutoffs that correspond to nothing in the epistemic norms.

260 2.2 A solution: vague credences

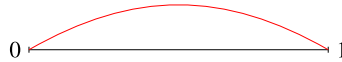
261 Finally, the musher might say that set-valued credences are a simplifying
 262 idealization. Rational credences are not only imprecise but also vague. Instead of
 263 having sharp cutoffs, as intervals and other sets do—

Footnote 18 continued

incomplete information about one's belief state. Walley offers no theory of (complete) imprecise credences that is not susceptible to arbitrariness worries.



—a more accurate way of modeling rational credences would represent a gradual drop-off.



266 The sharp cutoff was generated by the conception of imprecise credences as
 267 simply representable by sets of real numbers. Set membership creates a sharp cutoff:
 268 a probability is either in or out. But evidence is often too murky: there isn't a bright
 269 line between probabilities ruled out by the evidence and probabilities ruled in.

270 So imprecise credences should be represented with some sort of mathematical
 271 object more structured than mere sets of probabilities. We need a model that
 272 represents their vagueness. The natural way to achieve this is to introduce a
 273 weighting. We should allow some probabilities to have more weight than others. In
 274 the example above, probabilities nearer to 0 or 1 receive less weight than
 275 probabilities closer to the middle.¹⁹

276 What does the weighting represent? It should just represent a vague version of
 277 what set membership represents for non-vague imprecise credences.²⁰ What is
 278 represented about an agent by the claim that a probability function c is in an agent's
 279 representor C ? Joyce (2010) writes: "Elements of C are, intuitively, probability
 280 functions that the believer takes to be compatible with her total evidence" (288).

281 It's not clear what Joyce means by "compatible with her total evidence." He
 282 certainly can't mean that these probability functions are rationally permissible for
 283 the agent to adopt as her credence function. After all, the elements of C are all
 284 precise.

285 So we have to interpret "compatible with total evidence" differently from
 286 "rational to adopt." Leave it as an open question what it means for the moment. If
 287 imprecise credences are to be vague, we should assume this "compatibility" comes
 288 in degrees. Then we let the weighting over probabilities represent degrees of
 289 "compatibility."

290 Note that imposing some structure over imprecise credences can solve problems
 291 beyond the arbitrariness challenge I've presented. One of the big challenges for fans
 292 of imprecise credences is to give a decision theory that makes agents behave in
 293 ways that are intuitively rational. An uncontroversial decision rule, E-Admissibility,
 294 permits agents to choose any option with maximal expected utility according to at
 295 least one probability in her representor. But, as Elga (2010) argues, E-admissibility

¹⁹ Of course, the distribution of weights need not be symmetric or smooth.

²⁰ More accurately, it should represent the non-vague grounding base for the vague version of what set membership represents.

296 permits agents to accept a variant of a diachronic Dutch book. Generating a decision
297 rule that avoids this problem is difficult.²¹

298 The added structure of weighted imprecise credences makes Elga's pragmatic
299 challenge easier to avoid. Probabilities that have greater weight within an imprecise
300 credence plausibly have more effect on what actions are rational for an agent to
301 perform. Probabilities with less weight have less bearing on rational action. We can
302 use the weighting to generate a weighted average of the probabilities an imprecise
303 credence function assigns to each possible state of the world. Then our decision rule
304 could be to maximize expected utility according to the weighted average
305 probabilities.²²

306 Summing up: we've pieced together a model for imprecise credences that
307 insulates the musher view from the same arbitrariness worries that beleaguer precise
308 credences. We did so by representing imprecise credences as having added
309 structure: instead of mere sets of probabilities, they include a weighting over those
310 probabilities. Indeed, the sets themselves become redundant: whichever probabil-
311 ities are definitively ruled out will simply receive zero weight. All we need is a
312 weighting over possible probability functions.

313 So far so good. But there is a problem. How should we interpret this weighting?
314 Where does it fit into our psychology of credences?

315 We introduced the weighting to reflect what Joyce calls "compatibility with the
316 evidence." That property isn't all-or-nothing; it admits of borderline cases. The
317 weighting reflects "degrees of compatibility." But we already had something to fill
318 this role in our psychology of credences, independently of imprecision. The ideally
319 rational agent has credences in propositions about compatibility with the evidence.

320 And so the sharper will conclude: this special weighting over probability
321 functions is just the agent's credence in propositions about what credences the
322 evidence supports. In other words, it represents the agent's normative uncertainty
323 about rationality. A probability gets more weight just in case the agent has higher
324 credence that it is compatible with the evidence.²³

325 The structured musher view adds nothing new. It's just an unnecessarily
326 complicated interpretation of the standard precise credence model.

327 Now, there might be other strategies that the musher can pursue for responding to
328 the arbitrariness worry. Perhaps there is some fundamental difference in the mental
329 states associated with the weighted musher view and its sharper cousin. I don't
330 claim to have backed the musher into a corner.

²¹ Γ -Maximin—the rule according to which one should choose the option that has the greatest minimum expected value—is not susceptible to Elga's objection. But that decision rule is unattractive for other reasons, including the fact that it sometimes requires turning down cost-free information (see Seidenfeld 2004; thanks to Seidenfeld for discussion). Still other rules, such as Weatherson's (2008) rule "Caprice" and Williams's (2014) rule "Randomize," are committed to the controversial claim that what's rational for an agent to do depends not just on her credences and values, but also her past actions.

²² For this to work out as an expectation, we'll need to normalize the weighting such that the total weights sum to 1. Assuming the weighted averages are probabilistic—a plausible constraint on the weighting—the resulting recommended actions will be rational (or anyway not Dutch-bookable).

²³ The idea of reassessing imprecise evidence with higher-order probabilities is addressed in Savage (1972), 81 and Walley (1991), 258–261.

331 I draw attention to the strategy of imposing weightings over probabilities in
 332 imprecise representors for two reasons. First, I think it's the simplest and most
 333 intuitive strategy the musher can adopt for addressing the arbitrariness challenge.
 334 Second, it brings into focus an observation that I think is important in this debate,
 335 and that has been so far widely ignored: uncertainty about what's rational can play
 336 the role that imprecise credences were designed for. Indeed, I'm going to try to
 337 convince you that it can play that role even better than imprecise credences can.

338 3 The precise alternative

339 3.1 The job of imprecise credences

340 Before showing how this sort of sharper view can undermine the motivations for
 341 going imprecise, let's see what exactly the view amounts to.

342 Instead of having imprecise credences, I suggested, the rational agent will have a
 343 precise credence function that includes uncertainty about its own rationality and
 344 about the rationality of alternative credence functions. The weighting of each
 345 credence function c is just the agent's credence that c is rational.

346 Take the set of probability functions that might be compatible with the evidence:
 347 c_1, c_2, \dots, c_n . The musher claims you should have the belief state $\{c_1, c_2, \dots, c_n\}$.
 348 The sharper view I want to defend says: you should have specific attitudes toward
 349 c_1, c_2, \dots, c_n . A key difference: the musher demands a special kind of attitude (a
 350 special vehicle of content). The sharper suggests an ordinary attitude toward a
 351 particular kind of content.

352 Imprecise credences are thought to involve a particular kind of uncertainty:
 353 whatever form of uncertainty is appropriate to maintain in the face of ambiguous or
 354 unspecific evidence. It is often said that whatever this form of uncertainty makes it
 355 impossible to assign a probability to some proposition. I'll discuss three forms of
 356 uncertainty about probability that may fit the bill.

357 First: uncertainty about objective probabilities.²⁴ The Mystery Coin case is a case
 358 where objective probabilities are unknown. In such cases, one's uncertainty goes
 359 beyond the uncertainty of merely not knowing what will happen: for example,
 360 whether the coin will land heads.

361 Second, uncertainty about subjective probabilities. Examples like the Jellyfish
 362 case reveal that it can sometimes be difficult for ordinary agents like us to introspect
 363 our own credences. If I do, in fact, assign a precise credence to the proposition that
 364 the next item from the bag will be a tube of toothpaste, I have no idea what that
 365 credence is. Similarly if I assign an imprecise credence: I still don't know. Of
 366 course, agents like us have introspective limitations that ideally rational agents may
 367 not have. It's not obvious what ideal rationality requires: it might be that in some

²⁴ In economics, it's common to distinguish "risk" and "uncertainty," in the sense introduced in Knight (1921). Knightian "risk" involves known (or knowable) objective probabilities, while Knightian "uncertainty" involves unknown (or unknowable) objective probabilities. This is not the ordinary sense of "uncertainty"—i.e. the state of not being certain—that I use throughout this paper.

368 cases, rational agents should be uncertain about what their own credences are.²⁵ I
 369 leave this as an open possibility.

370 Finally, uncertainty about epistemic probabilities. In some cases, it's unclear
 371 which way the evidence points. Epistemic probabilities are probabilities that are
 372 rationally permissible to adopt given a certain body of evidence.²⁶ They
 373 characterize the credences, or credal states, that are rational relative to that
 374 evidence. In the Jellyfish example, it's hard to see how one's evidence could point to
 375 a unique precise assignment of probability. It's also hard to see how it could point to
 376 a unique imprecise assignment of probability. Indeed, the evidence may not
 377 determinately point at all.

378 So: there are three factors that may be responsible for the intuition that, in cases
 379 like Jellyfish and Mystery Coin, the evidence is imprecise: uncertainty about
 380 objective probabilities, uncertainty about subjective probabilities, and uncertainty
 381 about epistemic probabilities. Some cases of (putative) imprecise evidence may
 382 involve more than one of these forms of uncertainty. My primary focus will be on
 383 objective and epistemic probabilities.

384 3.2 Precise credences do the job better

385 On the musher view, I suggest, rational uncertainty about the objective and
 386 epistemic probabilities of a proposition *A* are reflected in an agent's attitude toward
 387 *A*. But plausibly, the rational agent has doxastic attitudes not just toward *A*, but also
 388 toward propositions about the objective and epistemic probabilities of *A*. When
 389 evidence is imprecise, she will also be uncertain about these other propositions.

390 In the case of objective probabilities this should all be uncontroversial, at least to
 391 the friend of precise credences. But I want to defend a more controversial claim: that
 392 genuinely imprecise evidence, if there is such a thing, demands uncertainty about
 393 epistemic probabilities. This is a form of normative uncertainty: uncertainty about
 394 what's rational. The norms of rationality are not wholly transparent even to ideally
 395 rational agents.

396 It's compatible with even ideal rationality for an agent to be uncertain about
 397 what's rational [this thesis, which sometimes goes under the name **Modesty**, is
 398 defended in Elga (2007)].²⁷ An ideally rational agent can even be somewhat
 399 confident that her own credences are not rational. For example: suppose a rational
 400 agent is faced with good evidence that she tends to be overconfident about a certain
 401 topic and good competing evidence that she tends to be underconfident about that
 402 same topic. The appropriate response to this sort of evidence is to become relatively

²⁵ A brief argument: introspection may be a form of perception (inner sense). Our perceptual faculties sometimes lead us astray. Whether introspection is a form of perception is arguably empirical. Rationality doesn't require certainty about empirical psychology. So it's possible that ideal rationality doesn't require perfect introspection. And it's possible that ideal rationality does require perfect introspection but doesn't require ideal agents to know that they can introspect perfectly.

²⁶ Note: this is not an interpretation of epistemic probability that presupposes objective Bayesianism.

²⁷ Modesty is further discussed in Christensen (2010), Williamson (2007), Elga (2013), Lasonen-Aarnio (2014), and Sliwa and Horowitz (2015).

403 confident that her credences do not match the epistemic probabilities (if the two
 404 competing forms of evidence are balanced, she won't need to adjust her first-order
 405 credences at all. And no adjustment of them will reassure her that she's rational). So
 406 a rational agent may not know whether she has responded correctly to her
 407 evidence.²⁸

408 I claim that imprecise evidence, if it exists, requires uncertainty about epistemic
 409 probabilities. This is, of course, compatible with continuing to have precise
 410 credences.²⁹ Once we take into account uncertainty about objective and epistemic
 411 probabilities, it's hard to motivate imprecision.

412 For example, consider the argument Joyce (2010) uses to support imprecise
 413 credences in the Mystery Coin case:

414 f_U [the probability density function determined by POI] commits you to
 415 thinking that in a hundred independent tosses of the [Mystery Coin] the
 416 chances of [heads] coming up fewer than 17 times is exactly 17/101, just a
 417 smidgen (= 1/606) more probable than rolling an ace with a fair die. Do you
 418 really think that your evidence justifies such a specific probability assignment?
 419 Do you really think, e.g., that you know enough about your situation to
 420 conclude that it would be an unequivocal mistake to let \$100 ride on a fair die
 421 coming up one rather than on seeing fewer than seventeen [heads] in a
 422 hundred tosses? (284)

423 Answer: No. If the Mystery Coin case is indeed a case of imprecise evidence,
 424 then I may be uncertain about whether my evidence justifies this (epistemic)
 425 probability assignment. If so, I will be uncertain about whether it would be an
 426 unequivocal mistake to bet in this way. After all, whether it's an unequivocal
 427 mistake is determined by what credences are rational, not whatever credences I
 428 happen to have.

429 A rational agent might independently know what's rational to believe. She might
 430 know the higher-order chances; she might learn from the Oracle what's rational.³⁰ If
 431 the Oracle tells an agent that a certain precise credence is rational, then the agent
 432 should adopt that credence. In such cases, there's no good reason to think of the
 433 evidence as imprecise. There's nothing murky about it. Chances are just one of the
 434 many facts that evidence might leave unsettled.

435 So: contra the musher orthodoxy, I conclude that evidence that underdetermines
 436 chances isn't necessarily imprecise evidence. What is distinctive about imprecise

²⁸ There are also some cases, too complex to discuss here, where an ideally rational agent might simply not be in a position to know what her evidence is, and therefore be uncertain about epistemic probabilities. See (Williamson 2007; Christensen 2010), and (Elga 2013).

²⁹ A caveat: it's compatible with the view I'm defending that there are no such bodies of evidence. It might be that every body of evidence not only supports precise credences, but supports certainty in the rationality of just those precise credences.

³⁰ It might even be that the Mystery Coin example is not really an example of a case where it's not clear what credence to have. Credence .5 is the obvious candidate, even without the principle of indifference to bolster it. If you had to bet on heads in the Mystery Coin case, I suspect you'd bet as though you had credence .5.

437 evidence? I claim: the distinctive feature of imprecise evidence is that it rationalizes
438 uncertainty about rationality.

439 Whatever imprecise evidence might require of our attitudes about what's
440 rational, it only places a weak constraint: uncertainty. A question looms: *which*
441 precise credences are rational? After all, the musher's initial challenge to the sharper
442 was to name any first order credences in the Jellyfish case that could seem rationally
443 permissible.

444 The kind of view I'm advocating does not and should not offer an answer to that
445 question. The view is that, in cases of imprecise evidence, rationality requires
446 withholding judgment about which credences are rational. The view would be self-
447 undermining if it also tried to answer the question of what credences are rational.³¹

448 One might wonder: how does normative uncertainty of this sort translate into
449 rational action? How should I decide how to behave when I'm uncertain what's
450 rational? The most conservative view is that normative uncertainty doesn't affect
451 how it's rational to act, except insofar as it constrains what first-order credences are
452 rational.³² A simple example: suppose you are offered a bet about whether *A*. Then
453 you should choose on the basis of your credence in *A*, not your credence in the
454 proposition that your credence in *A* is rational. Of course, if it's rational to be
455 uncertain about whether your credences are rational, then even though your
456 decisions are rational, you might not be in a position to know that they are.³³

457 Now we have a sharper view on the table. This view shows the same sensitivity
458 to imprecise evidence as the musher view. It does all the epistemic work that the
459 musher view was designed for. What's more, the best response that the musher can
460 give to address arbitrariness worries and decision theoretic questions will in effect
461 collapse the musher view into a notational variant of this sharper view.

462 So are there any reasons left to go imprecise? In the remainder of this paper, I'm
463 going to argue that there aren't.

³¹ I'll mention some possible constraints that uncertainty about rationality places on our other credences. What's been said so far has been neutral about whether there are *level-bridging norms*: norms that link one's beliefs about what's rational with what is in fact rational. But a level-bridging response, involving something like Christensen's (2010) principle of Rational Reflection, is a live possibility (see Elga 2013) for a refinement of the principle). According to this principle, our rational first-order credences should be a weighted average of the credences we think might be rational (on Elga's version, conditional on their own rationality), weighted by our credence in each that it is rational. This principle determines what precise probabilities an agent should have when she is rationally uncertain about what's rational.

Note, however, that a principle like this won't provide a recipe to check whether your credences are rational: whatever second-order credences you have, you may also be uncertain about whether your second-order credences are the rational ones to have, and so on. And so this kind of coherence constraint doesn't provide any direct guidance about how to respond to imprecise evidence.

³² And except in the rare case where, e.g., you're betting on propositions about what's rational.

³³ Thanks to Julia Staffel for pressing me on this point.

464 **4 Undermining imprecision**

465 There is a lot of pressure on mushers to move in the direction of precision. For
 466 example, there is pressure for mushers to provide decision rules that make agents
 467 with imprecise credences behave as though they have precise credences. Preferences
 468 that are inconsistent with precise credences are often intuitively irrational.³⁴ Various
 469 representation theorems purport to show that if an agent's preferences obey intuitive
 470 rational norms, then the agent can be represented as maximizing expected utility
 471 relative to a unique precise probability function and utility function (unique up to
 472 positive affine transformation).

473 Suppose rational agents must act as though they have precise credences. Then on
 474 the widely presupposed interpretivist view of credences—that whatever credences
 475 the agent has are those that best explain and rationalize her behavioral
 476 dispositions—the game is up. As long as imprecise credences don't play a role in
 477 explaining and rationalizing the agent's behavior, they're a pointless
 478 complication.³⁵

479 But the musher might simply reject interpretivism. Even if rational agents are
 480 disposed to act as though they have precise credences in all possible situations, the
 481 musher claims, epistemic rationality nevertheless demands that they have imprecise
 482 credences. Imprecise credences might play no role in determining behavior. Their
 483 role is to record the imprecision of one's evidence.

484 This bullet might be worth biting if we had good reason to think that epistemic
 485 norms in fact do require having imprecise credences. So the big question is: is there
 486 any good motivation for the claim that epistemic norms require imprecise
 487 credences?

488 I'm going to argue that the answer is *no*. Whatever motivations there were for the
 489 musher view, they equally support the sharper view that I've proposed: that
 490 imprecise evidence requires uncertainty about rationality. I'll consider a series of
 491 progressive refinements of the hypothesis that imprecise evidence requires
 492 imprecise credences. I'll explain how the motivations for each can be accommo-
 493 dated by a sharper view that allows for uncertainty about epistemic and objective
 494 probabilities. This list can't be exhaustive, of course. But it will show that the
 495 musher view has a dangerously low ratio of solutions to problems.

³⁴ Again, see Elga (2010).

³⁵ Hájek and Smithson (2012) argue that interpretivism directly favors modeling even ideal agents with imprecise credences. After all, a finite agent's dispositions won't determine a unique probability function/utility function pair that can characterize her behavioral dispositions. And this just means that all of the probability/utility pairs that characterize the agent are equally accurate. So, doesn't interpretivism entail at least the permissibility of imprecise credences? I find this argument compelling. But it doesn't tell us anything about epistemic norms. It doesn't suggest that evidence ever makes it rationally required to have imprecise credences. And so this argument doesn't support the musher view under discussion.

496 **4.1 Do precise credences reflect certainty about chances?**

497 In motivating their position, mushers often seem to identify precise credences with
 498 full beliefs about objective probabilities. Here's an example:

499 A...proponent of precise credences...will say that you should have some sharp
 500 values or other for [your credence in drawing a particular kind of ball from an
 501 urn], thereby committing yourself...to a **definite view about the relative**
 502 **proportions** of balls in your urn...Postulating sharp values for [your
 503 credences] under such conditions amounts to pulling **statistical correlations**
 504 out of thin air. (Joyce 2010, 287, boldface added)

505 Another example:³⁶

506 [H]aving precise credences requires having **opinions** that are so rich and
 507 specific that they pin down a single estimate $c(X)$ of the truth-value of every
 508 proposition X that you are aware of. And this really is *incredibly* rich and
 509 specific. It means, *inter alia*, that your **comparative beliefs—judgments** of the
 510 form X is at least as probable as Y —must be *total*. That is, you must either
 511 **think** X is at least as probable as Y , or vice versa, for any propositions X and
 512 Y that you are aware of. **No abstaining from judgment**. Same goes for your
 513 **conditional comparative beliefs**. You must either **think that** X given D is at
 514 least as probable as Y given D' , or vice versa, for any propositions X and Y , and
 515 any potential new data, D and D' . Likewise for your preferences. You must
 516 either **think that** bet A is at least as choiceworthy as bet B , or vice versa, for
 517 any A and B . (Konek, forthcoming, 2, boldface added)

518 Joyce and Konek seems to be endorsing the following principle:

519 **CREDENCE/CHANCE:** having credence n in A is the same state as, or
 520 otherwise necessitates, having credence 1 that the chance of A is (or at some prior
 521 time was) n .³⁷

522 A flatfooted objection seems sufficient here: one state is a partial belief, the content
 523 of which isn't about chance. The other is a full belief about chance. So surely they
 524 are not the same state.

525 More generally: whether someone takes a definite stance about the chance of
 526 A isn't the kind of thing that can be read locally off of her credence in A . There are
 527 global features of an agent's belief state that determine whether that credence
 528 reflects some kind of definite stance or whether it simply reflects a state of
 529 uncertainty.

³⁶ Konek is more circumspect about the kind of probability at issue. My objections to this view apply equally well if we sub some other form of probability in for chance.

³⁷ This is a relative of what White (2009) calls the *Chance Grounding Thesis*, which he attributes to a certain kind of musher: "Only on the basis of known chances can one legitimately have sharp credences. Otherwise one's spread of credence should cover the range of chance hypotheses left open by your evidence" (174).

530 For example, in the Mystery Coin case, someone whose credence in HEADS is .5
 531 on indifference grounds will have different introspective beliefs and different beliefs
 532 about chance from someone who believes that the objective chance of HEADS is .5.
 533 The former can confidently believe: *I don't have any idea what the objective chance*
 534 *of HEADS is; I doubt the chance is .5*; etc. Obviously, neither is rationally
 535 compatible with taking a definite position that the chance of HEADS is .5.

536 Another example: a credence $c(A) = n$ doesn't always encode the same degree of
 537 *resiliency* relative to possible new evidence. The resiliency of a credence is the
 538 degree to which it is stable in light of new evidence.³⁸ When an agent's credence in
 539 A is n because she believes the chance of A is n , that credence is much more
 540 stubbornly fixed at n . Precise credences formed under uncertainty about chances are
 541 much less resilient in the face of new evidence.³⁹ For example, if you learn that the
 542 last three tosses of the coin have all landed heads, you'll substantially revise your
 543 credence that the next coin will land heads. (After all, three heads is some evidence
 544 that the coin is biased toward heads). But if your .5 credence comes from certainty
 545 that the chance of heads is .5, then your credence should be highly resilient in
 546 response to this evidence.

547 In short: the complaint against precise credences that Joyce seems to be offering
 548 in the passages quoted above hinges on a false assumption: that having a precise
 549 credence in a hypothesis A requires taking a definite view about chance. Whether an
 550 agent takes a definite view about the chance of A isn't determined by the precision
 551 of her credence in A .

552 For rational agents who have credences in propositions about chances and about
 553 rationality, imprecise credences are *redundant*. They represent a form of uncertainty
 554 about objective probability that's already represented in the agent's attitudes toward
 555 other propositions.

556 The imprecise representation of uncertainty about probabilities is not merely
 557 redundant; it's also a worse representation than higher-order credences. Consider
 558 the Mystery Coin case. Using imprecise credences in heads to represent uncertainty
 559 about objective chances is effectively to revert to the binary belief model. It is to
 560 treat uncertainty about whether the coin lands heads as a binary belief about the
 561 objective chance of heads. The belief rules out every epistemically impossible
 562 chance, rules in every epistemically possible chance, and incorporates no
 563 information whatsoever about the agent's relative confidence in each chance
 564 hypothesis. This loss of information can be repaired only by introducing higher-
 565 order credences or credal states, which leaves the first-order imprecision otiose.

566 4.2 Are precise credences are "too informative"?

567 The second motivation for imprecise credences is a generalization of the first. Even
 568 if precise credences don't encode full beliefs about objective probabilities, they still
 569 encode some information that they shouldn't.

³⁸ See Skyrms (1977).

³⁹ See also White (2009), 162–164.

570 Even if one grants that the uniform density is the *least* informative sharp
 571 credence function consistent with your evidence, it is still *very* informative.
 572 Adopting it amounts to pretending that you have lots and lots of information
 573 that you simply don't have. (Joyce 2010, 284)
 574 If you regard the chance function as indeterminate regarding X , it would be
 575 odd, and arguably irrational, for your credence to be any sharper...How would
 576 you defend that assignment? You could say "I don't *have* to defend it—it just
 577 happens to be my credence." But that seems about as unprincipled as looking
 578 at your sole source of information about the time, your digital clock, which
 579 tells that the time rounded off to the nearest minute is 4:03—and yet believing
 580 that the time is in fact 4:03 *and 36 seconds*. Granted, you may just happen to
 581 believe that; the point is that you have no business doing so. (Hájek and
 582 Smithson 2012, 38–39)

583 Something of this sort seems to underpin a lot of the arguments for imprecise
 584 credences.

585 There's a clear sense in which specifying a set of probability functions can be
 586 less informative than specifying a unique probability function. But informative
 587 about what? The argument in Sect. 4.1 generalizes. The basic challenge to the
 588 musher is the same: if they claim that precise credence $c(A)$ is too informative about
 589 some topic B , then they need to explain why it isn't adequate to pair $c(A)$ with total
 590 suspension of judgment about B .

591 Imprecise credences do encode less information than precise credences. But we
 592 should ask:

- 593 (1) What kind of information is encoded?
 594 (2) Is it irrational to encode that information?

595 The answers: (1) information about agents' mental states, and (2) no.

596 Precise credences are not more informative about things like coin tosses. Rather,
 597 *ascriptions* of precise credences are more informative about the psychology and
 598 dispositions of the agent. This is third-personal, theoretical information about an
 599 agent's attitudes, not information in the contents of agent's first-order attitudes.⁴⁰

600 Suppose you learn that I have credence .5 in A . Then you know there's a precise
 601 degree to which I'm disposed to rely on A in my reasoning and decisions on how to
 602 act. Just as there is a precise degree to which I feel pain, or a precise degree to which
 603 I am hungry, on some psychologically appropriate scale.

604 Of course, we don't always have access to these properties of our psychology.
 605 But it's reasonable to expect that our psychological states are precise, at a time, at
 606 least up to a high degree of resolution. I might have the imprecise degree of pain
 607 [.2, .21], but I won't have the imprecise degree [0, 1] on a zero-to-one scale. What
 608 imprecision there might be results from the finitude of bodies. I claim the same is
 609 true of our credences. And in both cases, it is not out of the question that these

⁴⁰ Of course, the rational agent may ascribe herself precise or imprecise credences and so occupy the theorist's position. But in doing so, the comparative informativeness in her ascription of precise credences is still informativeness about her own psychological states, not about how coin-tosses might turn out.

610 quantities are precise. Finite bodies can determine precise quantitative measure-
611 ments: for example, blood pressure.⁴¹

612 Precise credences provide unambiguous and specific information about rational
613 agents' psychology and dispositions. But why would there be anything wrong with
614 being informative or unambiguous or specific in this way?

615 *Objection 1.* In the Mystery Coin case, if your credence in heads is .5, then you
616 are committed to claiming: "The coin is .5 likely to land heads." That counts as
617 information about the coin that the agent is presuming to have. Any precise
618 credence will commit an agent to some claim about likelihood; imprecise credences
619 do not.

620 *Reply.* What does it mean, in this context, to affirm: "The coin is .5 likely to land
621 heads"? It doesn't mean that you think the *objective probability* of the coin landing
622 heads is .5; you don't know whether that's true. It doesn't even mean that you think
623 the *epistemic probability* of the coin landing heads is .5; you can be uncertain about
624 that as well.

625 In fact, "It's .5 likely that heads" doesn't express any commitment at all. It just
626 expresses the state of having .5 credence in the coin's landing heads.⁴² But then the
627 .5 part doesn't tell us anything about the coin. It just characterizes some aspect of
628 your psychological state.

629 *Objection 2.* If the evidence for a proposition *A* is genuinely imprecise, then there
630 is some sense in which adopting a precise credence in *A* means not withholding
631 judgment where you really ought to.

632 *Reply.* If an agent's credence in *A* is not close to 0 or 1, then the agent is
633 withholding judgment about whether *A*. That's just what withholding judgment is.
634 The musher seems to think that the agent should double down and withhold
635 judgment *again*.

636 In short: there's just no reason to believe that imprecise evidence requires
637 imprecise credences. Why should our attitudes be confusing or messy just because
638 the evidence is? (If the evidence is unimpressive, that doesn't mean our credences
639 should be unimpressive). What is true is that imprecise evidence should be reflected
640 in our credences somehow or other. But that can amount to simply believing that the
641 evidence is ambiguous and unspecific, being uncertain what to believe, having non-
642 resilient credences, and so on.

643 5 Evidential indeterminacy

644 5.1 Imprecise evidence as indeterminate confirmation

645 We'll consider one final refinement of the motivation for going mushy. This
646 refinement involves breaking down the claim that precise credences are overly
647 informative into two premises:

⁴¹ Thanks to Chris Meacham for discussion and to Graham Oddie for this example.

⁴² Cf. Yalcin (2012).

648 **Imprecise Confirmation:** The confirmation relation between bodies of
 649 evidence and propositions is imprecise.
 650 **Strict Evidentialism:** Your credences should represent only what your
 651 evidence confirms.

652 These two claims might be thought to entail the musher view.⁴³

653 According to the first premise, for some bodies of evidence and some
 654 propositions, there is no unique precise degree to which the evidence supports the
 655 proposition. Rather, there are multiple precise degrees of support that could be used
 656 to relate bodies of evidence to propositions. This, in itself, is not a claim about
 657 rational credence, any more than claims about entailment relations are claims about
 658 rational belief. So in spite of appearances, this is not simply a denial of the sharper
 659 view, though the two are tightly related.

660 In conjunction with Strict Evidentialism, though, it might seem straightforwardly
 661 impossible for the sharper to accommodate Imprecise Confirmation.

662 Of course, some sharpeners—in particular, objective Bayesians—will consider it no
 663 cost at all to reject Imprecise Confirmation. They consider this a fundamental
 664 element of the sharper view, not some extra bullet that sharpeners have to bite.

665 But whether rejecting Imprecise Confirmation is a bullet or not, sharpeners don't
 666 have to bite it. The conjunction of Imprecise Confirmation and Strict Evidentialism
 667 is compatible with the sharper view.

668 It's clear that Imprecise Confirmation is compatible with one form of the sharper
 669 view: precise permissivism, e.g. subjective Bayesianism. If there are multiple
 670 probability functions that each capture equally well what the evidence confirms,
 671 then according to precise permissivists, any of them is permissible to adopt as one's
 672 credence function. Permissivism was essentially designed to accommodate Impre-
 673 cise Confirmation.

674 Some mushers might think that adopting a precise credence function entails
 675 violating Strict Evidentialism. But this requires the assumption that precise
 676 credences are somehow inappropriately informative. I've argued that this assump-
 677 tion is false. Precise permissivism is compatible with both claims.

678 Perhaps more surprisingly, precise impermissivism is also compatible with both
 679 claims. If Imprecise Confirmation is true, then some bodies of evidence fail to
 680 determine a unique credence that's rational in each proposition. And so epistemic
 681 norms sometimes don't place a determinate constraint on which credence function is
 682 rational to adopt. But this doesn't entail that the epistemic norms require adopting
 683 imprecise credences. It might just be that in light of some bodies of evidence,
 684 epistemic norms don't place a determinate constraint on our credences.

685 Suppose this is right: when our evidence is imprecise, epistemic norms
 686 underdetermine what is rationally required. Instead of determinately permitting
 687 multiple credence functions, though, we interpret the norms as simply underdeter-
 688 mining the normative status of those credence functions. They are neither
 689 determinately permissible nor determinately impermissible. This is compatible

⁴³ Thanks to Wolfgang Schwarz, R.A. Briggs, and Alan Hájek for suggesting this formulation of the motivation.

690 with the sharper view: it could be supervaluationally true that our credences must be
 691 precise. Moreover, this is compatible with impermissivism: it could be supervaluationally
 692 true that only one credence function is permissible. How could it be
 693 indeterminate what rationality requires of us? There are cases where morality and
 694 other sorts of norms don't assign determinate normative statuses. Here is an
 695 analogy: suppose I promise to pay you a £100 if you wear neon yellow trousers to
 696 your lecture tomorrow. You bashfully show up wearing trousers that are a
 697 borderline case of neon yellow. Am I bound by my promise to pay you £100?⁴⁴

698 According to the precise permissivist, it's determinately permissible for me not to
 699 pay you, and it's also determinately permissible for me to pay you. According to the
 700 precise impermissivist, it's indeterminate whether I'm obligated to pay you.
 701 According to the musher, it's determinately obligatory that I somehow indetermi-
 702 nately pay—perhaps by hiding £100 somewhere near you?

703 The upshot is clear: Indeterminacy in obligations doesn't entail an obligation to
 704 indeterminacy.⁴⁵

705 Analogously: even if epistemic norms underdetermine what credences are
 706 rational, it might still be the case that there's a unique precise credence that's
 707 rational to adopt in light of our evidence. It's just indeterminate what it is.

708 5.2 Indeterminacy and uncertainty

709 A common concern is that there's some analogy between the view I defend and
 710 epistemicism about vagueness. Epistemicism is the view that vague predicates like
 711 "neon yellow" have perfectly sharp extensions. We simply don't know what those
 712 extensions are; and this ignorance explains away the appearance of indeterminacy.

⁴⁴ Roger White suggested an analogous example to me in personal communication.

⁴⁵ This point extends to another argument that has been given for imprecise credences. According to Hájek and Smithson (2012), there could be *indeterminate chances*, so that some event E 's chance might be indeterminate—not merely unknown—over some interval like $[.2, .5]$. This might be the case if the relative frequency of some event-type is at some times .27, at others .49, etc.—changing in unpredictable ways, forever, such that there is no precise limiting relative frequency. Hájek & Smithson argue that the possibility of indeterminate objective chances, combined with a generalization of Lewis's Principal Principle, yields the result that it is rationally required to have imprecise credences. Hájek & Smithson suggest that the following generalization of the Principal Principle captures how we should respond to indeterminate chances:

PP* Rational credences are such that $C(A | Ch(A) = [n, m]) = [n, m]$ (if there's no inadmissible evidence).

But there are other possible generalizations of the Principal Principle that are equally natural, e.g. PP†:

PP † Rational credences are such that $C(A | Ch(A) = [n, m]) \in [n, m]$ (if there's no inadmissible evidence).

The original Principal Principle is a special case of both. (Note that PP† only states a necessary condition on rational credences and not a sufficient one. So it isn't necessarily a permissive principle). Hájek & Smithson don't address this alternative, but it seems to me perfectly adequate for the sharper to use for constraining credences in the face of indeterminate chances. So it's not obvious that indeterminate chances require us to have indeterminate credences.

713 One might think that precise impermissivism with uncertainty about epistemic
 714 probabilities amounts to something like an epistemicism about imprecise evidence.
 715 Instead of allowing for the possibility of genuine indeterminacy, the thought goes,
 716 this view suggests we simply don't know which sharp credences are determinately
 717 warranted. Still, though, the credences that are determinately warranted are
 718 perfectly sharp.

719 But a sharper view that countenances genuine indeterminacy (that is, indeter-
 720 minacy that isn't merely epistemic) is fundamentally different from epistemicism
 721 about vagueness. The supervenient hypothesis I mentioned above is obviously
 722 analogous to supervenientism about vagueness. The supervenientist about
 723 vagueness can hold that, determinately, there is a sharp cutoff point between neon
 724 yellow and non-neon yellow; it just isn't determinate where that cutoff point lies.

725 Similarly, the precise impermissivist who accepts Imprecise Confirmation
 726 accepts that for any body of evidence, there is determinately a precise credence
 727 function one ought to have in light of that evidence; it's just indeterminate which
 728 precise credence function is the right one.

729 Is this proposal—that imprecise evidence fails to determine certain rational
 730 requirements—a competitor to the view I've been defending—that imprecise
 731 evidence requires normative uncertainty? It depends on an interesting theoretical
 732 question: what attitude we should take toward indeterminate propositions.

733 It might be that, for some forms of indeterminacy, if it's indeterminate whether
 734 *A*, then it's rational to be uncertain whether *A*. For example, perhaps it's rational to
 735 be uncertain whether a hummingbird is blue or green, even though you know that
 736 it's indeterminate. (Uncertainty simply involves middling credence: it doesn't
 737 commit you to thinking there's a fact of the matter). And perhaps it's rational to be
 738 uncertain about whether it's rational to assign a particular credence to a proposition
 739 when the normative status of that credence is indeterminate.

740 There are plenty of reasonable worries about this view, though. For example,
 741 *prima facie*, this seems to make our response to known indeterminacy indistin-
 742 guishable from our response to unknown determinate facts. It's outside the scope of
 743 this paper to adjudicate this debate.

744 If uncertainty is not a rational response to indeterminacy, then the two proposals
 745 I've defended—normative uncertainty and indeterminate norms—may be competi-
 746 tors. But if uncertainty is a rational response to indeterminacy, then the two fit
 747 together nicely. Similarly for if they are appropriate responses to distinct forms of
 748 imprecise evidence: evidence that warrants uncertainty about rationality and
 749 evidence that leaves indeterminate what is rationally required.

750 6 Conclusion

751 The musher claims that imprecise evidence requires imprecise credences. I've
 752 argued that this is false: imprecise evidence can place special constraints on our
 753 attitudes, but not by requiring our attitudes to be imprecise. The musher's view rests
 754 on the assumption that having imprecise credences is the only way to manifest a sort
 755 of epistemic humility in the face of imprecise evidence. But the forms of uncertainty

756 that best fit the bill have a natural home in the sharper framework. Imprecise
 757 evidence doesn't demand a new kind of attitude of uncertainty. It just requires
 758 ordinary uncertainty toward a particular group of propositions—propositions that
 759 we often forget rational agents can question.

760 The kind of sharper view I defend can accommodate all the intuitions that were
 761 taken to motivate the mushier view. So what else does going imprecise gain us? Only
 762 vulnerability to arbitrariness worries and decision theoretic challenges. Better to
 763 drop the imprecision. When it's unclear what the evidence supports, it's rational to
 764 question what to believe.

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