What kind of Intensional Logic do we really want/need?

Toward a Modal Metaphysics

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Some Definitions

metaphysics (noun — functioning as sing)

1. the branch of philosophy that deals with first principles, esp of being and knowing
2. the philosophical study of the nature of reality, concerned with such questions as the existence of God, the external world, etc
3. See: descriptive metaphysics
4. (popularly) abstract or subtle discussion or reasoning

descriptive metaphysics (noun — functioning as sing)

1. the philosophical study of the structure of how we think about the world

ontology (noun)

1. (philosophy) the branch of metaphysics that deals with the nature of being
2. (logic) the set of entities presupposed by a theory

Source: Collins English Dictionary - Complete & Unabridged - 2012

Meta-ontology concerns itself with the nature and methodology of ontology, with the interpretation and significance of ontological questions. The problem of **ontological commitment** is a problem in meta-ontology rather than ontology proper.
The **metaontologist** asks (among other things): *What entities or kinds of entity exist according to a given theory or discourse, and thus are among its ontological commitments?*

Source: Stanford Encyclopedia of Philosophy, Phillip Bricker - 2014

Zermelo-Fraenkel Set Theory

Wikipedia: *Zermelo–Fraenkel set theory* with the axiom of choice, named after mathematicians Ernst Zermelo and Abraham Fraenkel and commonly abbreviated **ZFC**, is one of several axiomatic systems that were proposed in the early twentieth century to formulate a theory of sets free of paradoxes such as Russell's paradox. Today **ZFC** is the standard form of axiomatic set theory and as such is the most common foundation of mathematics.

ZFC provides the **Ontology** for Mathematics.

The idea for having set theory as foundations is a **Metaphysics** for Mathematics.

If set theory is to be the ultimate framework, one is still left with the conceptual mystery of why all mathematical objects *should* be sets. Michael Hallett - 1986

Intensional Logic

Wikipedia: *Intensional logic* is an approach to predicate logic that extends first-order logic, which has quantifiers that range over the individuals of a universe (*extensions*), by additional quantifiers that range over terms that may have such individuals as their value (*intensions*). The distinction between extensional and intensional entities is parallel to the distinction between *sense* and *reference*.

Modal logic is historically the earliest area in the study of intensional logic, originally motivated by formalizing "*necessity*" and "*possibility*"
(recently, this original motivation belongs to alethic logic, just one of the many branches of modal logic).

Suggestion: As the two main **Wikipedia** articles are written by one Imre Ruzsa, members of this seminar may wish to edit or suggest expansions of the entries.

A Modal Set Theory?

Taking *Modal Logic* seriously, let's focus on on the very popular Lewis System S4 as giving us a clean theory of

metaphysical necessity

for the sake of experimentation. We expand the use of this logic from propositional — to predicate logic — to higher-order logic. Keeping in mind the differences between *extensional* and *intensional* predicates, we base the understanding of **MZF** on the plan of *iterating* the intensional powerset into the transfinite. What we need to show, then, is whether **MZF** can provide a rich enough ontology for *Intensional Logic*.

This was the set theory put forward in Scott's 2009/10 lectures at Stanford, Berkeley, Oxford, and Edinburgh.

Avoiding Possible Worlds

Possible-worlds semantics works well for ***modal propositional logic***.

For ***predicate logic***, however, there are many hard-to-answer questions:

- **What are individuals?**
- **How does identity between worlds behave?**
- **What is the relation between individuals and concepts?**
- **How best to pass to higher-order logic?**

The Project Proposal

Investigate how *other* Boolean Algebras, other than the powerset algebras, $\mathcal{P}(W)$, can be used to model modalities and abstract entities.

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