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Thermodynamic Correlation Tests Between Magnetostrictive and Magnetomechanical Effects in 2% Mn Pipeline Steel

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Abstract—**The thermodynamic correlation between magnetostrictive and magnetomechanical effects is tested using reversible measurements of the uniaxial isostress magnetostrictive and isofield magnetomechanical coefficients, all measured on samples of the same 2% Mn pipeline steel, using both tensile and compressive stresses. The agreement is considered to be good considering the high sensitivity required for the measurements.**

INTRODUCTION

THE APPLICATION of Le Chatelier's principle gives a thermodynamic relationship between the isofield stress-induced magnetization change in a ferromagnet (the magnetomechanical effect) and the strain accompanying isostress magnetization (the magnetostrictive effect) [1]. However, this is complicated by the fact that equilibrium thermodynamic relationships cannot be applied directly to hysteretic systems and can only be used successfully under conditions of strict reversibility. There have been few attempts to verify the thermodynamic correlation in the past [2], [3], because this necessitates the identification and separation of the reversible magnetization components from the total measured magnetization changes. The results of Bozorth's correlation tests, using data for 45 Permalloy under tensile stress only, were encouraging, although the data were taken on different specimens annealed in different ways. This encouraging correlation may have been partly due to the relatively large and well-behaved magnetostriction in 45 Permalloy.

In recent years we have made measurements of both reversible and irreversible magnetization stress and strain effects on samples of the same sample of 2% Mn pipeline steel, where the effects are smaller and more complex than on 45 Permalloy. Here we summarize the relevant reversible results and use them to test their thermodynamic correlation.

We begin from thermodynamic first principles by deriving the correlation between the magnetostriction and magnetomechanical effects following Bozorth's analysis. The resultant correlation is then tested using the appropriate data obtained for our pipeline steel sample under isofield and isostress conditions.

THERMODYNAMIC RELATION

The magnetic Gibb's function (G) for a ferromagnet can be written as

$$G = U - TS - \mu_0 HM \tag{1}$$

where U is the total energy, T the temperature, S the entropy, μ_0 the permeability of free space, M the magnetization, and H the external magnetic field. In differential form, (1) can be expressed as

$$dG = dU - T\,dS - S\,dT - \mu_0 H\,dM - \mu_0 M\,dH \tag{2}$$

and since, for a reversible process,

$$T\,dS = dU - \mu_0 H\,dM \tag{3}$$

we have

$$dG = -S\,dT - \mu_0 M\,dH \tag{4}$$

where the changes are confined to heat and magnetization. When there is also elastic work due to mechanical stress σ and strain ϵ, the relationship becomes

$$dG = -S\,dT - \mu_0 M\,dH - \epsilon\,d\sigma. \tag{5}$$

Under isothermal conditions this reduces to

$$dG = -\mu_0 M\,dH - \epsilon\,d\sigma \tag{6}$$

and, applying the condition for an exact differential, this gives

$$[d\epsilon/dH]_\sigma = \mu_0 [dM/d\sigma]_H \tag{7}$$

where $[d\epsilon/dH]_\sigma$ is the reversible isostress magnetostrictive coefficient and $[dM/d\sigma]_H$ is the reversible isofield magnetomechanical coefficient.

Equation (7) is then the thermodynamic correlation to be tested.

EXPERIMENTAL DATA

We have previously [4] given examples of uniaxial isostress magnetostrictive measurements made on both the

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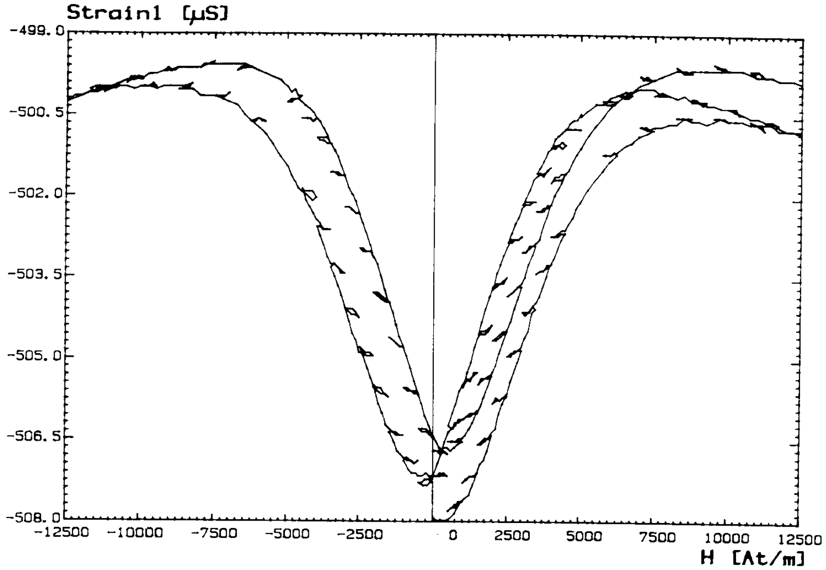

Fig. 1. 100-MPa compressive isostress magnetostrictive measurements as a function of field. Measurements are made along the initial magnetization curve and around a symmetric major hysteresis loop to about 75-percent saturation and include numerous small-amplitude minor hysteresis loops indicating the reversible component of magnetostriction. Note that magnetostriction at high field is nearly fully reversible but that, at low field, there is a substantial irreversible component.

initial and anhysteretic curves. Magnetostriction under tensile or compressive isostress is similar for both initial and anhysteretic magnetizations. Although these appear to be nearly reversible our more recent higher resolution results show slight hysteresis. We have, therefore, measured the reversible component by making small field reversals, as described previously [5].

Fig. 1 shows a typical experimental plot of isostress magnetostrictive strain as a function of field for a large field cycle with some small field reversals included. When using small field reversals the magnetostrictive strain during these small minor hysteresis cycles is essentially reversible and can be used to determine $[d\epsilon/dH]_\sigma$. These incremental strains, which are needed to separate the reversible component of magnetostriction are rather small, typically of the order of a hundred nanostrain total. We, therefore, need a resolution of a few tens of nanostrain in order to measure the derivative. These measurements, therefore, require patience and careful experimental technique.

It is apparent from Fig. 1 that, at high field, the reversible and total incremental magnetostriction are nearly identical indicating that, under these conditions, magnetostriction is almost fully reversible. At low fields, there are big differences between reversible and total incremental magnetostriction showing that there is a large irreversible component. The maxima in total magnetostriction and in the reversible component appear to occur at close but not identical points.

We can only make a few of these minor cycles during a full symmetric major cycle without drift becoming inconveniently large. Fig. 2 shows a cycle with four reversals, such as we have used for analysis. Fig. 3 shows the same magnetostrictive data plotted as a function of magnetization. This shows that, although hysteresis is less, it is still detectable and, therefore, that magnetostriction is not a simple reversible process.

We have also made previous measurements of the effect of uniaxial cyclic isofield stress on magnetization [6]. We have expanded and refined these to derive $[dM/d\sigma]_H$, the reversible isofield magnetomechanical coefficient. These measurements have been used for $[dM/d\sigma]_H$.

RESULTS AND DISCUSSION

Table I summarizes the derived data used to test the thermodynamic correlation given by (7). The left- and right-hand sides of the equation are evaluated at selected fields for isostress levels between ±150 MPa. Uncertainty estimates are shown. The fields are selected to represent the low-field region, the peak magnetostriction region, high, and near-saturation fields. The ratio, which should be unity, is also tabulated. It is apparent that, within estimated experimental uncertainty, there is generally good agreement. Although the uncertainties are large, there is also good agreement in the general field-dependent behavior. Both derivatives are positive at low field. They become negative at a field somewhat before the maximum total magnetostriction is reached, as shown in Fig. 1. Both decrease again as saturation is approached. Over the range investigated, the effects of stress are less and appear to diminish at high field.

In addition to the experimental uncertainties the following additional possible systematic uncertainties should be noted:

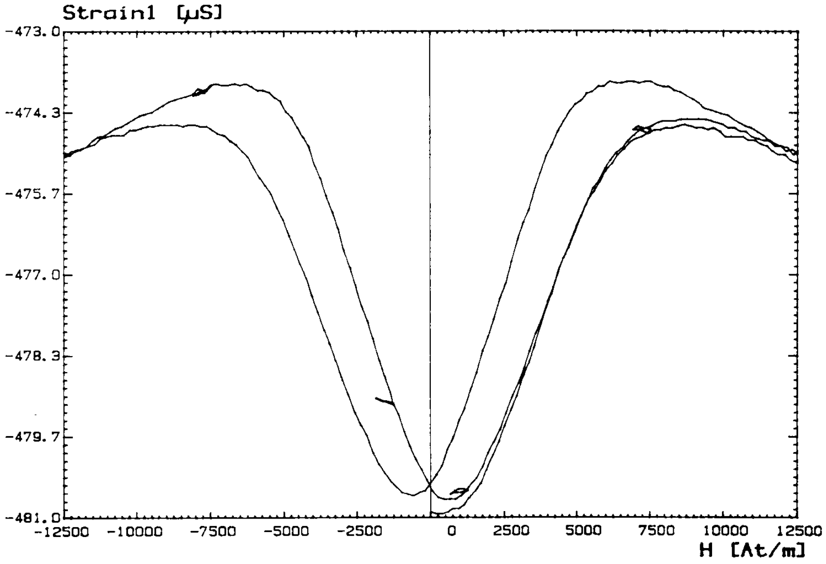

Fig. 2. 100-MPa compressive isostress magnetostrictive measurements as a function of field. Four small-amplitude minor hysteresis loops are used to determine the reversible component of magnetostriction.

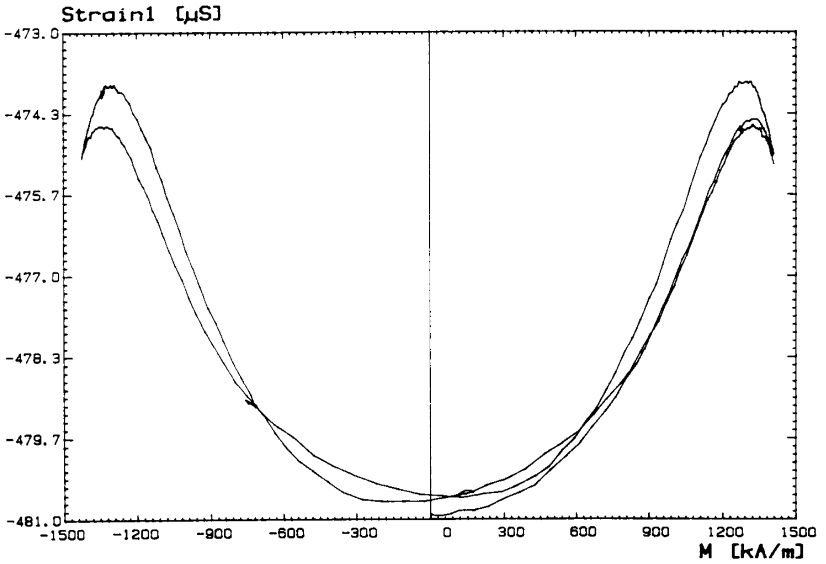

Fig. 3. The same isostress magnetostrictive data as Fig. 2 plotted as a function of magnetization showing reduced but still noticeable hysteresis.

1) The assumption of isothermal conditions. Our samples use an approximately 9-mm^2 cross section and, even though most measurements are made over periods of several seconds, their thermal condition is likely to be intermediate between isothermal and adiabatic.

2) The tacit assumption of constant elasticity (i.e., neglect of the ΔE effect) and the neglect of the volumetric effects of both magnetization (the volumetric magnetostrictive effect) or stress.

3) The uncertainties in deriving the magnetization M, because the field H can only be measured outside the sample. These are by comparison smaller but still significant uncertainties.

4) No consideration has been given to microstructure, domain configuration, or possible anisotropy.

CONCLUSIONS

This is believed to be the first attempt to measure the required reversible parameters under tensile and compres-

TABLE I
THE RIGHT- AND LEFT-HAND SIDES OF (7) AND THEIR RATIOS MEASURED AT VARIOUS FIELDS AND STRESSES, TABULATED TO TEST THE CORRELATION BETWEEN THE REVERSIBLE COMPONENTS OF MAGNETOSTRICTIVE AND MAGNETOMECHANICAL EFFECTS

σ (MPa)	H (kA/m)	$\mu_0[dM/d\sigma]_H$ (× 10ex − 10 Wb/N)	$[d\epsilon/dH]_\sigma$ (× 10ex − 10 m/A)	Ratio (Nm/WbA) = 1
150	50000	−0.97 ± 0.1	−0.84 ± 0.11	0.9 ± 0.2
100	50000	−0.92 ± 0.1	−0.54 ± 0.2	0.6 ± 0.5
50	50000	−1 ± 0.1	−0.43 ± 0.15	0.4 ± 0.4
0	50000		−0.56 ± 0.12	
−50	50000	−0.73 ± 0.1	−0.38 ± 0.07	0.5 ± 0.3
−100	50000	−0.33 ± 0.1	−0.48 ± 0.1	1.5 ± 0.5
−150	50000	−0.45 ± 0.1	−0.25 ± 0.15	0.6 ± 0.8
150	20000	−2.9 ± 0.2	−3.4 ± 0.5	1.2 ± 0.2
100	20000	−2.9 ± 0.2	−2.9 ± 0.2	1.0 ± 0.1
50	20000	−2.8 ± 0.2	−2.9 ± 0.2	1.0 ± 0.1
0	20000		−2.9 ± 0.2	
−50	20000	−2.5 ± 0.2	−2.9 ± 0.2	1.2 ± 0.1
−100	20000	−3 ± 0.2	−3.1 ± 0.4	1.0 ± 0.2
−150	20000	−2.8 ± 0.2	−2.7 ± 0.3	1.0 ± 0.2
150	7500	−3.3 ± 0.4	−4.7 ± 1.5	1.4 ± 0.4
100	7500	−2.7 ± 0.2	−3.3 ± 1.1	1.2 ± 0.4
50	7500	−2.4 ± 0.2	−3.9 ± 1.5	1.6 ± 0.5
0	7500		−2.8 ± 0.5	
−50	7500	−1.8 ± 0.8	−3 ± 0.8	1.7 ± 0.7
−100	7500	−1.3 ± 0.3	−2.1 ± 0.7	1.6 ± 0.6
−150	7500	−0.9 ± 0.7	−1.6 ± 0.4	1.8 ± 1.0
150	1250	2.5 ± 2.5	−2 ± 1.5	−0.8 ± 0.3
100	1250	1.5 ± 1	1.4 ± 0.7	0.9 ± 1.2
50	1250	1.5 ± 0.5	2.8 ± 1	1.9 ± 0.7
0	1250		1.6 ± 0.6	
−50	1250	2 ± 0.5	2 ± 0.6	1.0 ± 0.6
−100	1250	2.5 ± 1	1.4 ± 0.4	0.6 ± 0.7
−150	1250	5 ± 2	1.1 ± 0.5	0.2 ± 0.9

sive stresses and on the same material sample, but still more careful control is needed.

The results of our tests of the thermodynamic relationship give encouraging support for the quoted thermodynamic correlation.

Our results help to show the necessity for separating magnetic changes into reversible and irreversible components.

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