Cooperation, Conflict, and Power in the Absence of Property Rights

By STERGIOS SKAPERDAS*

This paper examines interaction in the absence of property rights when agents face a trade-off between productive and coercive activities. In this setting, conflict is not the necessary outcome of one-time interaction, and cooperation is consistent with domination of one agent over another. Other things being equal, an agent's power, a well-defined concept in this paper, is inversely related to an agent's resources when resources are valued according to marginal-productivity theory. Some implications for the evolution of property rights are drawn. The model is applicable to a variety of situations in which directly unproductive activities are prevalent. (*JEL* C70, D23, D30, D74)

Without complete assignment of enforceable property rights, as assumed in the competitive paradigm of economics, voluntary exchange may not exist; any single agent cannot be prevented from coercing another agent. No such constraints as property rights exist in the philosopher's hypothetical state of nature, where coercion and conflict are the presumed results. Unlike the competitive paradigm or the state of nature, property rights in actual societies are incompletely specified and enforced, and social norms or shared ethical beliefs, which could serve the same function, do not cover every contingency. Thus, opportunistic behavior and either tacit or open coercion can and do still exist. Economists have for some time recognized the need to move away from the stringent assumption of complete property rights, and many of the developments in economics over the last two decades have been partly motivated by that need.[1]

Yet little consideration has been given to the extreme case in which property rights are completely absent.[2] In this setting, agents face the classic trade-off between productive and coercive activities in its purest form, free of any institutional constraints. Its systematic study could prove useful in understanding situations in which this trade-off exists but is either subtle or limited by institutional constraints. Insights relevant to the evolution of property rights and institutions in general, currently being pursued under the more abstract goal of "the evolution of cooperation" in repeated versions of the prisoner's dilemma, can also be expected to follow from a concrete analysis of this state.

With these more distant objectives as background, this paper illuminates two issues about interaction in the absence of property rights. First, the possibility of cooperation in a static setting, its characteris-

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[1]The main developments I have in mind are transactions-cost economics, the principal–agent literature, and the theory of incomplete contracts and of missing markets.

[2]The closest exceptions are Winston C. Bush and Lawrence S. Mayer (1974) and, with appropriate interpretation, Jack Hirshleifer (1988, 1990). In its original form, Bush and Mayer's model does not provide much additional insight over the prisoner's dilemma. Dan Usher (1989), however, has employed a variation of that model to examine the rise and fall of despotic societies, a topic not directly related to the issues explored in this paper. For more details and other references see footnote 3.

tics, and the conditions under which it prevails are explored. Second, an important source of asymmetric power distribution is identified. I examine a model with two agents, each of whom can either transform her initial resources into a useful good or into "arms." The agents' choices in useful production determine the total available product, while the relative levels of arms investment determine each agent's probability of winning a war. The winner receives the total product as prize, or under an alternative interpretation, the agents could divide the total product before war takes place in proportion to their respective probabilities of winning.[3]

Despite the dual absence of property rights and long-term relationships in this model, cooperation is shown to be possible. In one type of cooperation, called "full cooperation," neither of the players invests in arms. An important condition for such an outcome to be an equilibrium is that it is sufficiently difficult for an agent to increase her probability of winning in conflict. This and other conditions and characteristics under which full cooperation prevails (developed in the body of this paper) are of descriptive and normative value for other contexts: for example, for situations in which rent-seeking and influence activities are possible.

Given the rules of interaction, there are additional (constrained) efficient outcomes, which are said to induce "partial cooperation," in which only one agent refrains from investing in arms. It is shown that the only partially cooperative outcome that could be sustained as an equilibrium is such that one agent receives his best possible payoff. The other agent refrains from investing in arms because, despite the low payoff, her marginal contribution to total product is too high to justify such an investment.

One feature of the model is that an agent's equilibrium win probability (or the share of total product) represents a clear index of the agent's power. Power is an important concept in the other social sciences, lately receiving increasing attention in economics.[4] It is, however, a notoriously difficult concept to define and investigate. When it is defined, it is frequently viewed as an exogenous parameter with unspecified determinants.[5] The implicit assumption is that linkages with the socioeconomic context are too numerous and murky to disentangle. The advantage in considering a primitive environment, like the one of this paper, is analytical as well as conceptual clarity. To the extent that coercion, either open or tacit, can be reasonably considered the main ingredient in the exercise of power, important sources of power in the primitive environment could be present as tendencies in more complicated ones.

[3]The choice the agents face between directly productive and appropriative activities is a feature of the model shared with a number of recent papers. In addition to those of Bush and Mayer (1974) and Hirshleifer (1988, 1990), this choice is present in Goran Skogh and Charles Stuart (1982), Michelle R. Garfinkel (1990), and Herschel I. Grossman (1991). Skogh and Stuart examine a model similar to Bush and Mayer's, and they focus on the feasibility of a social contract emerging out of the state of nature. Garfinkel is concerned with the dynamics of arming between nations, demonstrating the endogeneity of arms expenditures. The static counterpart to her model (i.e., the stage game) could be interpreted as a model of the state of nature, but to study the main questions my paper addresses it would have to be modified, especially by abandoning symmetry. Grossman constructs a model of insurrections with peasants dividing their labor between production, insurrection, and soldiering. My model is closest to that of Hirshleifer (1988) who, in addition to Nash equilibria, explores the implications of alternative solution concepts. The possibility of cooperation and the role of the conflict technology in its implementation [see Proposition 1a(i)] was first observed by Hirshleifer. For the other results of this paper, a cake that is asymmetric in the productive contributions of the two agents is required. Therefore, these results cannot be obtained in Hirshleifer's model, because the cake is a symmetric additive function of the agents' productive contributions (see, however, Hirshleifer [1990] for complementary insights on the issue of power).

[4]For an insightful survey of the literature on the concept of power as it relates to economics, see Pranab Bardhan (1988). Starting with Robert A. Dahl (1957), the subject has evolved into a field on its own in political science over the last 30 years.

[5]This is especially true in studies of union–firm bargaining in which power is measured by the weights in the Nash bargaining solution (see e.g., Paul Grout, 1984). Kenneth G. Binmore et al. (1986) is the only example that I know of with an endogenous determination of power.

It is shown that, with both agents having equal ability in the production of arms, the more powerful agent always possesses less-valuable initial resources for useful production when resources are valued according to the marginal-productivity theory of distribution.

Section I describes the model and the set of possible outcomes. Section II discusses the technology of conflict and introduces assumptions that guarantee existence and uniqueness of equilibrium; because some of the assumptions are empirically plausible but nonstandard, these results are of independent interest. Section III explores the conditions under which each outcome, cooperative or conflictual, is an equilibrium. Section IV examines the connection between power and the valuation of resources. Some implications for the evolution of property rights and for situations in which unproductive activities take more benign forms are briefly discussed in Section V. Section VI contains the conclusion.

I. The Model

Each of two players, labeled 1 and 2, possesses one unit of a player-specific and inalienable resource. Player 1 can transform his unit of resource into two different inputs, x_1 and y_1, with $1 = x_1 + y_1$. Similarly, player 2 can transform her unit of resource into x_2 and y_2 with $1 = x_2 + y_2$.[6] The inputs x_1 and x_2 are used in the production of the single consumption good valued by the players. The inputs y_1 and y_2 are directly unproductive and can be thought of as arms. The *production technology* $C(x_1, x_2)$ transforms the two productive inputs into the single consumption good and is assumed to have constant returns to scale and to be twice differentiable.[7] A player's marginal product is positive and decreasing. That is, letting the subscript i denote the partial derivative with respect to x_i, I assume $C_i > 0$, $C_{ii} < 0$, and $C_{12} > 0$. For technical reasons, it is also assumed that the ratio of marginal products, C_1 / C_2, is finite and nonzero.

The second main ingredient of the model is the *conflict technology* which, for any given value of y_1 and y_2, determines each player's probability of winning sole possession of both productive inputs in the event of war. For player i this probability is represented by $p(y_i, y_j)$ $(i \neq j)$, which is assumed to be a differentiable function of y_i and y_j with

$$(1) \qquad p(y_1, y_2) + p(y_2, y_1) = 1.$$

Since both players have access to the same conflict technology, the probability of winning is $\frac{1}{2}$ for either player when $x_1 = x_2$ (and $y_1 = y_2$).

The players make simultaneous choices of x_i and y_i that satisfy the resource constraints. These choices are then revealed. The production and conflict technologies, the win probabilities, and the size of the "pie" to be won in the event of war after the choices are revealed are all common knowledge. If either player declares war,[8] and assuming that both players are risk-neutral expected-utility maximizers, the

[6] Normalizing the initial resources of each player to unity is restrictive to the extent that there is a one-to-one rate of transformation between the two inputs (i.e., $1 = x_i + y_i$), which implies that the initial resources are equally good in the production of arms across the two players. The latter property makes it easier to concentrate on other sources of asymmetry in the model. Its relaxation does not change the qualitative results of the paper, although there are additional insights to be gained in the general case (see also the discussion in Section IV).

[7] Alternatively, the inputs x_1 and x_2 can be interpreted as consumption goods with both players having identical homothetic preferences represented by the homogeneous function $C(\cdot, \cdot)$ which implies transferable utility (see Theodore C. Bergstorm and Hal Varian, 1985). This alternative interpretation is as legitimate as the production interpretation maintained in the rest of the paper.

[8] The statement "declares war" means that the two players engage in open conflict with a probabilistic result. Alternatively, it may be possible for the two players to settle, under the threat of war, and divide the available surplus. Also the term "war" does not imply that the model only applies to a situation in which physical conflict is possible; see Subsection V-B for other interpretations.

payoffs are

$$\text{(2a)} \quad V^1(y_1, y_2) = p(y_1, y_2)C(1-y_1, 1-y_2)$$

$$\text{(2b)} \quad V^2(y_1, y_2) = [1-p(y_1, y_2)]C(1-y_1, 1-y_2).$$

Risk neutrality together with the constant-returns-to-scale property of $C(\cdot,\cdot)$ implies that these payoffs can have an alternative interpretation. Suppose that right before either player declares war, the two players exchange the productive inputs so that player 1 receives a $p(y_1, y_2)$ share of each input with the rest going to player 2.[9] Then, it can be shown easily that the payoffs would be identical to those in the event of war. Either player could still declare war unilaterally, even after the exchange takes place, but both players would be indifferent between the two alternatives. Moreover, this ratio of exchange is the only one that neither party can unilaterally improve upon by going to war; consequently, it is the only ratio that is viable with the threat of war. Thus, (2a) and (2b) describe the payoff functions under either war or the just-described exchange arrangement.[10] The natural solution concept to employ is Nash equilibrium, which henceforth will simply be called "equilibrium." For obvious reasons, the equilibrium win probability of a player will also be referred to as the *power* of that player.

[9]A third interpretation of (2) is that the final good is produced jointly by the two players and then divided in proportion to the win probabilities.

[10]With either increasing returns to scale in production or risk-seeking players, there would not be a possibility for such an exchange arrangement. With either decreasing returns to scale or risk-averse players (see Skaperdas [1991] for the latter possibility), there would never be war, and the settlement region would not just be a single point but would be a compact and convex set with the payoffs in (2) describing the disagreements points. Clearly, these alternative specifications can provide additional insights on the topics examined here.

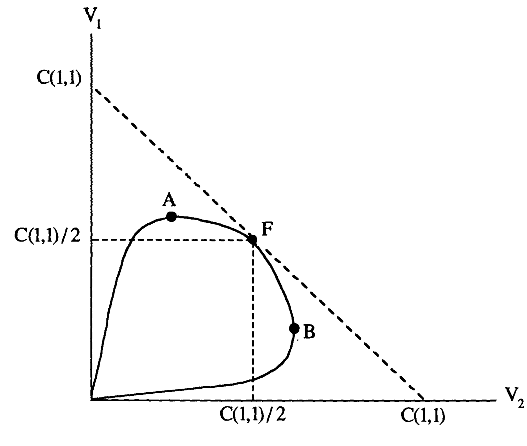

FIGURE 1. PAYOFF POSSIBILITIES SET

In game-theoretic models, efficient outcomes are usually identified with cooperation, while inefficient ones are usually identified with conflict. Although this general distinction will be maintained here, cooperative outcomes will be further subdivided into those that do not involve any waste of resources and those that do but are efficient relative to the rules of the game. In Figure 1, the dashed straight line represents the set of efficient consumption points for which $y_1 = y_2 = 0$. However, according to the payoff functions in (2), the midpoint F is the only achievable (fully) efficient outcome—ruling out *ex post* transfers between the parties—which will be called *full cooperation* from now on. Using (2), the payoff possibilities set expressing 1's payoff as a function of 2's payoff could be derived in principle. Its frontier would necessarily contain the point of full cooperation, but as the figure indicates, it may also contain points (those between A and B) that are efficient relative to the rules of the game. Any of these (constrained) efficient points will be said to induce *partial cooperation*. Finally, any outcome in the interior of this payoff possibilities set will be said to induce *conflict*.

Note that Figure 1 has been drawn so that the origin belongs to the frontier of the utility possibilities set. Such a shape can be derived from a production function for which the production of a positive quantity of the consumption good requires a positive quantity of both inputs: formally, $C(0, x_2) = C(x_1, 0) = 0$ for all x_1, x_2. In this case, since $C(1 - y_1, 1 - y_2) = 0$ when either $y_1 = 1$ or $y_2 = 1$, it follows from (2) that if one player, say 1, devotes all of his resources to the unproductive input ($y_1 = 1$), the other player's best response is the whole interval [0,1], and therefore, the strategy combination (1,1) is always an equilibrium. Because the class of production functions having this property includes many commonly utilized functional forms, showing the existence of a better equilibrium is a nontrivial issue, which is resolved in the next section.

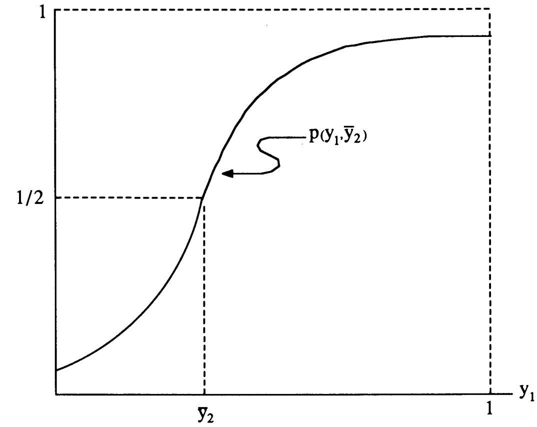

FIGURE 2. CONFLICT TECHNOLOGY

II. The Conflict Technology and Equilibrium

The conflict technology in my model relates arms investments to the probability of winning, but such functions have been used in many areas of economics relating effort levels (which need not be unproductive) to either the probability of winning or the share of the prize.[11] This section introduces assumptions on the conflict technology that are sufficient for the existence and uniqueness of pure-strategy equilibrium [excepting the "bad" (1,1) equilibrium]. Contrary to most other work utilizing conflict technologies, no specific functional forms are assumed. Instead, the assumptions restrict the signs of derivatives, a practice which economizes notation later on.

For any w and z in the interval [0,1], equation (1) can be rewritten as $p(w,z) = 1 - p(z,w)$. Letting the subscripts 1 and 2 denote the partial derivatives with respect to the first and second arguments of $p(\cdot,\cdot)$, respectively, it follows immediately that

$$\text{(3a)} \qquad p_1(w,z) = -p_2(z,w)$$

$$\text{(3b)} \qquad p_{11}(w,z) = -p_{22}(z,w)$$

$$\text{(3c)} \qquad p_{12}(w,z) = -p_{12}(z,w).$$

[11] Examples include Gordon Tullock (1980) for rent-seeking, Sherwin Rosen (1986) for tournaments, and Dixit (1987) for contests in general; see also Hirshleifer (1991) for a related perspective.

Note that for $w = y_1$ and $z = y_2$, $p(y_1, y_2)$ and $p(y_2, y_1)$ are, respectively, the win probabilities of players 1 and 2. Since they sum to 1 and the functions are identical, the partial derivatives at any point should have opposite signs and the same absolute value. For brevity's sake, $p(y_1, y_2)$ will be denoted by p. Assumption 1, below, will be maintained throughout the paper, with part (ii) illustrated in Figure 2.

ASSUMPTION 1:

(i) $0 < p_1 < \infty$ [*and by* (3a) $-\infty < p_2 < 0$]

(ii) $p_{11} \lesseqgtr 0$ *as* $y_1 \gtreqless y_2$ [*and by* (3b) $p_{22} \lesseqgtr 0$ *as* $y_1 \gtreqless y_2$]

(iii) $p_{12} \gtreqless 0$ *as* $y_1 \gtreqless y_2$

(iv) $0 < p < 1$.

Part (i) states that the win probability (or the power) of a player is increasing in that player's strategy and decreasing in the opponent's strategy. Part (ii) says that power is concave in a player's strategy when its value is greater than the value of the opponent's strategy and [by (3b)] convex otherwise. Thus, it is easier to increase one's power when it is lower than the opponent's and more difficult otherwise (see Fig. 2). The literature on contests (e.g., Avinash Dixit,

1987) usually assumes concavity in a player's strategy regardless of the strategy of the opponent. Many of the results would follow under this alternative specification.[12] Part (ii) is weaker than concavity, and it is employed here because there is evidence of its applicability in military contexts (see Trevor N. Dupuy, 1987; Hirshleifer, 1989). Part (iii) implies that the marginal return on power increases (decreases) in the opponent's strategy when p is concave (convex) in y_1, which seems natural given (ii), while part (iv) is technically convenient.

The next assumption amounts to a restriction on the convexity a function $p(\cdot,\cdot)$ can exhibit with respect to its first argument whenever the value of the function is less than $\frac{1}{2}$; it would be automatically satisfied by a concave function. The main role this assumption plays is in guaranteeing existence of equilibrium.

ASSUMPTION 2:

$$p_{11}p < p_1^2$$

[*and by* (3a) *and* (3b) $-p_{22}(1-p) < p_2^2$].

The proofs of Theorem 1 below and of most other results to follow are in the Appendix.

THEOREM 1: *Under Assumptions 1 and 2, there exists a pure-strategy equilibrium other than (1,1).*

ASSUMPTION 3:

$$p(1-p)p_{12} + (2p-1)p_1p_2 = 0.$$

This is a sufficient condition for the uniqueness of equilibrium. It could be replaced by possibly weaker (but more complicated) conditions on both the conflict and production technologies.[13]

THEOREM 2: *Under Assumptions 1–3, there exists only one pure-strategy equilibrium other than (1,1).*

III. Full Cooperation, Partial Cooperation, or Conflict as Equilibria

This section examines the conditions under which each of the three types of possible outcomes is an equilibrium. These conditions involve two readily interpretable variables: the derivative of the conflict technology and the ratio of marginal products, with both of them evaluated at the efficient production point (i.e., at full cooperation). The first variable is a measure of the ease or difficulty of conflict, while the second represents the ratio of opportunity costs of arms investment. Subsection D contains a diagrammatic summary of these conditions. Other properties of the three types of possible equilibria are also examined. The most interesting among them is that any partially cooperative equilibrium is one-sided. Any formal statement of results to follow, unless otherwise noted, depends only on Assumption 1. The proofs are in the Appendix.

A. *Full Cooperation*

Recall from Section I that full cooperation obtains only when no one invests in arms, or only when the strategy combination (0,0) is played. Letting the subscripts 1 and 2 denote partial derivatives with respect to y_1 and y_2, respectively, the following condition is necessary and sufficient for (0,0) to be an equilibrium:

$$V_1^1(0,0) \le 0 \qquad (4)$$

$$V_2^2(0,0) \le 0.$$

This is true because each player's payoff function is strictly concave in that player's

[12]The exceptions are the uniqueness of equilibrium and some of the characterization results in Subsections III-B and III-C. To generate these results with concavity, additional assumptions would have to be introduced [like part (iii), which, contrary to the present case, would not appear to be compelling].

[13]It can be shown that Assumptions 1–3 are satisfied by the following functional form (discussed in Hirshleifer [1989])

$$p(y_1,y_2) = e^{ky_1}/(e^{ky_1}+e^{ky_2}) = 1/[1+e^{k(y_2-y_1)}]$$

where $k>0$.

strategy when the other player's strategy is 0 as a consequence of the following result.

LEMMA 1:

$$V_{11}^1 < 0 \qquad \text{if } y_1 \geq y_2$$

$$V_{22}^2 < 0 \qquad \text{if } y_2 \geq y_1.$$

This lemma implies concavity of a player's payoff in her own arms investment over the range of arms investments that are at least as high as those of her opponent, and for the purpose of establishing (4) as a necessary and sufficient condition, it implies that $V_{11}^1(y_1, 0) < 0$ and $V_{22}^2(0, y_2) < 0$ for all y_1 and y_2.

Differentiation of the payoff functions in (2) at any (y_1, y_2) yields

$$V_1^1 = p_1 C - pC_1 \tag{5a}$$

$$V_2^2 = -p_2 C - (1-p)C_2. \tag{5b}$$

By the symmetry of the conflict technology, $p(0,0) = \frac{1}{2}$, and by (3a), $p_1(0,0) = -p_2(0,0)$. Using these properties in (5a) and (5b), the inequalities in (4) can be transformed, respectively, into

$$p_1(0,0)C(1,1) - \tfrac{1}{2}C_1(1,1) \leq 0$$

$$p_1(0,0)C(1,1) - \tfrac{1}{2}C_2(1,1) \leq 0.$$

Moreover, the first-degree homogeneity of $C(\cdot,\cdot)$ yields $C(1,1) = C_1(1,1) + C_2(1,1)$. By substitution into the two inequalities above, rearrangement, and letting $C_i^0 \equiv C_i(1,1)$ $(i = 1,2)$ and $p_1^0 \equiv p_1(0,0)$ for easier reference, (4) reduces to

$$\frac{2p_1^0}{1-2p_1^0} \leq \frac{C_1^0}{C_2^0} \leq \frac{1-2p_1^0}{2p_1^0}. \tag{4'}$$

It is easily confirmed that, for (4′) to hold at all, it is necessary to have $p_1^0 \leq \frac{1}{4}$; p_1^0 can be thought of as a (local) measure of the effectiveness of the conflict technology at the point of full cooperation. In the remainder of this paper, a conflict technology will be called more (respectively, less) *effective*, the larger (smaller) the value of p_1^0 is, with the understanding that it applies at the point of full cooperation. Thus, one implication of (4′) is that for (0,0) to be an equilibrium, a sufficiently ineffective conflict technology is required. Another implication of (4′) is that for any given conflict technology satisfying $p_1^0 \leq \frac{1}{4}$, the ratio of marginal products (C_1^0/C_2^0) should be sufficiently close to 1. These results are summarized as follows.

PROPOSITION 1a: *(i) A sufficiently ineffective conflict technology is required for the existence of a fully cooperative equilibrium. (ii) A fully cooperative equilibrium is more likely, the closer to 1 is the ratio of marginal products (evaluated at the point of full cooperation) and the less effective is the conflict technology.*

In situations in which either of two parties can expropriate each other and, in addition, there is no possibility for a long-term relationship, when would both parties refrain from investing in arms? The answer provided by Proposition 1a has two parts. First, it is necessary that a comparatively large differential in armaments brings about only a small increase in win probability. However, this is not always enough to establish full cooperation. The opportunity cost of arms investment can be expected to differ for the two parties, and the party with a significantly lower opportunity cost could well find arms investment worthwhile despite its relative ineffectiveness. Thus, the second part of the proposition says that these opportunity costs should be sufficiently close to each other for any given conflict technology. The *possibility of cooperation* under the simultaneous absence of property rights and long-term relationships is contrary to the widely held presumption of constant warfare in the state of nature and is consistent with the anthropological evidence on the earliest societies in the evolutionary hierarchy.[14] To my knowledge,

[14]These are family-level societies in which, according to Allen W. Johnson and Timothy Earle (1987 p. 93) warfare is virtually nonexistent. For village-level

only Hirshleifer (1988) has observed the connection between the difficulty in the conduct of conflict and cooperation.[15]

The following lemmas help establish another property of a fully cooperative equilibrium, as well as some other results later on.

LEMMA 2: $V_j^i < 0$ $(i \neq j)$. *(Each player's payoff function is strictly decreasing in the opponent's arms investment.)*

LEMMA 3: $V^i(0,0) > V^i(y_1, y_2)$ *for all* (y_1, y_2) *such that* $y_i \neq 0$ *if and only if* $V_i^i(0,0) \leq 0$ $(i = 1, 2)$.

Since (4) is necessary and sufficient for full cooperation to be an equilibrium, Proposition 1b follows by Lemma 3.

PROPOSITION 1b: *Full cooperation is an equilibrium if and only if for both players this outcome payoff-dominates every other possible outcome.*

Figure 3 shows a typical payoff possibilities set when full cooperation is an equilibrium. The low effectiveness of the conflict technology severely reduces the range of payoff pairs that are possible; note the absence of any partially cooperative outcomes. Another implication of the proposition is that full cooperation would Pareto-dominate any other possible equilibrium.

B. *Partial Cooperation*

According to the definition in Section I, with the exception of full cooperation, all

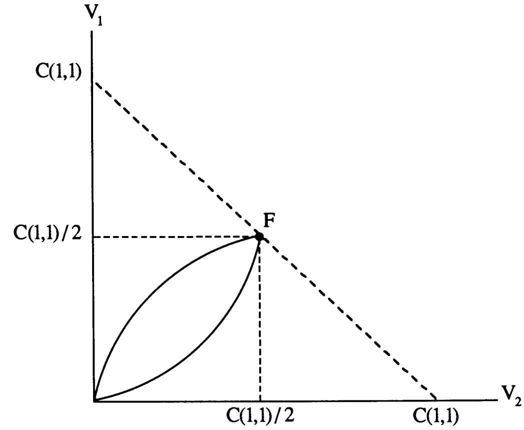

FIGURE 3. FULL COOPERATION

the payoff pairs that are efficient given the payoff functions in (2) belong to the set of partially cooperative outcomes. Lemma 4 reveals that at any partially cooperative outcome only one player invests in arms.[16]

LEMMA 4: *Let* (y_1', y_2') *be a partially cooperative strategy combination. Then, either* $y_1' = 0$ *and* $y_2' > 0$, *or* $y_1' > 0$ *and* $y_2' = 0$.

For specificity, let $(0, y_2^*)$ with $y_2^* > 0$ be a partially cooperative equilibrium, so that player 2 is the one who invests in arms. Then, for this strategy combination to be an equilibrium, the following condition is necessary:

$$V_1^1(0, y_2^*) \leq 0 \tag{6}$$
$$V_2^2(0, y_2^*) = 0.$$

societies, however, which are higher in the evolutionary hierarchy, like the well-known Yanomano people (see Napoleon Chagnon, 1983), warfare is endemic. The main difference between these two types of society is that population pressures and, hence, competition for food are more intense in the latter, which tends to make warfare cheaper relative to food production (and the conflict technology becomes more effective).

[15] In Hirshleifer's (1988) model, the production function is linear and symmetric. Thus, the opportunity cost of investing in arms is the same for the two players.

[16] There are two other papers I know of in which only one side invests in unproductive activities. Hirshleifer (1989) explores a contest with a fixed prize but with the players allowed to have different valuations of it, which can result in only one player making a positive effort in capturing the prize. Similarly, in Grossman (1991) it is possible for peasants not to expend any effort on insurrections while the ruler does hire soldiers. It also appears (but I have not formally shown this) that those equilibria are constrained-efficient in the same sense that partial cooperation is constrained-efficient.

Since $V^2(0, y_2)$ is strictly concave in y_2 by Lemma 1, and since $V^2(0, y_2) > V^2(y_1, y_2)$ for all $y_2, y_1 > 0$ by Lemma 2, the second part of (6) implies that $V^2(0, y_2^*)$ is the maximum possible payoff for player 2. Thus far, the following has been shown.

PROPOSITION 2a: *In a partially cooperative equilibrium, only one player invests in the unproductive input. That player also receives his best possible payoff.*

Although this equilibrium is constrained-efficient and its characterization as "cooperative" comes from that property, it is rather one-sided. In fact, for cases in which the equilibrium payoff pair is sufficiently far from the point of full cooperation, like that in Figure 4, it strongly resembles a *subjugational* arrangement: the outcome serves solely the interests of the "master" (player 2), who needs to have a requisite quantity of arms in order to enforce it and keep the "slave" at bay.

The next natural question to ask is: under what conditions can this type of outcome be an equilibrium? For the answer, Assumption 3 will be invoked (which has not been used in deriving Proposition 2a). It will be shown that, for player 1, full cooperation needs to dominate every other possible outcome as it does on Figure 4. This condition will be further reduced to one comparable to (4′). First, it can be shown that, for all $y_2 \in [0, y_2^*)$,

$$(6') \qquad V_1^1(0, y_2) < 0$$
$$V_2^2(0, y_2) > 0$$

is necessary for $(0, y_2^*)$ to be an equilibrium. [The proof is in the Appendix; the inequalities would be reversed for an equilibrium $(y_1^*, 0)$ such that $y_1^* > 0$.] Next consider the following special case of (6′):

$$(7) \qquad V_1^1(0,0) < 0$$
$$V_2^2(0,0) > 0$$

which is comparable to (4), but now it is

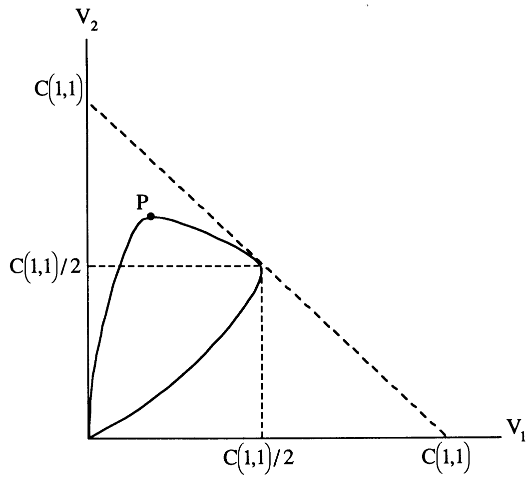

FIGURE 4. PARTIAL COOPERATION

only a necessary condition for a partially cooperative equilibrium $(0, y_2^*)$. Note that the first part of (7) readily implies, by Lemma 3, that full cooperation dominates every other outcome for player 1. By using (5a) and (5b) and following the same steps as in deriving (4′) from (4), (7) can be shown to be equivalent to

$$(7') \qquad \frac{C_1^0}{C_2^0} > \max\left\{\frac{2p_1^0}{1-2p_1^0}, \frac{1-2p_1^0}{2p_1^0}\right\}$$

provided $p_1^0 < \frac{1}{2}$. For an equilibrium $(y_1^*, 0)$, the condition is the same except that the ratio on the left-hand side should be reversed. Since it is necessary to have $p_1^0 < \frac{1}{2}$, the conflict technology must be sufficiently ineffective, but not as much as in the case of a fully cooperative equilibrium in which $p_1^0 \leq \frac{1}{4}$. Another implication of (7′) is that the ratio C_1^0/C_2^0 should be large enough when $(0, y_2^*)$ is the equilibrium and small enough when $(y_1^*, 0)$ is the equilibrium. Because the right-hand side of (7′) is greater than 1, note that the more powerful player (i.e., the one with $y_i^* > 0$) has lower marginal product at the point of full cooperation. This last implication will be discussed more fully in Section IV. The implications of (7′) are summarized below.

PROPOSITION 2b: *In a partially cooperative equilibrium, (i) the conflict technology is sufficiently ineffective but not necessarily as much as when full cooperation obtains, and (ii) the marginal products for the two players (evaluated at full cooperation) are sufficiently diverse for any given conflict technology, with the more powerful player having a lower marginal product (derived under Assumptions 2 and 3).*

C. *Conflict*

Since (4) is necessary and sufficient for a fully cooperative equilibrium and (7) is necessary for a partially cooperative equilibrium, it follows that

$$V_1^1(0,0) > 0 \qquad (8)$$

$$V_2^2(0,0) > 0$$

is a sufficient condition for the existence of a conflict equilibrium (again, under Assumptions 2 and 3). Lemma 3 implies by (8) that full cooperation is dominated by at least some other outcomes for both players. By going through similar steps as in the previous two cases, (8) can be shown to imply either

$$p_1^0 > \tfrac{1}{2} \qquad (8')$$

or

$$\frac{1-2p_1^0}{2p_1^0} < \frac{C_1^0}{C_2^0} < \frac{2p_1^0}{1-2p_1^0} \qquad (8'')$$

$$\text{with } \tfrac{1}{4} < p_1^0 \leq \tfrac{1}{2}$$

and in turn, either (8′) or (8″) implies (8). Proposition 3 summarizes the information contained in these two conditions.

PROPOSITION 3: *(i) For all conflict technologies beyond a minimum effectiveness level, there is a unique conflictual equilibrium. (ii) For conflict technologies below the minimum effectiveness level in (i), but above the maximum that can support cooperation, and with marginal product ratios (at full cooperation) sufficiently close to 1, there is also a unique conflict equilibrium (Assumptions 2 and 3).*

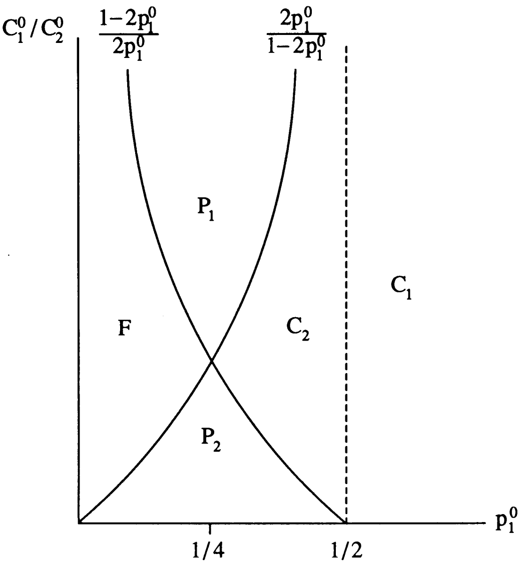

FIGURE 5. EQUILIBRIUM REGIONS

When investment in arms can easily increase power [$p_1^0 > \frac{1}{2}$ in (8′)] at the point of full cooperation, the marginal benefit of that investment ($p_1^0 C^0$) always exceeds its opportunity cost ($\frac{1}{2}C_i^0$) for both players. As a result, both players invest in arms in this case. When the conflict technology is less effective, however, the opportunity costs start to matter. One player's opportunity cost may be so high that he decides not to invest in arms, and as the effectiveness decreases, the range of values the opportunity-cost ratios can take in order to induce conflict narrows to point $p_1^0 = \frac{1}{4}$, beyond which conflict cannot be guaranteed anymore.

D. *A Diagrammatic Summary*

It has been shown that the type of outcome that emerges as an equilibrium depends on the relationship between the effectiveness of the conflict technology (p_1^0) and the ratio of marginal products (C_1^0/C_2^0). Figure 5 summarizes this relationship. The graphs of the two indicated functions of p_1^0, along with the horizontal line through $p_1^0 = \frac{1}{2}$, partition the figure into five regions. Every point in region F satisfies (4′), and therefore it leads to a fully cooperative equilibrium, and if an equilibrium is fully cooperative the corresponding parameter values must lie in F. Points in regions C_1 and C_2 satisfy (8′) and (8″), respectively. Finally, points in P_1 and P_2 satisfy (7′) and (7′) with its left-hand side inverted, respectively. Since (7′) is just necessary for partial cooperation and (8′) and (8″) are only sufficient for conflict, points in P_1 or P_2 could actually induce a conflict equilibrium.

IV. Power and the Valuation of Resources

With the exception of full cooperation, power is almost always asymmetrically distributed in equilibrium. Since full cooperation is assured for lower levels of conflict effectiveness, a fundamental requirement for the emergence of power asymmetry is the presence of a sufficiently high conflict effectiveness. Thus, this section concentrates on the other two types of possible equilibria, conflict and partial cooperation.

First, it will be shown that the more powerful player has lower marginal product than his opponent at the equilibrium point. Consider any conflict equilibrium (y_1^*, y_2^*) where $y_1^*, y_2^* > 0$.[17] Since this is an interior equilibrium, the following condition is necessary:

$$V_1^1(y_1^*, y_2^*) = 0$$

$$V_2^2(y_1^*, y_2^*) = 0$$

which, by (5), is equivalent to

$$p_1^* C^* - p^* C_1^* = 0$$

$$-p_2^* C^* - (1-p^*) C_2^* = 0$$

where an asterisk indicates evaluation at the equilibrium point. By eliminating C^* from these two equations and rearranging, one obtains

$$\frac{C_1^*}{C_2^*} = \frac{p_1^*(1-p^*)}{-p_2^* p^*}. \tag{9}$$

The following result helps establish the relationship between marginal products and power.

LEMMA 5: $p_1^*(1-p^*)/-p_2^* p^* > 1$ *if and only if* $p^* < \frac{1}{2}$ *(Assumption 2).*

By (9), Lemma 5 implies that the ratio of marginal products at the equilibrium point is inversely related to the ratio of powers, or that $C_1^*/C_2^* > 1$ (respectively, $C_1^*/C_2^* < 1$) is equivalent to $p^* < \frac{1}{2}$ ($p^* > \frac{1}{2}$). This is not surprising in the context of the model. In equilibrium, the marginal cost of the unproductive investment, $p^* C_1^*$ for player 1 and $(1-p^*)C_2^*$ for player 2, is equated to its marginal benefit, $p_1^* C^*$ for player 1 and $-p_2^* C^*$ for player 2. Suppose for the moment that the marginal benefits are equal for the two players, implying the equality of marginal costs or that $p^* C_1^* = (1-p^*)C_2^*$. Power in this case would then be inversely related to marginal productivity. Lemma 5 says that under Assumption 2 the marginal benefits are always sufficiently similar so as not to reverse this result.

Next it is shown that the more powerful player has lower marginal product not just at equilibrium, but at the efficient production point as well. For specificity, suppose $y_1^* < y_2^*$ (which implies $p^* < \frac{1}{2}$). Then, it follows that

$$C_1^* \equiv C_1(1-y_1^*, 1-y_2^*)$$

$$< C_1(1-y_1^*, 1-y_1^*)$$

$$C_2^* \equiv C_2(1-y_1^*, 1-y_2^*)$$

$$> C_2(1-y_1^*, 1-y_1^*).$$

However, since the production technology has constant returns, $C_i(1-y_1^*, 1-y_1^*) = C_i(1,1) \equiv C_i^0$ for both $i=1$ and $i=2$). Hence, the two inequalities above are

[17] The arguments for the case of partial cooperation are similar.

equivalent to $C_1^* < C_1^0$ and $C_2^* > C_2^0$, respectively, and $C_1^0/C_2^0 > C_1^*/C_2^* > 1$ (the last inequality follows from $p^* < \frac{1}{2}$). In a similar fashion, it can be established that $p^* > \frac{1}{2}$ implies that $C_1^0/C_2^0 < C_1^*/C_2^* < 1$. The results so far are summarized as the following proposition.

PROPOSITION 4: *At any equilibrium with power asymmetry, the more powerful player has lower marginal product than the less powerful player, both at the equilibrium point and at the efficient production point (Assumption 2).*

Does the player with greater power, and lower marginal product, possess more or less valuable resources? The only available method for comparing different types of resources, the marginal-productivity theory of distribution, yields an answer in this case. According to this theory, player 1's share at the efficient production point is C_1^0 $[\equiv C_1(1,1)]$ and player 2's share is C_2^0, with the sum of the two shares exhausting the total product by the assumption of constant returns to scale. Note, however, that $C_1^0 > C_2^0$ (so that 1's resource is worth more than that of 2) is equivalent, by Proposition 4, to player 1 having less power.[18] A sufficient condition in arriving at this result is that the two players have equal abilities to produce arms in the sense that there is a one-to-one rate of transformation between arms and each player's productive good, given the normalization of initial resources (remember that $y_i + x_i = 1$ for both $i = 1$ and $i = 2$). This result is stated as a corollary.

COROLLARY: *When agents have equal ability in the production of arms and resources are valued according to the marginal-productivity theory of distribution, the more powerful agent always possesses less valuable resources (Assumption 2).*

It could be argued that the player with more valuable resources for useful production should be better in the production of arms as well, thus contradicting the condition in the corollary. I do not deny that this may be true in certain cases, especially when the low marginal product is due to high past investments on the initial resources of a player, but the result would not necessarily be reversed under these conditions; to do so, the positive correlation between value of resources and arms productivity should be sufficiently high.[19] Moreover, it could be argued equally well that those with less valuable productive resources tend to specialize in arms production, thereby gaining greater ability in this endeavor, or conversely, those better in arms production tend to ignore the development of their abilities in useful production. In the latter case, the power of the less usefully productive party would be reinforced. This last case seems to accord well with the historical tendency of nomads and mountaineers, with marginal means of existence, subjugating peasants cultivating the richer lowlands (Franz Oppenheimer, 1972); it also happens that mountaineers and nomads are better in warfare than peasants.

V. Some Implications

A. *Domination and Contract*

Interaction in the state of nature is often described in terms of the prisoner's dilemma analogy. Analogies, though, are not as good in analyzing the nuances of a problem as they are in broadly defining it. The static

[18] The same result can be shown when productive resources are valued according to marginal productivity at an equilibrium point, instead of at the efficient production point. Additionally, under the "exchange" interpretation of the model (see footnote 7), this share of each player (at either equilibrium or the efficient point) would represent the player's utility at the competitive equilibrium.

[19] Thus, in Hirshleifer (1990) the players always have equal power at any interior equilibrium, regardless of initial resources, due to the symmetry of the production function (which is equal to $x_1 + x_2$, the sum of the productive inputs, which are denominated in the same units and are thus directly comparable). Although power is equal in this case, the "poorer" player devotes both absolutely and relatively fewer resources to useful production.

model of this paper provides a more refined view of this state. On the one hand, conflict is not the necessary outcome of one-time interaction in this setting; the use of force or the threat of its use may be too difficult or costly. On the other hand, if conflict were to occur, domination of one party over another one would be more typical than the symmetric outcome in the prisoner's dilemma.

Since repeated versions of the prisoner's dilemma are the primary vehicle in recent work on the evolution of cooperative institutions and social norms,[20] these caveats may bias significantly one's understanding of these processes. In particular, it is assumed, mostly implicitly, that the emergence of institutions like the state and property rights is achieved solely through a contractarian process (i.e., the interested parties jointly agree to a social contract, which is typically enforced through long-term relationships). Although this process is relevant and important in the propagation and development of these institutions once they are established, it sidesteps the role of coercion in their establishment, which is common, if not universal, in practice (see e.g., Robert L. Carneiro, 1970; Oppenheimer, 1972).

With primitive means of warfare and widely dispersed population (which also reduce the effectiveness of conflict), cooperation is not a problem, and the need for institutions is absent. Conflict becomes a problem when the conduct of warfare is more sophisticated. Actual warfare need not occur, however. Parties can settle under the threat of war (as can be done in the model), thereby establishing a semblance of property rights which can become institutionalized in a dynamic setting.[21] Conflict, in its metaphorical sense as waste of resources due to the continued production of arms, could still persist, but it could be reduced or eliminated altogether through credible contracting. Nevertheless, any contract would have the original settlement under the threat of war as its starting point, which would almost always favor one party.

In fact, the results of the previous section suggest that any initial asymmetries could be compounded over time. At any point in time, each party's resource can be expected to be an increasing function of past payoffs. With an unequal initial distribution of payoffs, the dominant party would reproduce its own resource faster in relation to its opponent. One effect of this is the further reduction of the dominant party's marginal product at the efficient point of production, which reduces the opportunity cost of arms investment and results in greater power than before. This process could be arrested eventually by the effect of third parties or other factors, but there is no guarantee that these effects would significantly alter the trend toward domination.

B. *Influence Activities and Bargaining*

The trade-off between production and coercion in the state of nature is formally similar to the trade-off that can appear when property rights over economic resources are unassigned. Individual agents in these situations, instead of investing in overtly coercive activities, can engage in directly unproductive activities so as to gain an advantage over their opponents. Thus, the model has some formal affinity to models of rent-seeking and related activities.[22]

In particular, the results, and to a lesser extent the modeling specifics, resemble those in Paul Milgrom (1988) and Milgrom and John Roberts (1988), who have examined influence activities in hierarchical organizations. Managers are usually subject to these activities from their employees. Often, but not always, this practice involves unpro-

[20] Recent research employs models with many agents who are randomly matched in each period. Examples with a traditional game-theoretic approach are provided by Michihiro Kandori (1988) and Joseph E. Harrington, Jr. (1989). Those with an evolutionary stability approach include Robert Axelrod (1984) and Robert H. Frank (1987).

[21] This idea and the discussion that follows are developed more extensively in section VII of Skaperdas (1990).

[22] For examples, in addition to the two references discussed in this section, see Tullock (1980) for rent-seeking and Ronald Findlay and Stanislaw Wellisz (1982) for tariff-lobbying.

ductive use of time. The model could be reinterpreted to describe the limiting case whereby the remuneration of employees is solely based on the amount of influence exerted on the manager,[23] given the output achieved by the organizational unit in question, with the conflict technology now renamed "influence technology." Then, for the superior caring only about total output, it would be advisable to limit the effectiveness of influence activities. This can be achieved directly, by limiting access to the manager, or indirectly, by using preannounced alternative compensation schemes. The result on power also suggests that the more unproductive employees would be more influential, which can have other deleterious effects on an organization. The degree of pay equity, seniority-based promotion rules, and other similar practices in organizations which are at odds with traditional theories can partly be explained as attempts to limit influence activities.

The model explored here could also be interpreted as a bargaining model with endogenously determined bargaining costs, given a bargaining-costs technology. Since Ariel Rubinstein (1982), most bargaining models include exogenously fixed costs. However, even under conditions of complete information, bargaining agents could have access to commitment devices with variable costs, or if these do not exist, they may be artificially introduced. In general, when there is indeterminacy in the terms of trade, the interested parties can be very creative in reducing it, each in her or his favor and at some cost of course. John F. Nash's (1953) variable-threat bargaining has similar motivations, although bargaining costs are not explicitly introduced there.

VI. Conclusion

There are two sets of issues about interaction in the absence of property rights that this paper has examined. First came an exploration of the possibility of cooperation and its characteristics in a static setting. The agents cooperate in equilibrium when conflict is ineffective, that is, when win probabilities are significantly different only for large arms differentials and, in addition, when their marginal contributions to useful production are similar. These results have descriptive as well as normative implications, the latter for situations in which the effectiveness or costs of conflict can be controlled by a principal. Cooperation itself can also be consistent with domination of one party over another.

As for the issue of power, it has been shown that ceteris paribus the more powerful agent always possesses less valuable productive resources when resources are valued according to the marginal-productivity theory. The argument driving this result is simple, and it runs as follows. Since the more powerful agent invests more in arms, he or she must have lower opportunity cost for that investment. With the relative contributions to total product being the sole source of asymmetry in the model, the marginal cost of arms investment is inversely related to these marginal contributions.

Allowing for interaction among more than two agents without the possibility of coalition formation would be a straightforward but uninteresting exercise; the qualitative results of this paper would be unaffected. Taking into account the possibility of coalition formation, however, would be an important, but difficult, undertaking especially for investigating the robustness of the relationship between power and the value of resources. Would the more powerful—and less productive—agent in the two-person case tend to attract third parties as partners in the multiperson case, or would alliances form against such agents? I do not have strong intuition for the likelihood of either of these two extremes or for intermediate cases. In turn, this a priori agnosticism reflects the difficulties not just in solving but in simply formulating the long-standing problem of coalition formation.

To investigate further the topics touched upon in the last section, the model could be extended in several other directions. For the study of environments with directly un-

[23]Thus, it is implicitly assumed that the manager cannot monitor individual performance of employees with any reliability.

productive activities, static extensions could yield additional insights about the effect of decreasing or increasing returns to scale in production, of different attitudes toward risk, or of alternative game rules. Nontrivial dynamic extensions of the model, which are important for exploring the evolution of property rights and related institutions, would be harder to pursue without drastic simplifications.

APPENDIX

PROOF OF LEMMA 1:

Letting the subscripts 1 and 2 denote partial derivatives with respect to y_1 and y_2, respectively, differentiation of the payoff functions in (2) at any (y_1, y_2) yields

$$V_1^1 = p_1 C - pC_1 \tag{5a}$$

$$V_2^2 = -p_2 C - (1-p)C_2 \tag{5b}$$

$$V_{11}^1 = p_{11}C - 2p_1 C_1 + pC_{11} \tag{A1a}$$

$$V_{22}^2 = -p_{22}C + 2p_2 C_2 + (1-p)C_{22}. \tag{A1b}$$

Consider equation (A1a). The second term is negative because $C_1 > 0$ and $p_1 > 0$, and the third term is negative because $C_{11} < 0$ and $p > 0$. By part (ii) of Assumption 1, $p_{11} \leq 0$ when $y_1 \geq y_2$, which implies that the first term is nonpositive. Then, the first part of the lemma follows immediately, with its second part following from symmetric arguments on (A1b).

PROOF OF LEMMA 2:

Differentiation of V^1 and V^2 with respect to y_2 and y_1, respectively, yields

$$V_2^1 = p_2 C - pC_2 \tag{A2a}$$

$$V_1^2 = -p_1 C - (1-p)C_1. \tag{A2b}$$

Since $C_1, C_2 > 0$, and since it is true that $0 < p < 1$, $p_1 > 0$, and $p_2 < 0$ (by Assumption 1), the lemma follows easily.

PROOF OF LEMMA 3:

For specificity, let $i = 1$ and first consider $V_1^1(0,0) \leq 0$. By Lemma 1, $V^1(y_1, 0)$ is strictly concave, and thus $V^1(0,0) > V^1(y_1, 0)\ \forall y_1 \neq 0$. In addition, Lemma 2 implies $V^1(y_1, 0) \geq V^1(y_1, y_2)\ \forall\ y_2$. Combining these two inequalities yields the stated result. Next, suppose $V^1(0,0) > V^1(y_1, y_2)$ and $V_1^1(0,0) > 0$. By the latter inequality, Lemma 1 implies the existence of some $y_1' > 0$ such that $V^1(y_1', 0) > V^1(0, 0)$, which contradicts $V^1(0,0) > V^1(y_1, y_2)$ for all y_1 and y_2. Therefore, it must be that $V_1^1(0,0) \leq 0$ as stated in the lemma.

Lemmas A1–A3 contribute to the proof of Theorem 1. To minimize the number of cases examined, it will be maintained that $C(1-y_1, 1-y_2) = 0$ when either $y_1 = 1$ or $y_2 = 1$, so that $(1,1)$ is always an equilibrium (see discussion at the end of Section I).

LEMMA A1:

$$V_{ii}^i < 0 \textit{ if } V_i^i \leq 0 \textit{ and } y_j \neq 1$$

(by Assumption 2).

PROOF:

Suppose $V_1^1 \leq 0$. Then (5a) implies $C_1 \geq p_1 C/p$, and since $p_1 > 0$ (by Assumption 1), $-2p_1 C_1 < -2p_1^2 C/p$. Using this in (A1a) yields $V_{11}^1 < C[(p_{11}p - 2p_1^2)/p] + pC_{11}$. Because, by Assumption 2, $p_{11}p - 2p_1^2 < 0$, the first term of this expression is nonpositive $(C \geq 0)$. In addition, $C_{11} < 0$ and $p > 0$ imply that the second term is negative. Therefore, $V_{11}^1 < 0$, as stated in the lemma, with the proof for $i = 2$ following identical steps.

LEMMA A2: *Each player's payoff function is strictly quasi-concave in its own strategy, provided the opponent's strategy is less than* 1 *(by Assumption 2).*

PROOF:

To show strict quasi-concavity for a function of one variable, it is sufficient to show either strict monotonicity or that the function is first strictly increasing and then strictly decreasing over the interval on which it is defined. Consider $V^1(y_1, \bar{y}_2)$ for some $\bar{y}_2 < 1$. Since $V^1(0, \bar{y}_2) > 0 = V^1(1, \bar{y}_2)$ [since the case of $C(y_2, 0) = 0$ is considered here], there exists $\bar{y}_1 \equiv \min\{y_1 \in [0,1) | V_1^1(y_1, \bar{y}_2) =$

0}. It then follows that $V_1^1(y_1,\bar{y}_2)>0\ \forall y_1<\bar{y}_1$ and, by Lemma A1 above, $V_1^1(y_1,\bar{y}_2)<0\ \forall\ y_1>\bar{y}_1$. Thus, $V^1(y_1,\bar{y}_2)$ is either strictly decreasing in y_1 ($\bar{y}_1=0$) or first strictly increasing and then strictly decreasing ($\bar{y}_1>0$) as required.

When the opponent's strategy is 1, the payoff function is zero and therefore quasi-concave in that player's strategy. Then, together with the continuity of the payoff functions this lemma guarantees the existence of a pure-strategy equilibrium (see e.g., theorem 1 in Partha Dasgupta and Eric Maskin [1986]). However, as has been discussed already, (1,1) is an equilibrium strategy combination. This introduces a complication which, essentially, is resolved in part (iii) of Lemma A3, below.

LEMMA A3: *Let $r_i(y_j)$ be the best-response function of player i for $y_j\in[0,1)$. This function has the following properties:*

(i) it is single-valued and continuous (by Assumption 2),

(ii) $r_i(y_j)<1$,

(iii) $\lim_{y_j\to 1} r_i(y_j)<1$.

PROOF:

Part (i) follows immediately from Lemma A2. Part (ii) is also immediate: with $y_j<1$, any $y_i<1$ would yield a positive payoff, while $y_i=1$ always yields zero payoff. The proof of (iii) is more involved. For specificity let $i=1$ and $j=2$ and suppose, contrary to the statement, one wants to prove

$$\lim_{y_2\to 1} r_1(y_2)\equiv r^*=1$$

which implies that $r_1(y_2)>0$ for y_2 sufficiently close to 1. Therefore, at any such point, using (5a), one has

$$\text{(A3)}\quad \frac{p(r_1(y_2),y_2)}{p_1(r_1(y_2),y_2)} = \frac{C(1-r_1(y_2),1-y_2)}{C_1(1-r_1(y_2),1-y_2)}.$$

The left- and right-hand sides are continuous functions of y_2, whose limits are equal as $y_2\to 1$. The limit of the left-hand side, $p(r^*,1)/p_1(r^*,1)=p(1,1)/p_1(1,1)$, is positive by Assumption 1 [part (iv) for the numerator and part (i) for the denominator]. Thus, the limit of the right-hand side must also be positive. Note that, by Euler's Theorem,

$$C(1-r_1(y_2),1-y_2) = C_1(1-r_1(y_2),1-y_2)[1-r_1(y_2)] + C_2(1-r_1(y_2),1-y_2)(1-y_2).$$

Substitution of this expression into the right-hand side of (A3) implies that its limit is equal to

$$\lim_{y_2\to 1}[1-r_1(y_2)] + \lim_{y_2\to 1}\left[\frac{C_2(1-r_1(y_2),1-y_2)}{C_1(1-r_1(y_2),1-y_2)}\right] \times \lim_{y_2\to 1}(1-y_2).$$

Since C_2/C_1 is finite and since by supposition $r^*=1$, this expression is zero. However, this result contradicts the positivity of the left-hand side of (A3) and the initial supposition of $r^*=1$. Thus, it must be that $r^*<1$ [part (iii)] as required.

PROOF OF THEOREM 1:

Consider

$$R_i(y_j)=\begin{cases} r_i(y_j) & \text{if } y_j<1 \\ \lim_{y_j\to 1} r_i(y_j) & \text{if } y_j=1 \end{cases}$$

which is a continuous function on [0,1]. Then, by Brouwer's fixed-point theorem, the function $R_1\circ R_2:[0,1]\to[0,1]$ has a fixed point. Any such fixed point would belong to [0,1), as required, provided $R_i(y_j)<1$ for both i and all y_j. By Lemma A3, parts (ii) and (iii), this is always true.

LEMMA A4: *$V^1_{12} \geq pC_{12}$ when $V^1_1 \leq 0$ and $V^2_2 \geq 0$, and $V^2_{12} \geq (1-p)C_{12}$ when $V^1_1 \geq 0$ and $V^2_2 \leq 0$. Additionally, if $V^1_1 = V^2_2 = 0$, then $V^1_{12} = pC_{12}$ and $V^2_{12} = (1-p)C_{12}$ (by Assumption 3).*

PROOF:

The cross-partial derivatives at any point are as follows:

$$(A4a)\quad V^1_{12} = p_{12}C - p_1C_2 - p_2C_1 + pC_{12}$$

$$(A4b)\quad V^2_{12} = -p_{12}C + p_1C_2 + p_2C_1 + (1-p)C_{12}.$$

If $V^1_1 \leq 0$ and $V^2_2 \geq 0$, (5a) and (5b) imply, respectively, that $C_1 \geq p_1C/p$ and $C_2 \leq -p_2C/(1-p)$. Using these two inequalities, along with $p_1 > 0$ and $p_2 < 0$, yields

$$(A4a')\quad V^1_{12} \geq p_{12}C + \frac{p_1p_2C}{1-p} - \frac{p_1p_2C}{p} + pC_{12}$$
$$= C\left[\frac{p_{12}(1-p)p + (2p-1)p_1p_2}{p(1-p)}\right] + pC_{12}.$$

By Assumption 3, the numerator of the term inside the brackets is zero, and $V^1_{12} \geq pC_{12}$ follows immediately. In addition, if $V^1_1 = V^2_2 = 0$, (A4a′) holds as an equality, as stated in the lemma. The symmetric properties for V^2_{12} are similarly established using (A4b).

In showing the uniqueness of equilibrium, the notion of local stability will be employed. For an equilibrium $(y^*_1, y^*_2) \in [0,1)^2$ to be locally stable it is sufficient to have $r'_1(y^*_2)r'_2(y^*_1) < 1$ with $r'_i(y^*_j) = 0$ if $r'_j(y^*_i) < 0$.[24]

[24]Usually these conditions are considered to be sufficient for local stability only for interior points of the strategy space. It can be easily confirmed that local stability (for a definition, see Herve Moulin [1986 p. 134]) is also implied in my model by these conditions when either y^*_1 or y^*_2 is zero.

LEMMA A5: *If $(y^*_1, y^*_2) \in [0,1)^2$ is an equilibrium, then it is locally stable (by Assumptions 2 and 3).*

PROOF:

To minimize the notational burden, all the partial derivatives of the payoff functions appearing below should be understood to be evaluated at (y^*_1, y^*_2). By the strict quasi-concavity of the payoff functions (Lemma A2), it follows that $V^1_1 \leq 0$ and $V^2_2 \leq 0$. Two cases are considered.

Case A: $V^1_1 = V^2_2 = 0$.—Then, the following holds:

$$(A5)\quad r'_1(y^*_2) = -\frac{V^1_{12}}{V^1_{11}}$$
$$r'_2(y^*_1) = -\frac{V^2_{12}}{V^2_{22}}.$$

$V^1_1 = 0$, by (5a) implies that $2p_1C_1 = 2p^2_1C/p$. Using this expression in (A1a) yields $V^1_{11} = C(p_{11}p - 2p^2_1)/p + pC_{11}$. Since, by Assumption 2, $p_{11}p - p^2_1 < 0$, it follows that $V^1_{11} < pC_{11} < 0$. By going through similar steps for V^2_{22}, it follows that

$$(A6)\quad V^1_{11} < pC_{11} < 0$$
$$V^2_{22} < (1-p)C_{22} < 0.$$

$V^1_1 = V^2_2 = 0$ implies, by Lemma A4, that

$$(A7)\quad V^1_{12} = pC_{12} > 0$$
$$V^2_{12} = (1-p)C_{12} > 0.$$

Using (A6) and (A7) in (A5) yields:

$$(A5')\quad 0 < r'_1(y^*_2) < -\frac{C_{12}}{C_{11}}$$
$$0 < r'_2(y^*_1) < -\frac{C_{12}}{C_{22}}.$$

Since both derivatives in (A5′) are positive, it remains to be shown that $r'_1(y^*_2)r'_2(y^*_1) <$

1, or by (A5′) that

$$C_{12}^2/(C_{11}C_{22}) \leq 1. \tag{A8}$$

However, the first-degree homogeneity of $C(\cdot,\cdot)$ implies zero-degree homogeneity for $C_1(\cdot,\cdot)$ and $C_2(\cdot,\cdot)$. The application of Euler's theorem on those two partial derivatives yields $C_{12}^2 = C_{11}C_{22}$. Therefore, (A8) holds as an equality, and local stability always follows.

Case B: $V_1^1 < 0$ or $V_2^2 < 0$.—Then, $r_1'(y_2^*) = 0$ or $r_2'(y_1^*) = 0$, with stability following immediately.

PROOF OF THEOREM 2:

Note that the local stability of every equilibrium other than $(1,1)$ precludes the existence of a continuum of them, since otherwise for any neighborhood around any particular equilibrium there would exist some strategy combinations (i.e., other equilibria) that could not converge to it. Therefore, if multiple equilibria were to exist, they could only be isolated. Suppose (y_1^1, y_2^1) and (y_1^2, y_2^2) are two isolated equilibria such that $y_1^1 < y_1^2$ and no equilibrium exists for $y_1 \in (y_1^1, y_1^2)$. By the definition of equilibrium, $r_1(r_2(y_1^k)) = y_1^k$ $(k = 1,2)$. Note that

$$\frac{d}{dy_1}\left[r_1(r_2(y_1^1))\right] = r_1'(y_2^1)r_2'(y_1^1) < 1$$

with the inequality following from Lemma A2; since, by assumption, no equilibrium exists for $y_1 \in (y_1^1, y_1^2)$, it must be that $r_1(r_2(y_1)) < y_1$ for all y_1 in that interval. Similarly, $(d/dy_1)[r_1(r_2(y_1^2))] < 1$ by Lemma A5, which this time implies that $r_1(r_2(y_1)) > y_1$ for $y_1 \in (y_1^1, y_1^2)$, contradicting the inequality derived above. Therefore, there must exist a unique equilibrium in $[0,1)^2$.

PROOF OF LEMMA 4:

If $y_1' = y_2' = 0$, there is full cooperation. Then it suffices to show that one cannot have both $y_1' > 0$ and $y_2' > 0$. It will be initially supposed that this is true and then shown that it leads to a contradiction. Consider $y_1' > y_2' > 0$, which by (1) and (2) implies that $V^1(y_1', y_2') > V^2(y_1', y_2')$. By the definition of partial cooperation, $V^1(y_1', y_2') > V^1(0,0) = V^2(0,0) > V^2(y_1', y_2')$. By the supposition $y_2' > 0$, Lemma 3 implies $V^1(y_1', 0) > V^1(y_1', y_2')$ [$> V^1(0,0)$ from above]. Since $V^1(y_1, 0)$ is continuous in y_1 there must exist $y_1'' \in (0, y_1')$ such that $V^1(y_1'', 0) = V^1(y_1', y_2')$. As $y_1'' < y_1'$ and $0 < y_2'$, it follows that $C(1-y_1'', 1) > C(1-y_1', 1-y_2')$ which is equivalent to $V^1(y'', 0) + V^2(y_1'', 0) > V^1(y_1', y_2') + V^2(y_1', y_2')$. Together with the last inequality above, this implies $V^2(y_1'', 0) > V^2(y_1', y_2')$. Hence, $(y_1'', 0)$ weakly dominates (y_1', y_2'). The continuity of the payoff functions also ensures the existence of a y_1 in the neighborhood of y_1'' such that $(y_1, 0)$ strictly dominates (y_1', y_2'), with the constrained-efficiency of the latter thus contradicted. A symmetric argument establishes that partial cooperation cannot be induced when $y_2' > y_1' > 0$. Finally, if $y_1' = y_2' > 0$, then

$$V^1(y_1', y_2') = V^2(y_1', y_2') = C(1-y_1', 1-y_1')/2$$

$$< C(1,1)/2 = V^1(0,0) = V^2(0,0)$$

which contradicts efficiency also.

PROOF OF (6′):

The second part of (6′) follows from the second part of (6) and the strict concavity of $V^2(0, y_2)$ in y_2 (Lemma 1). To show the first, suppose initially that $V_1^1(0, y_2) \geq 0$ for some $y_2 < y_2^*$. Since by (6) $V_1^1(0, y_2^*) \leq 0$, there must exist at least one $y_2' \in (y_2, y_2^*)$ such that $V_{12}^1(0, y_2') \leq 0$ with $V_1^1(0, y_2') = 0$. However, this last equality, together with $V_2^2(0, y_2') > 0$ (just shown above), implies by Lemma A4 that $V_{12}^1(0, y_2') \geq pC_{12}(0, y_2') > 0$, which is a contradiction. Therefore, one cannot have $V_1^1(0, y_2) \geq 0$ for any $y_2 \in [0, y_2^*)$, as required.

PROOF OF LEMMA 5:

Note that $(\partial/\partial y_1)[p_1/p] = (p_{11}p - p_1^2)/p^2$ is negative by Assumption 2, and

p_1/p is decreasing in y_1. [Similarly, $(1-p)/(-p_2)$ is increasing in y_2.] Now suppose $p^* = p^*(y_1^*, y_2^*) < \frac{1}{2}$, where $y_1^* < y_2^*$. Then, by the property just shown, it follows that

$$\text{(A9)} \quad p_1^*/p^* \equiv p_1(y_1^*, y_2^*)/p(y_1^*, y_2^*)$$
$$> p_1(y_2^*, y_2^*)/p(y_2^*, y_2^*)$$
$$= -2p_2(y_2^*, y_2^*)$$

since $p(y_2^*, y_2^*) = \frac{1}{2}$, and by (3a), $p_1(y_2^*, y_2^*) = -p_2(y_2^*, y_2^*)$ (p_2 is always negative). By part (iii) of Assumption 1, $p_{12}(y_1, y_2^*) < 0$ for $y_1 < y_2^*$, implying that $p_2^* \equiv p_2(y_1^*, y_2^*) > p_2(y_2^*, y_2^*)$ since $y_1^* < y_2^*$. Thus, given that $1 - p^* < \frac{1}{2}$, one obtains

$$(1-p^*)/(-p_2^*) > 1/[-2p_2(y_2^*, y_2^*)].$$

This inequality combined with (A9) yields $p_1^*(1-p^*)/(-p_2^*p) > 1$, as required.

If $p^* > \frac{1}{2}$, similar steps yield

$$p_1^*(1-p^*)/(-p_2^*p) < 1$$

while for $p^* = \frac{1}{2}$ it is easily established that this ratio is equal to 1. The "only if" part of the lemma then follows.

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