The Armed Peace:
A Punctuated Equilibrium Theory of War

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2006

- military force as instrument (Clausewitz, Schelling)
- bargaining model of war:
  (a) private information + misrepresent
  (Fearon 1995, Powell 1996, Slantchev 2003b)

  (b) **dynamic commitment problem**
  (Fearon 1995, Powell 2004)

  (c) guns vs. butter: cost of peace
  (Powell 1993, Slantchev 2005)

  (d) coordination problem: multiple equilibria
  (Slantchev 2003a)

Why Commitment Explanation Is Important

(a) substantive objections to informational story:

- uncertainty “too” pervasive
- problem with long wars (e.g., civil wars)
- strained reading of history (e.g., Europe in 1939)

(b) Powell (2005): common general mechanism

- large rapid (exogenous) shifts of power
- preemptive & preventive war, issue indivisibility, sources of military power

However. . .

- how does fighting resolve the problem?

- two players, $i = 1, 2$
- bargain and fight in $t = 1, 2, \ldots$
- initial resource endowments: $0 < K_1 < K_2 < \infty$
- value of contested prize: $v > 0$
- probability of winning a battle:
  - if opposed: $p_i \in (0, 1)$
  - if unopposed: $p = p_1 + p_2 < 1$
- cost of battle: $c_i > 0$
- prize worth at least one battle: $p_i v > c_i$
- size of pie at t: $S(t) = v + k_1(t) + k_2(t)$
- fight payoff: $S(t) - (c_1 + c_2)$ if victory, 0 if defeat
- bargain payoff: x_i where $x_1 + x_2 = S(t)$
- arbitrary bargaining protocol for each round
- peace: neither attacks after bargain reached
- complete information

Features of the Model: War and Peace

(a) war is not a lottery but a process

- victory: collapse in battle or attrition
- peace: military victory or negotiation

(b) agreement does not automatically end game

- peace is endogenous
- different from Rubinstein-style models

(c) flexible negotiations during fighting

- protocol arbitrary/players can negotiate
- all available resources can be used

Features of the Model: Power and Costs

(a) potential for rapid power shifts

- tactical surprise in battle (not war)
- similar to first-strike advantage

(b) costs per engagement, not war

- costs accumulate during war
- may exceed the value of the stakes
- different from lottery war models

Total War:

- If players fight $T \geq 1$ periods with $(k_1, k_2; S)$, then they fight in the current period with $(k_1 + c_1, k_2 + c_2; S + C)$ if v and p are large enough (Lemma 1)

- If a player is about to collapse, then players fight provided $p_j S > C$ (Lemma 2)

- If conditions for Lemma 1 and Lemma 2 are met, then players never redistribute and fight to the end (Proposition 1)

Stalin was right but:

- trivial solution to commitment problem

- empirically rather rare

Lemma 1: show that in current period at least one player will reject any offer; first, show that next period cannot be peaceful no matter what distribution is achieved today; second, show that this places minimum bounds on payoffs that, if conditions are right, exceed the surplus today.

Lemma 2: show that player 2 will not agree to division that extends player 1's life, then show that any other distribution results in a fight.

Proposition 1: take no-distribution path to penultimate node; players fight there (Lemma 2), going up a node, players fight there too (Lemma 1). Now show that at that next node they will not redistribute either. Suppose that there's a bargain that leaves both players at least as well off as the current distribution (they fight at this node in any case). Let this bargain be $(x_1, x_2; S)$. We show that $(x_1 - c_1, x_2 - c_2; S - C)$ is also a feasible bargain at the following node. Since they fight today regardless of bargain, they have no strict incentive to redistribute (they can get the bargain tomorrow). This unravels the game.

Limited War – Steps to Demonstration

1) Peace requires credible threats to fight

2) Sufficiency condition for fight in any period (P)

3) Fighting eventually leads to violation of (P)

4) Peace can be attained when (P) fails

5) But SPE threats not credible b/c too costly

6) Players will attempt to renegotiate punishment

7) Threats may no longer be deterrent

8) Derive SPE with credible threats (W)

9) Numerical examples of Limited/Total War

Threat to fight crucial for peace: if (x_1, x_2) is peaceful in period t, then no peace is possible in $t+1$ if a player deviates and fights after redistribution in t (Lemma 3)

The more severe the threat, the stronger the incentives for peace: prolonged war is costly for both, reduces total size of benefit, willing to settle for less

Allow commitment to minmax strategies: strategies to fight to the end are Nash/SPE, and produce the most severe threat ⇒ peace most likely

$F_i(t|T)$: i's payoff from rejecting all offers and fighting from t to T (this payoff i can guarantee)

Fight to collapse of weaker player in $T+1$:

$$F_i(t|T) = p_i \sum_{n=0}^{T-t-1} (1-p)^n [S-(n+t)C] + (1-p)^{T-t} F_i(T|T)$$

$\sum_i F_i(t|T)$ is strictly decreasing in T (Lemma 4), and so the higher T, the smaller the minimum payoffs and the better the prospects for peace.

Longest possible total war for some S:

$$\overline{T} = \left\lfloor \frac{S}{C} \right\rfloor$$

Therefore, if players equalize resources, they can make the most mutually deterrent threats possible, which means environment most conducive to peace.

Offering i his minmax payoff is not enough to induce him to agree; offer must be high enough to induce him to forego advantage of surprise attack.

Sneak attack at $(x_1, x_2; S)$ yields *at least*:

$$A_i(x_1, x_2) = p(S - C) + (1 - p)F_i(2|T),$$

where $T = \min\{x_1/c_1, x_2/c_2\}$. $(p - p_i)(S - C)$ is i's surprise-attack advantage.

Hence, peace in $t = 1$ will not be possible if:

$$\sum_i A_i(x_1, x_2) > S \text{ for all } x_i \geq 0, x_1 + x_2 = S,$$

which simplifies (after tedious algebra) to:

$$p^2(\overline{T} - 1) + (1 - p)^{\overline{T}} > 1 \quad \text{(P)}$$

This is logic of Powell's (2004) sufficiency condition.

(P) is defined in terms of exogenous parameters. As S decreases, (P) will fail $\Rightarrow$ fighting makes peace possible, and peace can be had in SPE. . . but

(P) involves strategies that generally will not be credible even if subgame-perfect because once total war begins, players have incentives to *renegotiate* as soon as possible and end it.

But if peace becomes possible sooner,

- expected payoff from (less) fighting increases
- expected benefit from sneak attack increases
- temptation to deviate from peace today increases

(P) can fail and yet peace may NOT be achievable.

Hence, we want a condition with credible (subgame perfect *and* renegotiation-proof) threats only.

Players can only credibly threaten to fight until a period in which peace is possible under some bargaining protocol.

Hence, assume that a period in which peace is possible must end the war (because players can renegotiate and achieve that).

Use threats to fight until such a period to deter sneak attacks.

Necessary/sufficient condition for fighting at t:

$$2p(S - C) + (1 - p)\sum_i V_i(t + 1) > S, \tag{W}$$

where $V_i(t)$ is i's payoff from period t using credible threats. Since $V_i(t) \geq F_i(t|T)$, (P) implies (W) but not vice versa.

After redistribution, $v = 0$; since peace must be impervious to deviations to such subgames, analyze subgames of this type.

Players are *symmetric* if $K_1 = K_2$, $c_1 = c_2$, and $p_1 = p_2$. Assume $v = 0$ and symmetric players with $k_i = nc$, where $n \geq 1$. Lemma 5:

WLOG, players do not redistribute in equilibrium; so use backward induction for following result.

Suppose that they find a peaceful settlement when $k_i = nc$. Then:

(A) they fight with $k_i = (n+1)c$ if, and only if, $pn > 1$;

(B) if they fight with $k_i = (n + t)c$, $t = 1, \ldots, T$, then they fight with $k_i = (n + T + 1)c$ if, and only if,

$$p^2(n + T) + (1 - p)^{T+1} > 1.$$

Lemma 5: WLOG, they don't redistribute.

- (nc, nc) may involve realloc only when it is not peaceful (if peaceful, players won't reallocate b/c one of them will be worse off); hence, in any realloc (nc, nc) produces a fight in continuation game
- choose smallest such n (for all $k_i < nc$ they do not reallocate), so unless they realloc, they fight $t \geq 1$ and stop at (m_1c, m_2c); WLOG, $m_1 \geq m_2$
- If $t = n$, then they must equalize at some point (otherwise one would collapse sooner), but one with larger initial share won't agree, so $t < n$
- If $m_2 < 1$, then $t < n$ implies 2 won't agree to initial realloc since she can fight more under initial distribution, so $m_2 \geq 1$ and peace achieved while both still alive
- If $m_1 = m_2$, they are indifferent between initial alloc and realloc, so our WLOG claim holds, so assume $m_1 > m_2$
- since i can guarantee m_ic in this period by accepting initial realloc $(m_i + t)c$, rejecting all offers and fighting t periods, it is a weakly dominant strategy for each not to agree to anything less than $(m_i+t)c$ in the initial period, so WLOG assume they realloc at $(m_i + t)c$ initially
- if 2 disagrees with $(m_i + t)c$, our choice of n implies at least one fight; if peace next period, then her share will be $p_2[2(n-1)c] + (1-p)(n-1)c = (n-1)c$; note that $m_1 + t + m_2 + t = m_1 + m_2 + 2t = 2n$, and since $t \geq 1$, $(m_1 + m_2)/2 \leq n - 1$; since $m_1 > m_2$, this means $m_2 < n - 1$, and so 2 is better off disagreeing with initial realloc; so they must fight at least two periods
- since by choice of n, no realloc in continuation game, $V_i((n-1)c, (n-1)c) > V_i((m_1 + t - 1)c, (m_2 + t - 1)c)$ for some i; or else they would agree to realloc & continue with $m_i + t - 1$, contradicting choice of n
- but if i disagrees with initial realloc, his payoff is $p_i(n-1)2c + (1-p)V_i((n-1)c, (n-1)c) > p_i(n-1)2c + (1-p)V_i((m_1+t-1)c, (m_2+t-1)c) = V_i((m_1+t)c, (m_2+t)c)$, where equality follows from players fighting ≥ 1 periods after $(m_i+t)c$; so he is better off rejecting realloc!

Lemma 5 gives a powerful corollary: if players are symmetric and cannot achieve peace if they redistribute such that $k_i = nc$, then they cannot achieve peace under any alternative distribution.

We now can check for deviations only after distributions $(S/2, S/2)$ for any S: if peace cannot be sustained with such a distribution, it won't be achievable under any other distribution. Instead of computing continuation values for games after all $(x_1 - c, x_2 - c)$, we only check $(S/2 - c, S/2 - c)$.

So algorithm to unravel SPE: for any period, starting with the last, suppose players equalize resources and check if they will fight using Lemma 5. If they do, peace is impossible; if they do not, then peace obtains (protocol does not artificially preclude it). Use this outcome for previous node in the game tree, all the way to the start.

Parameters: $v = 12, K_i = 30, c_i = 1, p_i = .09$

t	$(k_1, k_2; S)$	$\sum_i A_i$	Lemma 5	(P)	outcome	$\sum_i V_i(t)$
1	(30,30;72)	74.516	1.226	1.135	fight	61.92
2	(29,29;70)	72.381	1.214	1.103	fight	60.14
3	(28,28;68)	70.296	1.207	1.070	fight	58.42
4	(27,27;66)	68.271	1.204	1.038	fight	56.75
5	(26,26;64)	66.320	1.209	1.006	fight	55.16
6	(25,25;62)	64.459	1.221	0.974	fight	53.66
7	(24,24;60)	62.707	1.244	0.942	fight	52.27
8	(23,23;58)	61.088	1.278	0.910	fight	51.01
9	(22,22;56)	59.632	1.327	0.879	fight	49.91
10	(21,21;54)	58.375	1.394	0.847	fight	49.02
11	(20,20;52)	57.360	1.482	0.816	fight	48.36
12	(19,19;50)	56.640	4.320	0.785	fight	48.00
13	(18,18;48)	47.939	0.994	0.754	**peace**	48.00
14	(17,17;46)	46.187	1.017	0.723	fight	38.27
15	(16,16;44)	44.568	1.051	0.693	fight	37.01
16	(15,15;42)	43.112	1.100	0.663	fight	35.91
17	(14,14;40)	41.855	1.167	0.634	fight	35.02
18	(13,13;38)	40.840	1.256	0.606	fight	34.36
19	(12,12;36)	40.120	3.060	0.579	fight	34.00
20	(11,11;34)	33.672	0.971	0.553	**peace**	34.00
21	(10,10;32)	32.415	1.037	0.528	fight	27.02
22	(9,9;30)	31.400	1.126	0.505	fight	26.36
23	(8,8;28)	30.680	2.340	0.483	fight	26.00
24	(7,7;26)	25.335	0.940	0.465	**peace**	26.00
25	(6,6;24)	24.320	1.029	0.449	fight	20.36
26	(5,5;22)	23.600	1.800	0.437	fight	20.00
27	(4,4;20)	19.600	0.964	0.429	**peace**	20.00
28	(3,3;18)	18.880	1.440	0.427	fight	16.00
29	(2,2;16)	14.880	0.899	0.431	**peace**	16.00
30	(1,1;14)	14.160	1.080	0.444	fight	12.00

(P) fails for all $t > 5$ but still war in $t < 13$.

Parameters: $v = 12, K_i = 30, c_i = 1, p_i = .15$

t	$(k_1, k_2; S)$	$\sum_i A_i$	Lemma 5	(P)	outcome	$\sum_i V_i(t)$
1	(30, 30; 72)	86.333	3.150	3.150	fight	65.33
2	(29, 29; 70)	83.734	3.060	3.060	fight	63.33
3	(28, 28; 68)	81.134	2.970	2.970	fight	61.33
4	(27, 27; 66)	78.534	2.880	2.880	fight	59.33
5	(26, 26; 64)	75.934	2.790	2.790	fight	57.33
6	(25, 25; 62)	73.334	2.700	2.700	fight	55.33
7	(24, 24; 60)	70.735	2.610	2.610	fight	53.34
8	(23, 23; 58)	68.135	2.520	2.520	fight	51.34
9	(22, 22; 56)	65.536	2.430	2.430	fight	49.34
10	(21, 21; 54)	62.937	2.341	2.340	fight	47.34
11	(20, 20; 52)	60.339	2.251	2.250	fight	45.34
12	(19, 19; 50)	57.741	2.161	2.160	fight	43.34
13	(18, 18; 48)	55.144	2.072	2.070	fight	41.34
14	(17, 17; 46)	52.549	1.982	1.980	fight	39.35
15	(16, 16; 44)	49.955	1.893	1.890	fight	37.36
16	(15, 15; 42)	47.365	1.805	1.801	fight	35.37
17	(14, 14; 40)	44.779	1.717	1.711	fight	33.38
18	(13, 13; 38)	42.198	1.630	1.621	fight	31.40
19	(12, 12; 36)	39.626	1.544	1.532	fight	29.43
20	(11, 11; 34)	37.065	1.460	1.442	fight	27.47
21	(10, 10; 32)	34.522	1.378	1.353	fight	25.52
22	(9, 9; 30)	32.002	1.300	1.265	fight	23.60
23	(8, 8; 28)	29.518	1.228	1.177	fight	21.72
24	(7, 7; 26)	27.082	1.162	1.090	fight	19.88
25	(6, 6; 24)	24.718	1.108	1.004	fight	18.12
26	(5, 5; 22)	22.454	1.068	0.920	fight	16.45
27	(4, 4; 20)	20.334	1.050	0.838	fight	14.93
28	(3, 3; 18)	18.420	1.063	0.760	fight	13.62
29	(2, 2; 16)	16.800	1.120	0.688	fight	12.60
30	(1, 1; 14)	15.600	1.800	0.622	fight	12.00

(P) fails for all $t > 25$ but war in all t anyway.

Parameters: $v = 12, K_i = 30, c_i = 1, p_i = .005$

t	$(k_1, k_2; S)$	$\sum_i A_i$	Lemma 5	(P)	outcome	$\sum_i V_i(t)$
1	(30, 30; 72)	70.70	0.35	0.700	**peace**	72.00
2	(29, 29; 70)	68.68	0.34	0.707	**peace**	70.00
3	(28, 28; 68)	66.66	0.33	0.714	**peace**	68.00
4	(27, 27; 66)	64.64	0.32	0.721	**peace**	66.00
5	(26, 26; 64)	62.62	0.31	0.728	**peace**	64.00
6	(25, 25; 62)	60.60	0.30	0.735	**peace**	62.00
7	(24, 24; 60)	58.58	0.29	0.743	**peace**	60.00
8	(23, 23; 58)	56.56	0.28	0.750	**peace**	58.00
9	(22, 22; 56)	54.54	0.27	0.757	**peace**	56.00
10	(21, 21; 54)	52.52	0.26	0.765	**peace**	54.00
11	(20, 20; 52)	50.50	0.25	0.773	**peace**	52.00
12	(19, 19; 50)	48.48	0.24	0.780	**peace**	50.00
13	(18, 18; 48)	46.46	0.23	0.788	**peace**	48.00
14	(17, 17; 46)	44.44	0.22	0.796	**peace**	46.00
15	(16, 16; 44)	42.42	0.21	0.804	**peace**	44.00
16	(15, 15; 42)	40.40	0.20	0.812	**peace**	42.00
17	(14, 14; 40)	38.38	0.19	0.820	**peace**	40.00
18	(13, 13; 38)	36.36	0.18	0.828	**peace**	38.00
19	(12, 12; 36)	34.34	0.17	0.836	**peace**	36.00
20	(11, 11; 34)	32.32	0.16	0.845	**peace**	34.00
21	(10, 10; 32)	30.30	0.15	0.853	**peace**	32.00
22	(9, 9; 30)	28.28	0.14	0.861	**peace**	30.00
23	(8, 8; 28)	26.26	0.13	0.870	**peace**	28.00
24	(7, 7; 26)	24.24	0.12	0.879	**peace**	26.00
25	(6, 6; 24)	22.22	0.11	0.887	**peace**	24.00
26	(5, 5; 22)	20.20	0.10	0.896	**peace**	22.00
27	(4, 4; 20)	18.18	0.09	0.905	**peace**	20.00
28	(3, 3; 18)	16.16	0.08	0.914	**peace**	18.00
29	(2, 2; 16)	14.14	0.07	0.923	**peace**	16.00
30	(1, 1; 14)	12.12	0.06	0.933	**peace**	14.00

$pv > C$ does NOT hold, so multi-period peace possible... but can war still happen?

... yes, war is possible even if $pv > C$ fails:

Parameters: $v = 12, K_i = 30, c_i = 1, p_i = .05$

t	$(k_1, k_2; S)$	$\sum_i A_i$	Lemma 5	(P)	outcome	$\sum_i V_i(t)$
1	(30, 30; 72)	72.122	1.006	0.373	fight	65.12
2	(29, 29; 70)	71.380	1.069	0.365	fight	64.58
3	(28, 28; 68)	70.800	1.140	0.358	fight	64.20
4	(27, 27; 66)	70.400	3.200	0.351	fight	64.00
5	(26, 26; 64)	63.322	0.966	0.344	**peace**	64.00
6	(25, 25; 62)	62.580	1.029	0.338	fight	56.58
7	(24, 24; 60)	62.000	1.100	0.332	fight	56.20
8	(23, 23; 58)	61.600	2.800	0.327	fight	56.00
9	(22, 22; 56)	55.980	0.999	0.322	**peace**	56.00
10	(21, 21; 54)	55.400	1.070	0.318	fight	50.20
11	(20, 20; 52)	55.000	2.500	0.315	fight	50.00
12	(19, 19; 50)	49.380	0.969	0.312	**peace**	50.00
13	(18, 18; 48)	48.800	1.040	0.310	fight	44.20
14	(17, 17; 46)	48.400	2.200	0.309	fight	44.00
15	(16, 16; 44)	42.780	0.939	0.308	**peace**	44.00
16	(15, 15; 42)	42.200	1.010	0.309	fight	38.20
17	(14, 14; 40)	41.800	1.900	0.312	fight	38.00
18	(13, 13; 38)	37.800	0.990	0.315	**peace**	38.00
19	(12, 12; 36)	37.400	1.700	0.320	fight	34.00
20	(11, 11; 34)	33.400	0.970	0.327	**peace**	34.00
21	(10, 10; 32)	33.000	1.500	0.335	fight	30.00
22	(9, 9; 30)	29.000	0.950	0.346	**peace**	30.00
23	(8, 8; 28)	28.600	1.300	0.359	fight	26.00
24	(7, 7; 26)	24.600	0.930	0.374	**peace**	26.00
25	(6, 6; 24)	24.200	1.100	0.392	fight	22.00
26	(5, 5; 22)	22.000	1.000	0.414	**peace**	22.00
27	(4, 4; 20)	19.800	0.900	0.439	**peace**	20.00
28	(3, 3; 18)	17.600	0.800	0.467	**peace**	18.00
29	(2, 2; 16)	15.400	0.700	0.500	**peace**	16.00
30	(1, 1; 14)	13.200	0.600	0.538	**peace**	14.00

(a) Vegetius was right: if you want peace, prepare for war (Lemma 3)

(b) more decisive military technology + high stakes ⇒ prospects for peace dim (Proposition 1)

(c) commitment problem due to large rapid shifts of power:

- can be resolved through fighting: (P) will fail
- threats to sustain peace may be incredible
- this resolved through more fighting (W)

Size of the power shift is partially endogenous because it depends on strategies states play. To solve the credibility problem, we have to explain why players cannot choose strategies that minimize the size of the power shift. One such explanation is that they cannot credibly commit not to renegotiate at earliest possible opportunity, thereby undermining deterrent effect of threat to punish surprise attacks.

(a) common models assume war costs less than prize; else costs > value of victory ⇒ no war

(b) here, costs per battle, not war

(c) if players survive to make a peaceful settlement, war is net loss for both:

- at $t = 13$, pie is $S = 48$ out of 72: they paid war costs of 24 for a prize of 12
- at time of peace, players accept shares that are worse than conceding prize from the outset (each gets 24 here, whereas outright concession would have left him with 30)

(d) so, can be rational to escalate far out of proportion to the value of the issues at stake

(e) problem is that war costs are sunk at time of peace

(a) specific windows of opportunity to end war

- even though negotiations available throughout
- if windows closes, fighting until next
- “ripe for resolution” (Zartman 1985)
- but not because of “mutually hurting stalemate”

(b) frequency increases with duration of war

- more resources ⇒ longer fighting spells
- pie shrinks/players weaker ⇒ willing to end
- better terms of peace ⇒ worse its prospects

So:

- large, rapid power shifts lead to war
- size of shift depends on strategies
- strategies to minimize shift must be credible
- incentives for peace undermine that
- players unable to commit to small shift
- fighting can enable this through destruction