

1. Desiderata for a theory of properties

The aim is to develop a theory of properties, relations and propositions that will support the semantics of natural language.

Semantics provides a fundamental testing ground for a [philosophical] property theory because properties are the semantic counterparts of natural language expressions. For example, the sentence *John runs* says of John that he has or instantiates the property of running.

The three desiderata:

1.1 *Closure of properties under the logical connectives and quantifiers*

1.2 *Intensionality*. As far as properties are concerned, the view that their intensionality can be analyzed as functions from worlds into sets is at odds with the intuition that properties can be logically equivalent without being identical. For example, something is sold iff it is bought, but the properties *buy* and *sell* are not identical.

1.3 *Self-predication*. Unlike sets [e.g., as conceived in ZF], properties are not well-founded: they can truly be predicated of themselves. For example, the property of being *autoidentical* holds of itself. [But must worry about Russell’s heterological property.]

1.4 *Some views of the nature of properties*

The Fregean view (also Cocchiarella’s): properties are “incomplete” or “unsaturated” structures; an act of predication is their “completion” or “saturation”. C&T call the unsaturated structures information unit functions (by analogy with propositional functions).

On the other hand, properties also have an individual nature and as such can play the role of subjects in acts of predication. Represent this by positing individuals that are systematically correlated with information unit functions, i.e. form their nominalization. For example, *runs* is considered as an information unit function with the related noun phrase *running*, as in *running is fun*; that is the individual correlate of *runs*.

Other views: properties are a special sort of individuals, and those can be predicated of other individuals (e.g., Bealer). On this view we have a predication relation $\pi(x, y)$, “x has the property y”. Which role a property plays depends on its place in this relation. For example, *running is fun* would be represented by $\pi(\text{run}, \text{fun})$.

The property theory of C&T takes the Fregean perspective.

2. A theory of properties, relations and propositions

Sources for the theory PT_1 : Axiomatic part, Gilmore ('74) and Feferman ('84); model-theoretic part, Gupta ('82), Herzberger ('82) for theories of self-applicable truth. Carried out in detail in Turner '87.

[The Gupta-Herzberger 3-valued model is more complicated than Kripke's; why? And why not use the Aczel 2-valued model obtained from Kripke's as used in Feferman '84?]

2.1 The syntax of PT_1 .

Four basic sorts: u ("urelements"), nf ("nominalized functions"), i ("information units") and e ("entities", the universal sort). The only complex sort is $\langle e, e \rangle$ ("information unit functions"); functions whose values are information units are called "information unit functions".

The language contains the λ -operator, a nominalization operator $^{\wedge}$ and a predication operator $^{\cup}$ inverse to it. Finally, there is an operator $^!$ that forms information units from arbitrary entities; when applied to information units it is interpreted as asserting truth, and otherwise as falsity.

For each sort α , Var_α is a denumerable set of variables of sort α and $Cons_\alpha$ is a set of constants of that sort. There are no variables of sort $\langle e, e \rangle$.

The set ME_α of meaningful expressions of sort α is defined recursively; instead of writing $t \in ME_\alpha$ I write $t : \alpha$, and instead of $ME_\alpha \subseteq ME_\beta$ I write $\alpha \subseteq \beta$.

- (i) $Var_\alpha, Cons_\alpha \subseteq ME_\alpha$
- (ii) If $t : e$ and $x \in Var_e$ then $\lambda x[t] : \langle e, e \rangle$
- (iii) If $t : nf$ then $^{\cup}t : \langle e, e \rangle$
- (iv) If $f : \langle e, e \rangle$ then $^{\wedge}f : nf$
- (v) If $f : \langle e, e \rangle$ and $t : e$ then $f(t) : e$
- (vi) $i \subseteq u; u, nf \subseteq e$
- (vii) If $t : e$ then $^!t : i$
- (viii) If $\phi, \psi : i$ and $t, t' : e$ and $x \in Var_\alpha$ for any basic sort α then $(t = t'), (\neg\phi), (\phi \vee \psi), (\phi \& \psi), \exists x(\phi), \forall x(\phi), (\phi \rightarrow \psi), (\phi \leftrightarrow \psi)$ are all in ME_i .

The operation p of predication is expressed in this symbolism as follows:

$$p = \lambda x[\wedge \lambda z [\exists y_{nf} (x = y \& ^!\cup y(z))]]$$

where $x, z \in Var_e$ and $y \in Var_{nf}$. For each function f with nominalization y , $p(y)$ is an information unit that is true at z just in case z has the property f .

2.2 The axiomatic theory

The axioms of PT_1 are first of all, λ -conversion and secondly, axioms for the (self-applicable) truth operator. One has $^!\phi \leftrightarrow \phi$ only for atomic ϕ ; in general, $^!\phi \rightarrow \phi$. Negation has the special axiom $^!(\neg^!\phi) \leftrightarrow ^!\neg\phi$.

C&T indicate that consistency of the axioms is established by means of the Gupta-Herzberger revision theory of truth. From that it follows that neither the Russell's paradoxical term RP nor its negation is derivable from the axioms.

2.3 The semantics of PT_1

A PT_1 frame is a structure $F = \langle O, I, P, S, \Delta, T \rangle$, where first of all, $O = \langle E, [E \rightarrow E], \gamma, \delta \rangle$ is a model of the λ -calculus, $[E \rightarrow E]$ is some set of functions in 1-1 correspondence with E via γ and its inverse δ . [The details of the rest of F are skipped].

We are given subsets E_r of E for each basic sort r , and take $E_{\langle e, e \rangle} = [E \rightarrow E]$.

Then $M = \langle F, a \rangle$ is F plus an assignment function a of constants of various sorts r to E_r . Let g be an assignment of E_r to variables of sort r . Then one defines $[[t]]^g$ recursively for all meaningful expressions t of PT_1 using the operations provided by M . For example, $[[\cup t]]^g = \delta[[t]]^g$ and $[[\cap t]]^g = \gamma[[t]]^g$.

We say that M is a PT_1 model if all the axioms of PT_1 are true in M .

[NB. No model M is specified in the paper; but the reader referred to a model construction using Gupta-Herzberger revision semantics in Turner '87.]

2.4 Meeting the desiderata

First, we have closure of properties under the logical operations both *qua* information unit functions and *qua* their nominalizations.

Second, we have self-predication, for example, one has a universal property V and so V holds of its nominalization v . This is expressed via the predication operator p above.

Finally, one has intensionality, if information units are regarded as “structured meanings” along the lines of Creswell '83. For example, one interprets sentences as denoting structures obtained from syntactic analysis trees, chopping off the leaves and replacing them by their semantic values.

Alternatively, C&T suggest construing information units as structured meanings by interpreting them as the formulas themselves, replacing symbols by their semantic values. There are other approaches, but C&T leave the choice open in the end.

3. Types in semantics

In 3.1-3.4 advantages of type theory that a theory of properties should account for in one way or another are summarized.

3.5 Wrap-up

The introduction gave reasons for rejecting a type-theoretic approach to predication in natural language. The problem is how to get type-theory's advantages—especially providing general classificatory criteria for semantic domains—in a property-theoretic approach.

4. A fragment of English

The purpose of this section is to develop an extension of PT_1 with modal and temporal operators and an expanded sortal system that can be used to show how the general framework inherits all the positive features of Montague semantics.

NB. Since this is done for comparison, it should not be taken to imply adherence to Montague's approach to syntax.

4.1 PT_2 , a modal extension of PT_1

The basic sorts are the same as for PT_1 (e, u, nf, i) plus pw ("possible worlds") and Q ("generalized quantifiers"). Complex sorts are of the form $\langle a_1, \dots, \langle a_n, b \rangle \rangle$, where the a_i and b are basic sorts. This replaces n-ary functions $f(a_1, \dots, a_n)$ by their "curried" versions.

The set of meaningful expressions ME_r of sort r is defined recursively and, as before, I write $t : r$ instead of $t \in ME_r$, and $r \subseteq s$ for $ME_r \subseteq ME_s$.

- (i) $Var_\alpha, Cons_\alpha \subseteq ME_\alpha$
- (ii) If $t : e$ but not $t : pw$, and $x \in Var_e$ then $\lambda x[t] : \langle e, e \rangle$
- (iii) If $t : nf$ then $\cup t : \langle e, e \rangle$ and $\cup t : \langle e, i \rangle$
- (iv) If $f : \langle e, e \rangle$ then $\cap f : nf$
- (v) If $f : \langle a, b \rangle$ and $t : a$ then $f(t) : b$
- (vi) We have $i, pw \subseteq u$ and $u, nf \subseteq e$ and $Q \subseteq \langle e, i \rangle \subseteq \langle e, e \rangle$
- (vii) If $t : e$ then $\downarrow t : i$
- (viii) If $\phi, \psi : i$ and $t, t' : e$ and x is a variable of any basic sort then $t = t', \neg\phi, (\phi \vee \psi), (\phi \& \psi), \exists x(\phi), \forall x(\phi), (\phi \rightarrow \psi), (\phi \leftrightarrow \psi), \Box\phi, \Diamond\phi, H\phi$, and $W\phi$ are all : i
- (ix) If $\phi : i$ then $\wedge\phi : pw$.

The notion of a model M for PT_2 is expanded accordingly. [Note: pw only serves for the semantics of $\Box\phi$, not for intensionality.]

4.2 The grammar of the fragment

This section presents a fragment of English that extends Montague's in PTQ. It uses syntactic categories e_α, i_α , and $A/\alpha B$ where α is an "admissible feature bundle". A sample lexicon is given. The details of this are omitted in these notes.

Next, following Partee and Rooth '83, a multi-valued map k is defined from categories to types.

Then, each lexical entry β of category A is translated as a constant β' of sort $k(A)$. It is noted that English predicative expressions (verbs, common nouns, etc.) are treated as predicates in two forms: as information unit functions and as their nominalized counterparts. Hence one has a choice as to how to represent natural language predicates.

Finally, the syntactic and semantic rules are defined recursively in tandem.

5. Some consequences

5.1 Generalia

The fragment in sec. 4 is a slight extension of that in PTQ. Furthermore, all the proposed extensions and revisions of Montague's semantics (Dowty, Carlson, Bach and Partee, etc.) can also be straightforwardly implemented.

Several things are gained.

5.2.1 First, the fragment will assign to (1) both the readings given in (2) and (3), resp.

- (1) John believes that Mary likes Sue
- (2) $\text{believe}'(j, \text{like}'(m, s))$
- (3) $\text{believe}'(j, \wedge \text{like}'(m, s))$

Of these, (3) obtains just in case in every world compatible with John's beliefs, Mary likes Sue; on this construal, (3) entails that John believes all the logical consequences of *Mary likes Sue*. On the reading in (2), on the other hand, belief is cast as a relation of John to an information unit, which, arguably, captures the sense in which belief is a disposition to manipulate logical structures in deliberation and reasoning.

5.2.2 and 5.2.3 [skip]

5.3 Reference to properties in natural language

There are various ways of referring to entities in natural language; some of them are quantificational as in (1a-c):

- (1) a. John reads *every thing that Mary reads*;
- b. John reads *the thing that Mary reads*;
- c. John reads *whatever Mary reads*.

The grammar generates three readings:

- (2) a. $\forall x[\text{thing}'(x) \ \& \ \text{read}'(m, x)] \rightarrow \text{read}'(j, x)$,

and similarly for (2b) and (2c) [where we omit ' $\text{thing}'(x)$ ']. Given this analysis, any of the sentences in (1a-c) in conjunction with (3a) will entail (3b):

- (3) a. Mary reads Principia; hence,
- b. John reads Principia.

Consider now:

- (4) a. John tries *everything that Mary tries*;
- b. John tries *the same thing that Mary tries*;
- c. John tries *whatever Mary tries*.

It is intuitively clear that any one of these sentences in conjunction with (5a) will entail (5b):

- (5) a. Mary tries reading {alt., to read} Principia; hence,
b. John tries reading {alt., to read} Principia.

And the grammar predicts that. For example, the readings of (4c), (5a) and (5b), resp., are:

- (6) a. $\forall x[\text{try}'(m, x) \rightarrow \text{try}'(j, x)]$;
b. $\text{try}'(m, \text{read}'(\text{Principia}'))$; hence,
c. $\text{try}'(j, \text{read}'(\text{Principia}'))$.

What is crucial in obtaining this result is that properties may be treated as individuals and that infinitives and gerunds denote properties.

There is an alternative, “propositional view”, of infinitives and gerunds that “appears to be held currently by a majority of generative linguists.” That would represent (5a) by something like:

- (7) $\text{try}'(m, \text{read}'(m, \text{Principia}'))$.

Then that in conjunction with (6a) would imply

- (8) $\text{try}'(j, \text{read}'(m, \text{Principia}'))$,

in other words, John tries to bring about a situation where Mary’s reading of Principia occurs. But that is just not true. So reasoning patterns as in (4) and (5) are a total mystery on a propositional view.

Finally, consider the descriptive generalization given in (9a) and illustrated in (9b, c):

- (9) a. Finite VP’s never occur as arguments of other VP’s
b. *John forces Mary leaves
c. *John tries leaves.

On the present “Fregean” approach, a simple hypothesis suggests itself: finite VP’s are information unit functions and as such they cannot serve as arguments of other information unit functions. Their individual correlates, however, can occur in argument position; the latter are realized as infinitival VP’s.

Note, the brief section 5.3 *Final remarks*, on p. 298, should be numbered 5.4.

References. See the original paper.