The Dynamics of Alliances in Anarchy *

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Abstract

In this paper, we study the dynamics of alliances in a world of anarchy by taking states' outside options seriously. We investigate how the existence of outside options affects the decision of producing a collective good within an existing alliance. Our analysis shows that anarchy plays an important role in mitigating the incentive to free-ride on alliance partners, and that the incentive to invest political capital in an alliance arises endogenously. Additionally, we show that there is a non-monotonic relationship between the cost of searching for a new ally and the probability of breaking the old alliance and forming a new one.

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1 Introduction

For as long as sovereign polities have existed, there have been alliances. The ubiquitous nature of alliances has lead to numerous studies on why alliances are formed, how alliances are managed, and under what conditions alliance agreements are honored in anarchy. To date, however, these questions concerning the politics of alliances have been treated independently. A useful way of categorizing previous work is to sort them by the key questions they look to answer. What drives the logic of alliances in an international system with anarchy as its defining feature? How does the incentive for allies to free-ride on the efforts of their partners influence a country's willingness to contribute resources to an alliance?

The earliest literature on alliances focused on the importance anarchy in world politics. Anarchy is commonly equated to the ubiquitousness of security threat. However, such a threat is a consequence of the absence of exogenous measures to enforce agreements. We conceptualize anarchy in our model as implying choices, and that any international agreement must be "self-enforcing." In other words, acting within the agreement must be a best strategy, given the behavior of one's alliance partner and the requirements of the agreement. The realist literature, where alliances are viewed as tools for aggregating states' capabilities, provides the first explanation for why alliances exist (Morgenthau 1960, Waltz 1979). This view of alliances follows from balance of power theory and argues that alliances arise due to a common interest among the alliance members to aggregate their capabilities against a common threat. As such, mutual threat, and therefore mutual interest, explains why alliances are formed and why they may dissolve.

Fundamental to relationship between anarchy and alliance behavior is the idea of choice. Choice allows countries to either honor their alliance agreements or act outside of the stipulations of the alliance relationship. Therefore, any model of alliances must provide countries the choice to break them.

Noting that in the anarchic international system there are no enforceable contracts, an alternative line of research has argued that the fundamental problem facing states in an alliance is the commitment problem. While drawing insights from both the realist theory, work in this vein focuses on explaining how reputation and punishment strategies influence the nature of alliance interactions. A primary purpose of our analysis is to explain why a state with outside options would make such policy choices, and why creating political costs for acting outside an existing alliance may be beneficial even to those who must pay the cost for opting out. In this literature institutionalized political costs play an important role in explaining alliance behavior. Whether these political costs are created through investment in the alliance relationship or through reputational consequences of a country's behavior, the institutional theory of alliances is consistent with the observation that alliance agreements are often formed such that they produce opportunity costs for opt-ing out. These opportunity costs come from infrastructure investment among allies. For example, allies often build joint military bases and participate in joint military planning. As a result of this integration, the act of searching for an ad hoc partner would likely be detrimental to the allies' political relationship.

Another recurring theme in the alliance literature looks at the burden-sharing given the idea that alliances are international agreements that represent a common interest among some states to act in concert, the economic theory of alliances analyzes how an alliance may function once in existence. This literature, following from Olson & Zeckhauser (1966) and Smith (1980), takes the alliance agreement as an enforceable contract to produce the collective good and asks how the incentive states may have to free-ride influences their willingness to contribute resources to achieving the alliance's objectives.(Snyder & Diesing 1977, Morrow 1994, Haftendorm, Keohane & Wallander 1999).[1]

In this paper, we take a step in the direction of integrating these separate views of alliance

[1]Alternatively, an extensive literature developed on the economic theory of alliances that incorporated the private benefits and positive externalities of defense spending. For example, see Sandler (1988), O'neal (1990), and Palmer (1990). This literature grew out of the

dynamics within a single theoretical framework.[2] We take the view that alliances agreements, like any other international agreement in anarchy, can only be binding if states wish to fulfill them. We then ask questions such as: why might states choose to form alliances? How is the incentive to free-ride affected by the existence of outside options in anarchy?

What we find is that there exists an important interaction between the incentive to create and contribute to an alliance and the fact that these international alliances exist in an environment of anarchy. First, in a situation where the alliance agreement is not enforceable, an alliance with institutional investment by both players is optimal for a state. Second, anarchy works to decrease the incentive to free-ride in collective production in an alliance, and members will contribute more resources to the collective action than in the absence of anarchy. Finally, we find that there is a non-monotonic relationship between the opportunity cost of searching for a new ally and the probability of breaking the old alliance and forming a new one. These results have implications beyond the specific international institution we study in the paper, where international institutions are viewed as producing

observation the the pure collective goods model was inconsistent with NATO burden sharing from the late 1960's to the end of the Cold War. We would note that this research largely maintained the working hypothesis that alliances are, in some sense, binding and asked how private benefits from defense spending change the hypotheses that follow from a pure collective goods model. Here we revert to the pure collective goods assumption, but add the qualification that alliances are not enforceable. Such a framework maintains the intuitively pleasing description of alliances as producing a collective good, but also–as will be shown below– is able to explain why we see departures from the predictions of the pure collective goods model in terms states' propensity to free ride.

[2]A similar attempt to integrate multiple aspects of the alliance problem into a single theoretical frame work was done by Smith (1995). Smith's work differs from ours in that it focuses on the effects of alliance commitments on the likelihood of conflict, specifically in the setting of defensive agreements.

a collective good (Russett & Sullivan 1971, Sandler 1993).

The rest of the paper is organized as follows. Section two presents a model of collective good production in an alliance. We analyze the model in section three, and discuss the implications of the results in sections four and five. Section six concludes.

2 Model

To analyze the interaction of common interest and burden-sharing in international alliances, we present an alliance game with outside options.[3] Our model is closely related to Lee (1994), who studies the role of searching for outside options in bargaining between a buyer and a seller over the price of an indivisible good. We extend the model to a context in which a collective good is to be produced for a known cost, and where the good can be produced by different combinations of states. The model captures three important aspects of alliance relationship that are identified by previous work on alliances. First, the nature of an alliance is perceived to be producing a collective good for its members (Olson & Zeckhauser 1966); second, that the frequent source of conflict in an alliance is burden-sharing (Snyder 1997); third, that since international agreements are made in the shadow of anarchy, the fundamental problem states face is deciding whether or not to take a joint action with its ally, or exercising their outside option of forming an ad hoc coalition.[4]

More specifically, consider an alliance facing a bilateral collective good production prob-

[3]We view alliances along the lines of Theis (1987) where alliances are, for the most part, "temporary, ad hoc agreements organized for a specific purpose," but our assumptions are general enough to capture the special cases where alliances take on the form enduring institutions, like NATO.

[4]While few have analyzed the effects of outside options in international relations (Voeten 2001), there exists a significant literature in economics on the subject (Binmore 1985, Lee 1994, Muthoo 1999, Muthoo 1995, Shaked 1994).

lem, where for some fee, ϕ, the allies can produce a good that provides each member of the partnership a benefit b.[5] Normalize the benefit such that if the collective good is produced each player gets a payoff of 1, but gets 0 otherwise. Also assume $1 < \phi < 2$, such that no state wants to produce the good on its own, but if two states contribute to its production both are better off. Since we normalize the players' benefit , ϕ can be interpreted as the relative cost of taking the foreign policy action that produces the collective good. This collective good game emphasizes the basic strategic trade-off faced by members of an alliance. On the one hand, both states have a common interest in producing the policy outcome, modelled here as a unit payoff; on the other hand, both allies desire to "free ride" on the contribution of their alliance partner in order to maximize their individual gain from the alliance's action.

The game is played for two periods, which adds important dynamic considerations to the players' calculations. In particular, we analyze the following sequential interactions. In each period, one alliance partner—called player 1—is asked to propose some resource contribution r_i $(i = 1, 2)$ to the production of the good. Since no state has an incentive to contribute an amount of resource greater than 1, $r_i \leq 1$. Also, in each period player 1's ally, player 2, is presented with the option of producing the good with player 1, or searching the international community for a partner that shares his interest in producing the good, and who is also willing to contribute more toward the production cost to receive the resulting benefit.

Formally, the game begins with an offer from player 1, r_1, that player 2 may subtract from the fee for the production of the good. Given r_1, player 2 can decide whether or not to produce the collective good with player 1. If the good is produced within the alliance, player 1 gets $1 - r_1$ and player 2 gets $1 - \phi + r_1$. Obviously, player 2 will only produce the good if $1 - \phi + r_1 \geq 0$. As an alternative, player 2 may either choose to "search" for contributions

[5]For convenience, player 1 will sometimes be referred to as "she" and player 2 will be referred to as "he."

from potential partners outside the alliance, or to advance the game to the second period and ask player 1 to make a second offer. If player 2 decides to search in the first period, he pays a cost c ($c > 0$) and then calls for a contribution from a state outside the alliance. For simplicity, we assume that the contribution offered by the potential ad hoc ally, denoted as x_1, is drawn from a uniform distribution on $[0, 1]$.[6] If at that point player 2 decides to accept either r_1 or x_1,[7] the game ends and payoffs are realized. Otherwise, he moves the game to the second period by calling on his alliance partner, player 1, to make a second offer.

The second period is played in a similar fashion with player 1 making a new offer, r_2. Given r_2, player 2 can choose to produce the good with the alliance partner at a cost of $\phi - r_2$ or with the ad hoc partner at a cost of $\phi - x_1$, and end the game. Alternatively, he can search again and draw a second outside offer, x_2, at a cost c. After observing r_2, x_1, and x_2, player 1 must choose whether or not to produce the good, and with whom to produce it. The sequence of the game is also depicted in Figure 1.

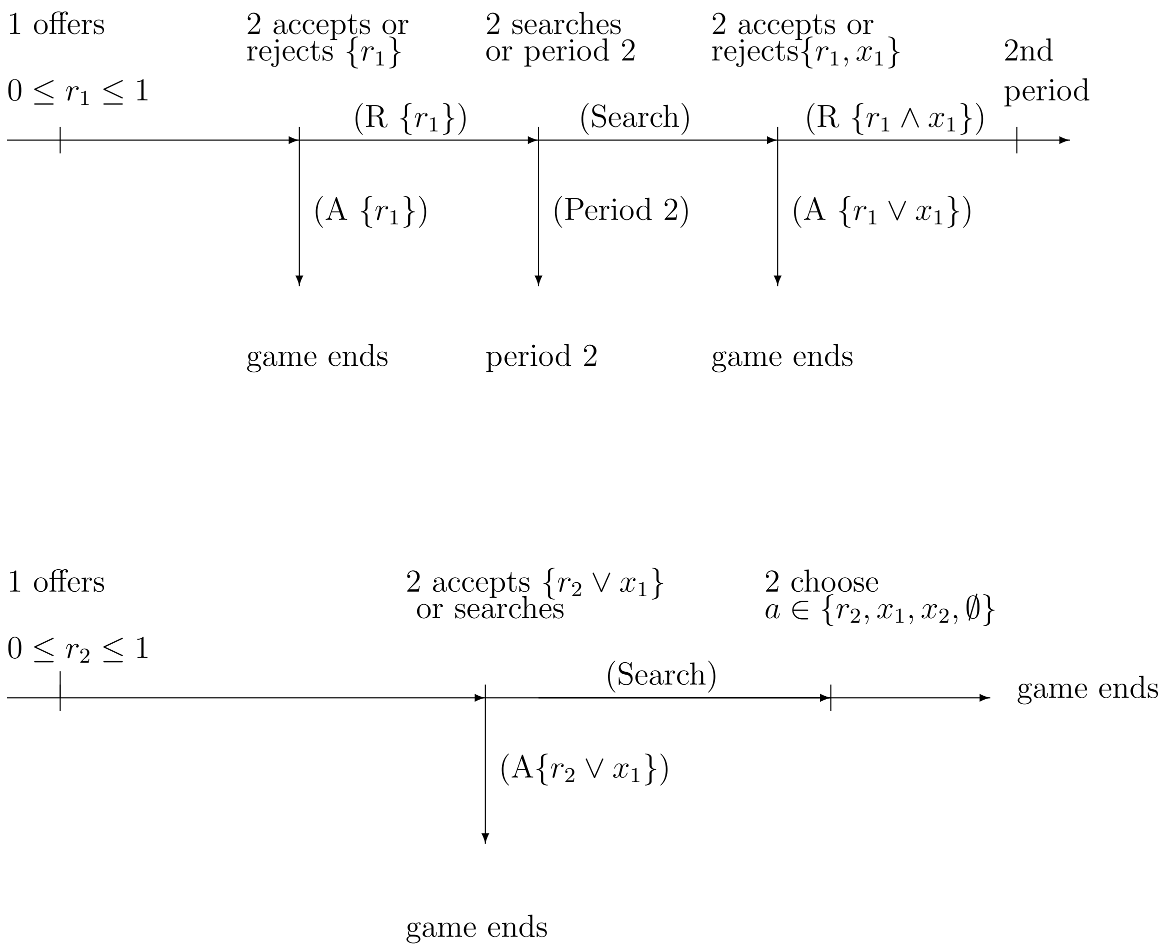
We now specify the players' utilities over different outcomes of the game. Players in this dynamic game are assumed to discount future payoffs with a common discount factor $0 \leq \delta \leq 1$. We can then write player 1's utility for a given strategy as $\delta^{i-1}(1 - r_i)$ if r_i is accepted in period i, and zero if her ally goes with a different partner. Similarly, player 2's utility for a strategy where he takes an action with his ally is $\delta^{i-1}(1-\phi+r_i)$, while his utility for taking the action with a new coalition is $\delta^{i-1}(1 - \phi + x_i)$. If such an action is taken in the second period, then $x_i = max[x_1, x_2]$.

Finally, we make three tie-breaking assumptions to obtain a unique equilibrium for different ranges of the parameter that we are interested in. First, we assume that if a player is indifferent between choosing an action that leads to a lottery and an action that produces

[6]We also note that the outside offer (x_i) is a "standing offer" and can be accepted in period 1 or 2.

[7]If player 2 does not search in the first period then, without loss of generality, we can set $x_1 = 0$.

Figure 1: Time Line of the Alliance Game



the same expected utility but with certainty, she chooses the action leading to the sure outcome. Substantively, this assumption implies that players are mildly "risk averse." Second, we assume that if player 2 is choosing a coalition partner when there are multiple proposals of equal value, then he chooses to produce the collective good with the partner who has made the most recent proposal. This assumption is reasonable because, given the sequential nature of the decision-making process, the potential partner making the latter offer could always ensure that his proposal is chosen by offering ϵ more than the previous offer. Finally, we assume that if a player is indifferent between taking an action that produces the collective good and an action that does not produce the good, they prefer the outcome where the collective good is produced. The assumption allows us to consider only pure strategy equilibrium of this game.

Given the information structure of the game, we apply the subgame perfection solution concept and use backward induction to solve the the equilibrium.

3 Results

Since our model is dynamic, a country's expectations about what their alliance partner will do in the second period has important effects on its decision in the first period. Therefore, our analysis starts by looking at the equilibrium strategies of the allies in the second period. We begin by distinguishing between the two possible histories that could lead to second period play. In the first case, we consider the second period interaction given that country 2 searched in the first period. Once the players strategies are defined for that subgame, we then turn to the second period strategies given that there was no first period search.

After the second period strategies are defined, we go back to the first period and describe the optimal actions of our allies, given that they know what their first period actions imply about how the second period will be played. The analysis of the first period then allows us to characterize the unique subgame perfect equilibrium to our game.

There is also the additional complication that the strategies of our players depend on the exogenous value of ϕ. We therefore start with the assumption that $\phi < 3/2$. This assumption requires us to solve the game in its most complicated form. That is, assuming $\phi < 3/2$ requires us to solve the game for the case that maximizes the number of factors our allies must consider. The logic driving the equilibrium strategies, however, is the same for all values of ϕ and become simpler as ϕ increases from 3/2 to 2. Finally, for ease of exposition the main text focuses on the building blocks of our model and illustrating the intuitions of our results, leaving the formal proofs of our propositions to the Appendix.

3.1 Period 2 Strategies

In the second period, player 1 makes the first move by offering a contribution r_2. Given player 1's offer, player 2 decides either to accept player 1's second period offer, accept x_1 if he searched in the first period, or search for a new offer. If he decides to search for an outside offer, player 2 moves again and decides to accept r_2, x_1, or x_1. If player 2 choose one of r_2, x_1, or x_1 the game ends with the collective good produced. If he rejects all offers, then the collective good is not produced.

3.1.1 Period 2 after first period search

Since player 2 moves last, we first consider his strategy. For player 2, there are three important decisions: when to produce the collective good, with whom to produce it, and whether or not to search in the second period.

Suppose player 2 *searched* in the first period. Let $\bar{s}_2 = max\{x_1, r_2, (\phi - 1)\}$. Player 2's expected payoffs from a search in the second period is then:

$$\theta(\bar{s}_2) = -c + \int_0^{\bar{s}_2} (1 - \phi + \bar{s}_2) f(x) dx + \int_{\bar{s}_2}^{1} (1 - \phi + x) f(x) dx. \tag{1}$$

With the uniform distribution of outside offers, equation 1 becomes

$$\theta(\bar{s}_2) = -c + 3/2 - \phi + (\bar{s}_2)^2/2. \tag{2}$$

Let r^* be the level of contribution that makes player 2 indifferent between searching and not searching in the second period. Then

$$1 - \phi + r^* = \theta(r^*). \tag{3}$$

Solving equation 3, we have $c = (r^* - 1)^2/2$. Since $0 \leq r_2 \leq 1$, it must be the case that $0 \leq c \leq 1/2$. For each $c \in [0, 1/2]$, r^* is unique and it is decreasing in c. Furthermore, note that $1 - \phi + r_2 < \theta(r_2)$ for all $r_2 < r^*$ and $1 - \phi + r_2 \geq \theta(r_2)$ for all $r_2 \geq r^*$. Therefore, in the second period player 2 will search if $\bar{s}_2 < r^*$, and not search if $\bar{s}_2 \geq r^*$.

Given player 2's search strategy, we now look at player 1's offer strategy in the second period. First, player 1's expected utility in the second period for an offer r_2 is:

$$u_2(r_2) = \begin{cases} 1 - r_2 & \text{if } r_2 \geq r^* \\ (1 - r_2)r_2 & \text{if } \phi - 1 \leq r_2 < r^* \\ 0 & \text{if } r_2 < \phi - 1, \end{cases} \tag{4}$$

as long as $r_2 \geq \max\{x_1, \phi - 1\}$.

To insure that we need only consider the case where $r_2 \geq \max\{x_1, \phi - 1\}$ is satisfied in the subgame where player 2 searched in the first period, we are able to prove that $r_2 \geq \max\{x_1, \phi - 1\}$ in any equilibrium. In other words, in equilibrium, player 1 will offer an amount in the second period that is at least as big as the minimal contribution required to jointly produce the good and the outside offer player 2 obtained in the first period.

Lemma 1. *If period two is reached in an equilibrium, then* $r_2 \geq \max\{x_1, \phi - 1\}$.

This result goes a long way toward simplifying our analysis, and the intuition behind it is straightforward. If r_2 were less than the $\max\{x_1,\, \phi - 1\}$ then there are two possibilities. In the first case, x_1 is the maximum and $r_2 < x_1$. Here player 2 is always better off accepting x_1, rather than r_2, independent of whether or not he searches. As a result, player 1's expected utility of offering $r_2 < x_1$ is zero. In the other case, where $\phi - 1$ is the maximum and $r_2 < \phi - 1$, player 2 would never produce the good with player 1. Again, regardless of whether or not player 2 searches in the second period, player 1's expected utility is zero. However, if player 1 were to match $\max\{x_1, \phi - 1\}$, i.e., $r_2 = \max\{x_1, \phi - 1\}$, then she would have a strictly positive expected utility, $(1 - r_2)r_2$, and be better off. Therefore, it must be that $r_2 \geq \max\{x_1, \phi - 1\}$ and the relevant utilities for player 1 in the second period are defined in equation (4).

More importantly, this Lemma implies two things about player 1's second period offer strategy. First, player 1 will always offer a large enough contribution in the second period to ensure the collective good is produced. Second, player 1 will never allow herself to be "out of the running" as an option for player 2. That is, she always makes an offer such that, if player 2's second period search goes poorly, the good will be produced within the alliance, not with the ad hoc partner from the first period.

We now proceed to fully characterize the players' equilibrium strategies in the second period after a search has taken place. It is easy to see that if player 1 offers an amount that induces player 2 to search, at most she gets 1/4, where we note that the offer $r_2 = 1/2$ is the maximizer of the lottery $(1 - r_2)r_2$. Player 1's best response, then, depends on the values of r^*, x_1, and 1/2. If $1/4 \leq 1 - r^*$, that is, if the most player 1 can get from inducing player 2 to search is less than what she receives from satisfying player 2 outright, then player 1 will offer $r_2 = max\{r^*, x_1\}$.[8] If $1/4 > 1 - r^*$, however, the strategies are more complicated.

Let $\tilde{r} > 1/2$ denote the offer by player 1 that gives her the same expected payoff as

[8]Ideally, player 1 would like to offer $r_2 = r^*$. However, by Lemma 1 we also know that $r_2 \geq \max\{x_1, \phi - 1\}$. It is easy to see that $r^* \geq \phi - 1$; what is left to satisfy, then, is that player 1 offers r^* or x_1, whichever is larger.

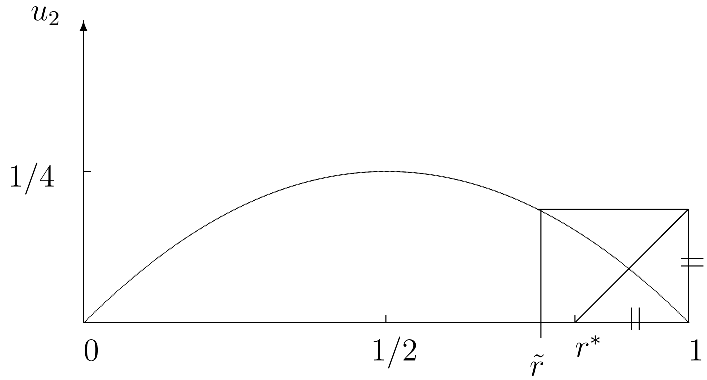


Figure 2: Player 1's utility for an offer r_2 in period 2.

offering r^*, by inducing player 2 to search. In other words, $\tilde{r}$ is the solution to the following equality, with the constraint that $\tilde{r} > 1/2$:

$$1 - r^* = (1 - \tilde{r})\tilde{r}. \tag{5}$$

The relationship between r^* and $\tilde{r}$ is shown in Figure 2. Since $u_2(r_2)$ is concave in r_2, $\tilde{r}$ is always going to be less than or equal to r^*. The values, $\tilde{r}$, r^*, and 1/2, mark the thresholds of different regions that x_1 may fall and to which player 1 will respond differently with her own optimal offer. A complete characterization of the players' strategies at this stage is presented in Table 1. [9]

The second period strategy of player 1 provides some insight into the dynamic effects of the outside offers on the alliance relationship. We see that the outcome of the first period search can have important effects on the ally's second period offer. For example, consider the case where $1 - r^* < 1/4$. If player 2's first period search goes "badly", then player 1's offering strategy is unaffected by the search. However, when the outside offer from the first period falls in the regions $(1/2, \tilde{r}]$ and $(r^*, 1]$, we see that the ally has an incentive to match that offer and increase the amount of resources they contribute to the joint foreign policy action. Furthermore, if the outside offer from the first period falls in the region $(\tilde{r}, r^*]$, then

[9]The derivation of the equilibrium strategies in Table 1 can be found in the Appendix.

Table 1: Equilibrium Strategies in Period 2 after a Search in Period 1.

	$1 - r^* \geq 1/4$			
Range of x_1	$[0, r^*]$		$(r^*, 1]$	
Player 1's offer	r^*		x_1	
Player 2's response	Accept		Accept	
	$1 - r^* < 1/4$			
Range of x_1	$[0, 1/2]$	$(1/2, \tilde{r}]$	$(\tilde{r}, r^*]$	$(r^*, 1]$
Player 1's offer	$1/2$	x_1	r^*	x_1
Player 2's response	Search	Search	Accept	Accept

there is an additional increase in the ally's second period offer, i.e, player 1 will offer *more* than the outside offer from the first period. As such, in the second period we can begin to see the incentive to free-ride, which characterizes the collective good problem, is curbed to some degree by anarchy and the presence of outside options. A successful search by player 2 decreases player 1's incentive to free-ride and, therefore, increase the resource contribution they are willing to make.

3.1.2 Period 2 play after no first period search

The preceding analysis assumes that player 2 searched in the first period. This is the more complicated case. If he did not search in the first period, then player 2's decision problem in the second period is similar to the above, but we need no longer consider the constraint that $r_2 \geq \max\{x_1, \phi - 1\}$. Therefore, if $1 - r^* \geq 1/4$, then Player 1 will offer r^* and player 2 will accept; If $1 - r^* < 1/4$, then player 1 will offer $1/2$ and player 2 will search.

With the players' second period strategies defined, we can now determine their equilibrium behavior in the first period.

3.2 Period 1 Strategies

In the first period, player 1 makes the first move by offering a contribution r_1. Given player 1's offer, player 2 decides either to accept, search, or move directly to the second period by asking for a second offer from player 1. If he decides to search for an outside offer, player 2 moves again and decides to accept r_1 or x_1, or reject both and move to the second period by asking for a new offer from player 1. Again, we start from player 2's second move by assuming that he searched and has received an outside offer in the first period and must decide to accept a first period offer $\{r_1, x_1\}$ or move the game to period 2.

3.2.1 Player 2's Strategy in Period 1 after a Search

To characterize player 2's strategy in the first period after a search, we need identify a value for r_1 that gives player 2 the same payoff as r^* in the second period. Let r_1^* be that value and it is the solution to the equation $1-\phi+r_1^* = \delta(1-\phi+r^*)$. Additionally, to fully characterize players' equilibrium strategies, we require that the discount factor is sufficiently large, such that $\delta \in [\underline{\delta}, 1]$, where $\underline{\delta}= \{\min_\delta : \tilde{r} < r_1^* \leq r^* \quad \wedge \quad 1-\phi+s < \delta\theta(s), \forall s < \tilde{r}\}$. The intuition for the assumption can be explained in simple terms.

First, the condition helps to locate the relative positions of thresholds that are necessary to characterize the equilibrium strategies, i.e., $\tilde{r} < r_1^* \leq r^*$. Second, the assumption implies that the discount factor would not change strategic incentives of player 2 in a significant way. Specifically, if an offer from player 1, s, induces player 2 to search in a one-period game $(1-\phi+s < \theta(s))$, then by the assumption, the same offer would lead to a search by player 2 in a two-period game as well $(1-\phi+s < \delta\theta(s))$, as long as the offer is sufficiently small $(s < \tilde{r})$. The assumption rules out the possibility that the time preference plays a decisive role in players' strategic calculations, and allows us to focus on the effect of outside options on the choices players make in equilibrium.

With $\delta \in [\underline{\delta}, 1]$, the full characterization of the equilibrium strategies at this stage is

Table 2: Player 2's Equilibrium Strategy in Period 1 after a Search.

		$1 - r^* \geq 1/4$		
Range of x_1	$[0, r_1^*]$	$(r_1^*, r^*]$	$(r^*, 1]$	
1's offer in pd 2	r^*	r^*	x_1	
2's response in pd 1	Accept r_1 if $r_1 \geq r_1^*$; else 2nd pd.	Accept $max\{x_1, r_1\}$	Accept $max\{x_1, r_1\}$	

		$1 - r^* < 1/4$			
Range of x_1	$[0, 1/2]$	$(1/2, \tilde{r}]$	$(\tilde{r}, r_1^*]$	$(r_1^*, r^*]$	$(r^*, 1]$
1's offer in pd 2	$1/2$	x_1	r^*	r^*	x_1
2's response in pd 1	Accepts r_1 if $r_1 \geq$ $\delta\theta(1/2) + \phi - 1$; else 2nd pd.	Accept r_1 if $r_1 \geq$ $\delta\theta(x_1) + \phi - 1$; else 2nd pd.	Accept r_1 if $r_1 \geq r_1^*$; else 2nd pd	Accept $max\{x_1, r_1\}$	Accept $max\{x_1, r_1\}$

presented in Table 2. [10]

3.2.2 Player 2's Strategy in Period 1 before a Search

Now we analyze player 2's strategy in the first period *before* a search. At this stage, player 2 decides if he will accept an offer from player 1, reject it and search, or reject it and move to the second period. Intuitively, if the offer r_1 is sufficiently large, then player 2 will accept; otherwise, he will look for a better outside offer or ask player 1 to make a second offer directly. Lemma 2 shows that, again, r^* is the offer that makes player 1 indifferent between accepting and rejecting in the first period. Additionally, lemma 2 says that when rejecting

[10]The derivation of the equilibrium strategies in Table 2 can be found in the Appendix.

an offer (r_1), player 2 will search, rather than moving to the second period directly.

Lemma 2. *In the first period, player 2 will accept an offer from player 1 if* $r_1 \geq r^*$*; otherwise, he will search in the first period.*

If an offer is greater than r^*, we know from player 2's strategy and payoffs afterward (Table 2) that he will accept the offer immediately and terminate the game. If an offer is smaller than r^*, player 2 will either search or move to the second period directly. Comparing the utility of searching in the first period and moving directly to the second period, it can be shown that for any $r_1 < r^*$, player 2 prefers to search in the first period.

3.2.3 Player 1's Strategy in Period 1

With player 2's equilibrium strategy in the first period fully characterized, we are at the point of specifying player 1's strategy in period 1 and the resulting equilibrium. The next two propositions characterize the unique equilibrium for each relevant range of parameter values.

Proposition 1. *If* $1 - r^* \geq 1/4$*, there is a unique equilibrium to this game where player 1 offers* $r_1 = r^*$ *in the first period and player 2 accepts immediately.*

The condition $1 - r^* \geq 1/4$ implies that the search cost c is sufficiently high such that r^*, which is a decreasing function of c, is relatively small. In other words, the level of contribution that would make player 2 indifferent between accepting and searching in the first period is small when the search cost is high. It is then optimal for player 1 to offer a large enough contribution to prevent player 2 from searching, avoiding the risk of being replaced by an ad hoc ally.

As the search cost decreases, r^* increases. This in turn makes player 1 less willing to satisfy player 2 outright, and more willing to gamble on player 2 having a bad draw from searching. Proposition 2 characterizes the unique equilibrium that results when ending the game immediately is too costly for player 1.

Proposition 2. *If $1 - r^* < 1/4$, there is a unique equilibrium to the game where player 1 offers $\bar{r}_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$ that maximizes (6), and player 2 searches in the first period.*

$$\omega(r_1) = \int_0^{x(r_1)} (1 - r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1 - x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \qquad (6)$$

The proposition states that player 1 will offer an amount that is less than enough to make player 2 satisfied outright, but will maximize her expected utility given that player 2 will search. What is interesting about this equilibrium is that all sorts of outcomes are possible. While player 2 will surely search under this equilibrium, if the outside offer is sufficiently small , i.e., $x_1 \in [0, 1/2]$, then player 2 will accept $\bar{r}_1$ in the first period. If, however, the offer is somewhat better, then player 2 moves the game to the second period and "forces" player 1 to match the outside offer. If the first period search goes "well", then player 2 can move the game to the second period and induce his ally to offer r^*, and player 2 gets a bonus from his ally's bid-jumping incentive. Finally, the first period search could go exceedingly well, in fact, so well that player 2 decides to break his alliance and produce the collective good with the outside partner. So, even in an alliance with complete information, if anarchy implies that states have outside options, then there is positive probability that the alliance will break down. On the other hand, the fact that states do have outside options reduces player 1's incentive to free-ride and, in equilibrium, she often offers a resources contribution greater than the minimum needed to make his ally willing to take the joint action.

4 Alliances vs. Ad-hoc Coalitions

In this and the next section we provide some analysis based on the equilibrium results. As stated in propositions 1 and 2, there are two possible equilibria. Proposition 1 suggests that when the cost of searching for a new ally is sufficiently high, it is the existing alliances that will be activated to produce a collective good. On the other hand, Proposition 2 suggests

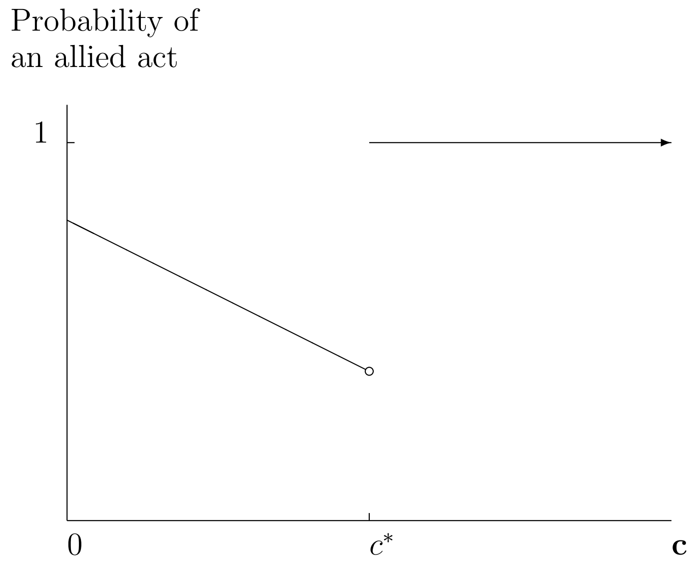


Figure 3: Probability of taking a foreign policy action with an ally as a function of c.

that when the search cost is sufficiently low, so that r^* is sufficiently large, player 1's offer will lead to a search. Accordingly, low search cost imply that there is a positive possibility that the old alliance breaks down and a new alliance is formed.

However, the relationship between the cost and the probability of forming a new alliance in the equilibrium identified by Proposition 2 is not monotonic. In particular, as the cost of searching decreases, the level of contribution that deters player 2 from searching, r^*, increases. This, in turn, moves upward the entire range from which player 1's first period offer is drawn.[11] Consequently, the equilibrium offer of player 1, $\bar{r}_1$, increases as well.[12] As $\bar{r}_1$ gets larger, it is more difficult to find an outside offer that is better, which means that

[11]That is, both $\delta\theta(1/2) - 1 + \phi$ and $\delta\theta(\tilde{r}) - 1 + \phi$ increase as r^* increases.

[12]The logic of the proof goes as follows. If the original maximizer is available after the decrease of the cost, then the new maximizer is at least as good as the old one. If the original maximizer is not available, then any new maximizer is larger than the original one, given that the entire region from which r_1 is drawn has moved up.

the probability of forming a new alliance decreases as the search cost becomes smaller. This gives us a non-monotonic relationship between the two, roughly captured by Figure 3.[13]

5 Optimal Alliance Integration

Our results also speak to more general questions of why alliances are credible and why they exist at all. Some argue that the consideration of reputations makes alliances credible (Snyder & Diesing 1977). In particular, it is argued that the repeated nature of states' interactions deters a state from reneging on its alliance commitments. Others argue that alliances are a signaling device of common interests among allies (Morrow 1994). Our model suggests an alternative explanation of the existence of alliances. That is, compared to what a state can receive from an ad hoc partner at the time of taking an action, they can do better if there is an existing alliance with institutionalized costs.

To see the logic of our argument, consider the optimality of a particular institution, given different possible institutional arrangements. As is clear from the analysis, what player 1 offers in the first period is a function of the search cost associated with the players' investment in their alliance, and consequently, the equilibrium payoff for player 2 is a function of that opportunity cost. We may then ask: given a menu of possible alliance institutions, which imply different search costs, what is the optimal level of c? Figure 4 shows that, for "small" ϕ, player 2– the player with the outside option– maximizes his expected utility by being

[13]We would also note that the logic that underlies this result does not depend on the the uniform distribution. As long as $\omega(\bar{r}_1) > 1 - r^*$, the proof of the non-monotonicity only requires that it is harder to find an outside offer better than $\bar{r}_1$ as $\bar{r}_1$ increases in the defined range. This will be true for any well-defined cumulative distribution function, continuous or discrete. If however, $\omega(\bar{r}_1) < 1 - r^*$, then, obviously, player 1 offers $r_1 = r^*$ for all values of c, and the equilibrium is similar to the one characterized by Proposition 1.

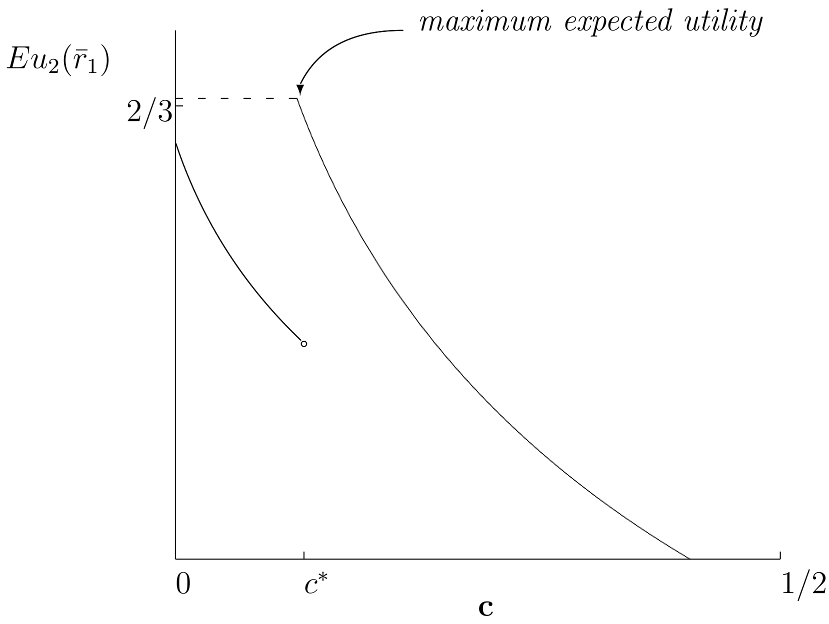


Figure 4: Player 2's expected utility as a function of c, given $\phi = 1.1$, and $\delta = .99$.

a member of an alliance with positive search costs. Under our assumptions, we see that an institution that induces a search cost of $c^* = 1/32$ gives player 2 the highest expected utility. This is interesting because, counter-intuitively, player 2 is better-off in an alliance that penalizes searching for an ad hoc partner than one in which he pays no cost for searching for the best possible outside offer. This implies an alternative explanation for the existence of alliances: players enjoy welfare improvement when they are members of an alliance than when they are not.

Additionally, our analysis shows that ϕ plays an important role in determining the optimal amount of investment a player is willing to make in developing alliance institutions. In our welfare analysis we start by assuming that $\phi - 1$ is "small". If $\phi - 1$ were "large,", for example near 3/2, even though the equilibrium strategies we characterize would not change, the optimal investment in an alliance does change for such a ϕ. More specifically, as ϕ gets larger, player 2's expected utility is greatest when $c = 0$, rather than when $c = c^*$. As such, there is a value of ϕ (call it ϕ^*), where for any production cost $\phi' \geq \phi^*$, player 2 is better-off by not being a member of an alliance that imposes political costs on searching for an ad hoc ally. This implies that determining the optimal alliance institution depends not only the players' investment in the institution, but also on what they believe the relative cost of future foreign policy actions to be. If those relative costs are believed to be low, then the optimal institution is one where the players invest in their alliance and generate positive costs for searching for an ad hoc partner. If, however, the relative costs of future foreign policy actions are believed to leave little net benefit for the actors involved, then states are better-off having flexibility in forming ad hoc coalitions. Under these conditions states should avoid making investments in alliance institutions.

If we interpret the search cost as the political cost of threatening what players have invested in the alliance, then the above analysis suggests that player 2 is most well-off if such cost is greater than zero. Such a cost creates the incentives for players to work within existing institutions, at the same time, it leaves open the option of going outside of the existing alliance. From the perspective of optional institutional design, the model suggests, counter-intuitively, that a positive investment in an alliance that deepens the relationship between allies is optional for player who must pay the cost to search.

Note, however, the existence of outside options is necessary in bringing a higher payoff to player 2. Having an alliance in and of itself does not have such an welfare effect. If there were no outside options, player 1 will provide just enough resources to make player 2 indifferent between producing and not producing the collective good. That is, without an outside option, whenever player 2 produced the good, he would receive a net benefit of 0, while player 1 receives the maximum net benefit. So here anarchy plays a critical role in the incentive to form alliances. If the alliance were an enforceable contract, then player 2 would be better off by not being a member of such an institution, because in an alliance he would have an expected payoff of 0, but without an alliance he could search for an ad hoc partner and have a strictly positive expected utility. From our analysis we see that anarchy can, to some degree, mitigate the incentive to free-rider when considering collective good production.

6 Conclusion

In this paper, we study the dynamics of alliances in an anarchic world by taking states' outside options seriously. The effect of outside options on player 2's payoff brings to our attention the familiar argument that, in the end, the constraining effect of any institution is limited in an anarchic world. The model points to an important source of this limitation: outside options. It has often been argued that the reason institutions (alliances) are ineffective, is that states have divergent interests and commitments are not credible. Our model shows that even when players share common interests an alliance may break down. In fact, the only way that alliances never breakdown is if the alliance partner is willing to sufficiently cater to the needs of the state with outside options. Additionally, the model shows that the termination of an alliance may not result from divergent interests; it may simply be a consequence of utility maximizing behavior of states in the face of outside options.

We also find, contrary to our intuitions, there is a non-monotonic relationship between the cost of searching for a new ally and the probability of breaking the old alliance and forming a new one. Correspondingly, the payoffs of players vary non-monotonically as a function of the cost.

Finally, our equilibrium results suggest that an alliance can be seen as an institution that focalizes a particular solution to the problem of collective good provision when multiple choices exist. Furthermore, the welfare analysis suggests that generating a moderate cost for breaking up an existing institution may, in certain situations, be the optimal institutional agreement. The paper thus speaks to the literature on alliances and literature on institutions more generally.

Clearly, our analysis is not the final word on the dynamics of alliances in anarchy. One interesting extension of this model would be to conceptualize states' payoffs from their foreign policy action as a “pie” that must be divided after it is produced. This could be done by adding a bargaining phase at the end of the game. Another extension would allow the players to choose their level of integration before the alliance acts. A third extension would be to make the potential ad hoc ally a strategic player. While under some conditions results from such extensions would not vary much from those found here, obviously, it could also be the case that new dynamics arise that would reverberate throughout the entire alliance interaction.

Appendix

Lemma 1 If period two is reached in an equilibrium, then $r_2 \geq \max\{x_1, \phi - 1\}$.

Proof. We first consider the case $x_1 < \phi - 1$. We show that $r_2 \geq \phi - 1$. Suppose not, i.e., $r_2 < \phi - 1$. Then player 1 gets the payoff of 0 with certainty. Consider her deviation to $r_2 = \phi - 1$. If the game is played such that player 2 searches in the second period, then player 1's expected payoff is

$$\pi(r_2 = \phi - 1) = (2 - \phi)(\phi - 1) > 0,$$

If player 1 does not search in the second period, then player 1's payoff is $2 - \phi > 0$. Clearly, $r_2 = \phi - 1$ is a profitable deviation for player 1. A contradiction.

Now suppose $x_1 \geq \phi - 1$. We show that $r_2 \geq x_1$. Suppose not, i.e., $r_2 < x_1$. Then player 1 gets the payoff of 0 with certainty. Consider her deviation to $r_2 = x_1$. If the game is played such that player 2 does not search in the second period, by the tie-breaking rule player 2 produces the good with player 1 and her utility is $1 - x_1 > 0$. If player 2 does search in the second period, by offering $r_2 = x_1$ player 1 gets

$$\pi(r_2 = x_1) = (1 - x_1)x_1 > 0.$$

Again, $r_2 = x_1$ is a profitable deviation for player 1. A contradiction. □

The derivation of players' equilibrium strategies in period 2 after player 1 searched in period 1 (Table 1) where $1 - r^* < 1/4$. [14]

Case 1: In the first case, player 2's first period search goes badly and x_1 is less than 1/2. So if $x_1 \in [0, 1/2]$, player 1 offers $r_2 = 1/2$ to maximize her expected payoff from inducing player 2 to search, and player 2 searches given the offer.

[14] Since the strategies when $1 - r^* \geq 1/4$ are discussed in the text, we omit them from the appendix.

Case 2: In the second case, player 2's first period search is large enough— x_1 is between $1/2$ and $\tilde{r}$— to get player 1 to match player 2's first period outside offer, but it is not large enough for player 1 to make an offer that deters player 2 from searching in the second period. So if $x_1 \in (1/2, \tilde{r}]$, then $r_2 = x_1$, and player 2 searches.

Case 3: In the third case, player 2's first period search goes well and it is sufficiently high to induce player 1 to outbid the outside offer from the first period, and to deter player 2 from searching in the second period. This "bid-jumping" occurs if $x_1 \in (\tilde{r}, r^*]$. As a result of such an outside offer, player 1 offers $r_2 = r^*$, and player 2 accepts.

Case 4: In the final case, player 2's outside offer is very good and if the second period is reached, then player 1 will again match the first period offer. So if $x_1 \in (r^*, 1]$, then $r_2 = x_1$, and player 2 accepts.

The derivation of player 2's equilibrium strategy in period 1 after he searched (Table 2).

As in the second period, we need to discuss players' strategies when $1 - r^* \geq 1/4$ and when $1 - r^* < 1/4$. First, suppose that $1 - r^* \geq 1/4$ and player 2 has already searched in the first period. Here we have two cases.

Case 1: Suppose $x_1 \in (0, r_1^*]$, then player 2 accepts r_1 if $r_1 \geq r_1^*$; otherwise, he moves to the second period and accepts r^*.

Case 2: Suppose $x_1 \in (r_1^*, 1]$, player 2 will accept $max\{x_1, r_1\}$ in the first period, produce the good, and end the game.

Next, suppose that $1 - r^* < 1/4$ and player 2 has already searched in the first period. We have four cases to consider.

Case 1: If $x_1 \in [0, 1/2]$, then player 2 accepts r_1 if $1 - \phi + r_1 \geq \delta\theta(1/2)$; otherwise, he moves the game to the second period, gets a second period offer of $1/2$ from player 1, and searches.

Case 2: If $x_1 \in (1/2, \tilde{r}]$, then player 2 accepts r_1 if $1 - \phi + r_1 \geq \delta\theta(x_1)$; otherwise, he moves to the second period, player 1 offers x_1, and player 2 searches.

Case 3: If $x_1 \in (\tilde{r}, r_1^*]$, player 2 accepts r_1 if $r_1 \geq r_1^*$; otherwise, he moves to the second period, player 1 offers r^*, and player 2 accepts.

Case 4: If $x_1 \in (r_1^*, 1]$, player 2 accepts $max\{x_1, r_1\}$ at the end of the first period and the game ends.

Lemma 2: In the first period, player 2 will accept an offer from player 1 if $r_1 \geq r^*$; otherwise he will search in the first period.

Proof. Suppose $r_1 \geq r^*$. Given player 2's strategy in the first period after a search (Table 2), we know that he will either accept r_1 immediately or accept $max\{x_1, r_1\}$ after a search. In either case, he will not go to the second period. So if $r_1 > r^*$, player 2's decision depends only on his utility from searching, $\theta(r_1)$, and not searching, $1 - \phi + r_1$. By the definition of r^*, we know $1 - \phi + r_1 \geq \theta(r_1)$, and player 2 will accept r_1 immediately, rather than search.

Now, suppose $r_1 < r^*$. If player 2 accepts r_1, he gets $1 - \phi + r_1$; if he rejects r_1 and searches, he gets at least $\theta(r_1)$. Since $1 - \phi + r_1 < \theta(r_1)$, searching is better than accepting. Where $r_1 < r^*$, however, player 2 must consider both the option of searching and the option of moving directly to the second period. We must consider two cases. First suppose $1 - r^* \geq 1/4$. If player 2 moves to the second period directly, he gets $\delta(1-\phi+r^*) = 1-\phi+r_1^*$. If he searches in the first period, his expected utility given r_1 is:

$$\begin{aligned}
\theta(r_1) &\geq -c + \int_0^{r_1^*} \delta(1 - \phi + r^*)dx_1 + \int_{r_1^*}^1 (1 - \phi + x_1)dx_1 \\
&= -c + \int_0^{r_1^*} (1 - \phi + r_1^*)dx_1 + \int_{r_1^*}^1 (1 - \phi + x_1)dx_1 \\
&= \theta(r_1^*)
\end{aligned}$$

Since $r_1^* < r^*$, from the inequalities above we have $\theta(r_1) \geq \theta(r_1^*) > 1 - \phi + r_1^*$. Therefore, the utility from moving to the second period is strictly less than the expected utility from searching and player 2 will search.

Now suppose $1 - r^* < 1/4$. If player 2 moves to the second period directly, he has to search and gets $\delta\theta(1/2)$. If he searches in the first period, his expected utility is:

$$
\begin{aligned}
\theta(r_1) &\geq -c + \int_0^{1/2} \delta\theta(1/2)dx_1 + \int_{1/2}^{\tilde{r}} \delta\theta(x_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1-\phi+r^*)dx_1 + \int_{r_1^*}^{1} (1-\phi+x_1)dx_1 \\
&> -c + \int_0^{1/2} \delta\theta(1/2)dx_1 + \int_{1/2}^{\tilde{r}} \delta(1-\phi+x_1)dx_1 + \ldots \\
&\qquad \int_{\tilde{r}}^{r_1^*} \delta(1-\phi+x_1)dx_1 + \int_{r_1^*}^{1} \delta(1-\phi+x_1)dx_1 \\
&> -c + \int_0^{1/2} \delta\theta(1/2)dx_1 + \int_{1/2}^{1} \delta(1-\phi+x_1)dx_1 \\
&> -c + \int_0^{1/2} \delta(1-\phi+1/2)dx_1 + \int_{1/2}^{1} \delta(1-\phi+x_1)dx_1 \\
&= \delta\theta(1/2)
\end{aligned}
$$

That is, $\theta(r_1) > \delta\theta(1/2)$. Therefore, the utility from moving to the second period directly is strictly less than the expected utility from searching and player 2 will search. □

Proposition 1: If $1 - r^* \geq 1/4$, there is a unique equilibrium to this game, where player 1 offers $r_1 = r^*$ in the first period and player 2 accepts immediately.

Proof. By lemma 2 player 2 will accept any offer $r_1 > r^*$. So player 1 will not offer anything greater than r^*. Suppose he offers $r_1 < r^*$. Then player 2 will search in the first period for sure. We consider two cases.

Suppose $r_1 < r_1^*$, then with probability $1 - r_1^*$ player 2 will produce the good with the ad hoc partner. With probability r_1^*, however, player 2 will not find a sufficiently large outside offer and the game reaches the second period. In the second period, player 1 will offer $r_2 = r^*$ and it will be accepted. Thus, the expected payoff for player 1 making offer r_1 is $\delta(1-r^*)r_1^*$, which is smaller than offering r^* to player 2 outright and getting $1 - r^*$.

Now suppose $r_1 \geq r_1^*$. Then player 2 will search in the first period but the game will not reach the second period. The expected payoff for player 1 is $r_1(1-r_1) \leq 1/4 \leq 1-r^*$, which means she is better off offering r^* outright. □

Proposition 2. If $1 - r^* < 1/4$, there is a unique equilibrium to the game where player 1 offers $\bar{r}_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, that maximizes (6), and player 2 searches in the first period.

$$\omega(r_1) = \int_0^{x(r_1)} (1 - r_1)dx_1 + \int_{x(r_1)}^{\tilde{r}} \delta(x_1(1 - x_1))dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{6}$$

Proof. First, note that player 1 will never offer anything more than r^*. By offering r^*, player 1 will guarantee the offer will be accepted and the game ends. Now the question is whether there is an offer $r_1 \in [0, r^*)$ that makes player 1 better off. To find such an offer, which will induce player 2 to search in the first period, we start by ruling out offers that are strictly dominated by other offers.

Suppose $r_1 \in [r_1^*, r^*)$. Player 1 will search and his expected utility from such an offer is $r_1(1 - r_1)$ (see Table 2). Since $\tilde{r} < r_1^* \leq r_1$, $r_1(1 - r_1) < \tilde{r}(1 - \tilde{r})$. We know that $\tilde{r}(1 - \tilde{r}) = 1 - r^*$, and it follows that $r_1(1 - r_1) < 1 - r^*$. In other words, the strategy of offering $r_1 \in (r_1^*, r^*)$ is strictly dominated by offering r^* outright.

Suppose $r_1 \in (\delta\theta(\tilde{r}) - 1 + \phi, r_1^*)$. First, note that since $1 - \phi + r_1^* = \delta(1 - \phi + r^*) = \delta\theta(r^*)$, $r_1^* = \delta\theta(r^*) - 1 + \phi > \delta\theta(\tilde{r}) - 1 + \phi$. Next, $r_1 > \delta\theta(\tilde{r}) - 1 + \phi > \delta\theta(x) - 1 + \phi$, $\forall x \leq \tilde{r}$. The total expected utility from offering r_1 is then

$$\int_0^{\tilde{r}} (1 - r_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{7}$$

Since (7) is an decreasing function of r_1, the maximum is achieved at $r_1 = \delta\theta(\tilde{r}) - 1 + \phi$. Therefore, offering $\delta\theta(\tilde{r}) - 1 + \phi$ strictly dominates any $r_1 \in (\delta\theta(\tilde{r}) - 1 + \phi, r_1^*)$.

Suppose $r_1 \in (0, \delta\theta(1/2) - 1 + \phi)$. Note that $\delta\theta(1/2) - 1 + \phi \in (1/2, \delta\theta(\tilde{r}) - 1 + \phi)$. For any r_1 in this range, it will always be rejected in the first period and the total expected utility from making such an offer is

$$\int_0^{1/2} \delta(1/4)dx_1 + \int_{1/2}^{\tilde{r}} \delta x_1(1 - x_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{8}$$

If, on the other hand, player 1 just offers $\delta\theta(1/2) - 1 + \phi$, she will get

$$\int_0^{1/2} [2 - \phi - \delta\theta(1/2)]dx_1 + \int_{1/2}^{\tilde{r}} \delta x_1(1 - x_1)dx_1 + \int_{\tilde{r}}^{r_1^*} \delta(1 - r^*)dx_1 \tag{9}$$

The difference between (8) and (9) is the first term, and it is easy to show that $2 - \phi - \delta\theta(1/2) > \delta(1/4)$:

$$2 - \phi - \delta\theta(1/2) > 2 - \phi - \theta(1/2) = c + 3/8 > \delta(1/4)$$

Thus, offering $\delta\theta(1/2) - 1 + \phi$ strictly dominates the strategy of offering any $r_1 \in (0, \delta\theta(1/2) - 1 + \phi)$.

After the above analysis, in terms of player 1's equilibrium strategy in the first period, one of two things must happen: player 1 offers r^*, or player 1 offers some $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$ that is better than any other offer in the region. Exactly which one is player 1's best response in the equilibrium depends on her expected utility from the two offers. If $r_1 = r^*$, then player 2 will accept the offer immediately and player 1's utility is $1 - r^*$. If player 1 offers $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, player 2 could have two potential responses: accept after a search, or reject after a search and go to the second period. More specifically, from player 2' equilibrium strategy in the first period, we know that for any $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$, there is a unique outside offer $x_1 \in (1/2, \tilde{r})$ that satisfies the equality $1 - \phi + r_1 = \delta\theta(x_1)$. Let $x(r_1)$ denote such an outside offer. Then, player 1's expected utility from offering $r_1 \in [\delta\theta(1/2) - 1 + \phi, \delta\theta(\tilde{r}) - 1 + \phi]$ is as (6).

Let $\bar{r}_1$ be the maximizer of equation (6), i.e., $\bar{r}_1 = argmax \;\; \omega(r_1)$. We show that $\omega(\bar{r}_1) \geq 1 - r^*$. The proof proceeds as follows. [15] First, let $\hat{r}_1 = \delta\theta(\tilde{r}) - 1 + \phi$, i.e., pick a point in this range that may or may not be the maximizer of equation (6). Then, calculate $\omega(\hat{r}_1)$ and compare $\omega(\hat{r}_1)$ with $1 - r^*$, given that $1 - r^* < 1/4$. We can show that $\omega(\hat{r}_1) > 1 - r^*$. Since

[15] Since the algebraic expressions of the proof is rather complicated, we only sketch the logic of the proof here.

$\omega(\bar{r}_1) \geq \omega(\hat{r}_1)$, it follows that $\omega(\bar{r}_1) > 1 - r^*$. Thus, in equilibrium, player 1 will offer $\bar{r}_1$ in the first period, which will lead player 2 to search. $\square$

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