Repeated Games and Bargaining

Show all steps and calculations in your answers, for proofs justify each step.
Questions marked with '*' are harder and result in extra points.

QUESTION 1. Consider the two strategic-form games in Figure 1. Find the perfect Bayesian Nash equilibria in the following games:
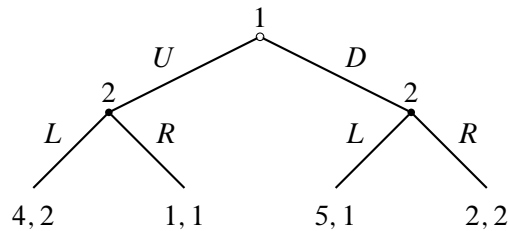
(a) Nature chooses the game to the left with probability $\pi \in (0, 1)$. Neither player observes the choice.

(b) Player 1 observes Nature's choice but player 2 does not.

		Player 2	
		L	R
Player 1	T	$1, 1$	$0, 0$
	B	$0, 0$	$0, 0$

		Player 2	
		L	R
Player 1	T	$0, 0$	$0, 0$
	B	$0, 0$	$2, 2$

Figure 1: Two Strategic-Form Games.

(Hint: construct the strategic forms.)

QUESTION 2. Consider the following extensive form game.

(a) Find a subgame perfect equilibrium to this game. Is it unique? Are there any other Nash equilibria?

(b) Suppose that player 2 cannot observe player 1's move. Write down the new extensive form. What is the set of Nash equilibria?

(c) Now suppose that player 2 observes player 1's move correctly with probability $p \in (0, 1)$ and incorrectly with probability $1 - p$. That is, if player 1 chooses U, then player 2 observes U with probability p and observes D with probability $1 - p$. Suppose that this facet of the situation and the value of p are common knowledge. What is the extensive form of this situation? Find

Turn over

all perfect Bayesian equilibria. (Hint: be very careful with the new extensive form; different actions by player 1 will lead to the same information set for player 2; think about what player 2 "sees" and how this relates to player 1's true action. Write out Bayes rule first and derive the best responses. To search for equilibria from these best responses, consider pooling first, then separating, and finally semi-separating equilibria in which both types mix or exactly one type mixes. There are many equilibria in this game.)