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Bargaining in the Shadow of Power*

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Often a bargainer can use some form of power—legal, military, or political—to impose a settlement. How does the "outside" option of being able to impose a settlement, albeit at some cost, affect the bargaining? And, how does the probability that the bargaining will break down vary with the distribution of power between the bargainers? These questions are examined by adding the option of imposing a settlement to Rubinstein's game of dividing a pie. Each actor can accept an offer, make a counteroffer, or try to impose a solution. Imposing a settlement is, however, costly and each bargainer has private information about its cost. *Journal of Economic Literature* classification number: C72.

1. INTRODUCTION

Bargaining is ubiquitous. But the settings in which it takes place vary immensely. Bargaining is often presumed to occur in a situation of voluntary exchange. A buyer, for example, cannot be compelled to pay more than his or her valuation of the good for sale. In situations of voluntary exchange, there will be no bargaining unless the bargainers believe that there may be mutual gains from exchange. In many other contexts, however, bargaining takes place in the shadow of power. If a bargainer becomes sufficiently pessimistic about the prospects of reaching a mutually agreeable resolution, that bargainer can often use some form of power—be it legal, military, or political—to try to impose a settlement.

In pretrial bargaining, for instance, a litigant can use the court to impose a settlement by letting the suit go to trial. In contractual disputes with a compulsory-

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arbitration clause, the parties can impose a settlement by forcing the dispute into binding arbitration. In a two-party coalition-government in which the parties are bargaining about the position the cabinet will take on an issue, each party can impose a resolution by withdrawing from the government and forcing new elections (Lupia and Strom, 1995). If an interest group becomes sufficiently pessimistic about the outcome of legislative bargaining, then in 23 of the states in the United States an interest group can use a popular initiative to try to secure a more favorable outcome (Gerber, 1995). In international negotiations about revising the territorial status quo, a state can use military force to try to impose a new distribution of territory if that state becomes too doubtful about the prospects of reaching a mutually acceptable resolution (Fearon, 1992). In the bargaining between different ethnic groups about the rights each group will have under a new constitution, each group may resort to force to effect a resolution (Fearon, 1993). In the bargaining between political parties during democratic transitions, the parties may turn to violence and take to the streets to secure a better outcome (Houantchekon, 1994).

How does the shadow of power affect bargaining? How, in particular, do changes in the distribution of power between the bargainers affect the probability that the bargaining will end with one of the bargainers' trying to impose a resolution? This essay examines these questions by adding the option of forcing a settlement to Rubinstein's (1982) bargaining model. In Rubinstein's game, two actors are bargaining about dividing a pie and alternate making offers until one of them accepts the other's offer. In the model developed below, each actor also has the option of trying to impose a solution whenever that actor is deciding whether to accept an offer on the table or to make a counteroffer. If an actor does try to impose a settlement, it wins the entire pie with probability p while the other actor wins the pie with probability $1 - p$. In effect, p represents the distribution of political power: the larger p, the greater one actor's expected payoff to trying to impose a solution and the lower the other actor's expected payoff. It is, however, costly to use power, and each actor has private information about its cost.

Two competing factors make the relation between the probability of breakdown and the distribution of power ambiguous. The weaker a bargainer, the lower its expected payoff in the event of an imposed settlement and, consequently, the more likely that bargainer is to accept any given offer. This factor suggests that bargaining is less likely to breakdown if the distribution of power is uneven so that one of the bargainers is weak. But the weaker one bargainer, the more the other bargainer is likely to demand and these greater demands are more likely to be rejected. This second factor suggests that bargaining is less likely to breakdown if there is an even distribution of power, because this distribution will moderate both bargainer's demands. The net effect of these two factors is unclear.

The factors affecting the probability of bargaining impasses are of general interest (see Crawford, 1982), and, especially, in the work on strikes, arbitration, pretrial bargaining, and the causes of war in international relations theory. The latter two are of particular interest here. Priest and Klein (1984, p. 15) and Cooter, Marks, and Mnookin (1982) study nonstrategic pretrial bargaining models. Priest and Klein find that clear-cut cases, i.e., cases in which the plaintiff is either very likely to win or lose, are least likely to go to court. Cooter, Marks, and Mnookin find that anything that increases one of the litigant's expected value of going to trial makes settlement less likely. (The effects of a change in the relative strength of the plaintiff's case thus seems ambiguous; the stronger the plaintiff's case, the higher the plaintiff's expected payoff to going to court and the lower the defendant's expected payoff.) Reinganum and Wilde (1986), Nalebuff (1987), Schweizer (1989), and Spier (1992) study strategic models in which one of the parties has private information about the case. In Reinganum and Wilde's and Nalebuff's models, the defendant has private information about its degree of liability and the plaintiff makes a single settlement offer which the defendant either accepts or rejects. Excluding the possibility of nuisance suits by assuming that the plaintiff strictly prefers going to court if the defendant rejects its demand, Reinganum and Wilde examine, among other things, how different ways of allocating court costs affect the probability of trial. Nalebuff allows for the possibility of nuisance suits and shows that higher litigation costs may increase or decrease the probability of settlement depending on whether a "credibility" constraint is binding. Spier studies the pattern of settlement over time in a model in which the defendant still has private information about its liability but the plaintiff can make multiple offers. Schweizer assumes that the plaintiff and defendant have private information about the strength of their case and the probability of winning in court. In this model, the defendant makes a single offer which the plaintiff either accepts or rejects by litigating the case. Schweizer uses this model to examine the effects of changes in the quality of litigant's information about the strength of the case and shows that the probability of litigation can increase if one of the parties receives more accurate private information. (See Cooter and Rubinfeld, 1989, and Kennan and Wilson, 1993, for more extensive reviews of bargaining over legal disputes.)

The game analyzed here can be interpreted as a model of pretrial bargaining in which the litigants agree on the expected award if the case goes to court, but each litigant has private information about its litigation costs which, under the American system, it will have to pay regardless of the verdict. The litigants also discount the future, so the plaintiff, at least, is eager to settle the dispute as soon as possible. In trying to reach a settlement, the parties alternate making offers until they agree on a settlement or until one of them becomes sufficiently pessimistic about the prospects of reaching an agreement that it goes to court.

In international relations theory, the relation between the distribution of power and the probability that war will be used to try to impose a settlement has been the focus of a long debate. Morgenthau (1966) and the balance-of-power school more generally (Wright, 1965) argue that an even distribution is more stable, i.e., bargaining is less likely to break down in war if the probabilities that either state will prevail are roughly equal. Blainey (1973), Organski and Kugler (1980), and the preponderance-of-power school more generally argue that a preponderance of power is more stable. That is, bargaining is less likely to break down in war if one or the other of the bargainers is very likely to prevail in the event that military force is used. (This assertion directly parallels Priest's and Klein's (1984, p. 15) claim that clear-cut cases are less likely to go to court.) Empirical efforts to resolve this debate have yielded conflicting results. Siverson and Tennefoss (1984) find an even distribution to be more stable. Moul (1988), however, finds the opposite, and Maoz (1983) finds no significant relation between the distribution of power and the probability that a dispute will end in war. (See Levy, 1989, for a review of this debate as well as a survey of the empirical attempts to settle it.) Bueno de Mesquita and Lalman (1992) and Fearon (1992) have studied this relationship formally. Bueno de Mesquita and Lalman do not allow for endogenous offers. Fearon studies a model in which one state can present the other with a take-it-or-leave-it demand in the form of a military *fait accompli* which the other state can either accept or go to war to overturn. Both of these analyses find the distribution of power to be unrelated to the probability of war.

The game studied here can be interpreted as one in which two states are bargaining about revising the territorial status quo. The states agree on the distribution of military power and, hence, on the division of territory expected to result from war, but they have private information about their costs of using force, or, more generally, about their willingness to use force. The states alternate making offers until they reach agreement or one of them becomes too pessimistic and tries to impose a settlement by force.

The game has a simple, unique equilibrium outcome. A player will be called dissatisfied if it prefers an imposed settlement to what it would obtain in the bargaining game if the options of imposing a settlement were not present. Because it is costly to impose a settlement, at most only one bargainer can be dissatisfied. In equilibrium, the satisfied bargainer makes what is effectively its optimal take-it-or-leave-it offer. The dissatisfied bargainer either accepts this offer or imposes a settlement. (If both bargainers are satisfied, the options of imposing a settlement have no effect and the game is equivalent to a game in which these options are not present.)

This simple equilibrium is used to trace the relation between the probability of breakdown and the distribution of power. This probability is zero if the expected allocation of the "pie" resulting from an imposed settlement is the same as the Nash bargaining outcome of the underlying bargaining game in which the options of imposing a settlement do not exist. In these circumstances, no one expects to gain by imposing a settlement. If the bargainers are risk neutral or if the bargaining problem is symmetric in a sense described below, then the probability of breakdown in nondecreasing in the disparity between the expected outcome of imposing a solution and the Nash bargaining solution; i.e., this probability is nondecreasing in the absolute value of the difference between these two outcomes. If, however, the bargaining problem is sufficiently asymmetric, then the probability of breakdown may not be a monotonic function of this disparity.

Viewing these results in the context of pretrial bargaining, the plaintiff is the potentially dissatisfied bargainer; only the plaintiff may have a positive expected payoff to going to court. Thus in the equilibrium of alternating-offer game analyzed here, the defendant makes its optimal take-it-or-leave-it offer which the plaintiff either accepts or rejects by going to court. The model also indicates that if the plaintiff and defendant are risk neutral, as is generally assumed in models of pretrial bargaining, then the stronger the plaintiff's case, the more likely the bargaining is to break down with the plaintiff's taking the case to trial. More precisely, the probability that a dispute will go to trial is nondecreasing in the probability that the plaintiff will prevail.

In the context of international politics, the results derived below conflict with the claims of both the balance-of-power and preponderance-of-power schools. In the former, the probability of war is smallest when both sides are equally likely to prevail; in the latter, the probability of war is smallest when one state or the other is very likely to prevail. In the present model, the probability of war is smallest when the territorial distribution expected from fighting approximates the status quo distribution of territory.

The present analysis focuses primarily on the relation between the distribution of power and the probability that the bargaining will break down in an imposed settlement. This focus accounts for the specific way that the payoffs to the outside option are defined. In the model, the values of the bargainers' outside options are uncorrelated, but the expected values of these options are inversely related. The more powerful a bargainer, the higher the expected value of its outside option and the lower the expected value of the other bargainer's. The characterization of the equilibria, however, does not depend on the precise specification of the payoffs of the outside option. Indeed, the model can be seen somewhat more generally as an infinite-horizon, alternating-offer game in which each bargainer has an outside option the exercise of which ends the game in a Pareto inefficient outcome. The bargainers discount the future and have private information about the (uncorrelated) values of their outside options.

From this more general perspective, the fact that the game has a unique equilibrium outcome may seem surprising for two reasons. First, bargaining models in which an informed party can make offers generally have a multiplicity of equilibria. Second, adding an outside option to bargaining models that have unique equilibria often creates a multiplicity of equilibria (Fudenberg, Levine, and Tirole, 1987; Osborne and Rubinstein, 1990). The unique equilibrium in the present model is due to a kind of bargaining shutdown. Consider any information set at which the potentially dissatisfied bargainer is deciding whether or not to accept an offer. Regardless of its belief at this information set, the dissatisfied bargainer will generally not reject an offer in order to make a counteroffer. It will either accept the offer on the table or exercise its outside option. At the beginning of the game, therefore, the satisfied bargainer's best response to this strategy is to make its optimal initial take-it-or-leave-it offer. A similar kind of bargaining shutdown occurs in models in which the players have outside options, the values of which are common knowledge, and pay a fixed per-period cost to bargaining rather than discount the future (Fudenberg and Tirole, 1983, p. 246; Perry, 1986; Fudenberg, Levine, and Tirole, 1985, pp. 87–89). This shutdown also appears in models in which the players discount but also have to pay a known entry fee before beginning to bargain (Fudenberg and Tirole, 1991, pp. 414, 429).

The rest of the paper is organized as follows: Section II presents the model. Section III characterizes the dissatisfied bargainer's behavioral strategy at any information set at which it is considering an offer from the satisfied bargainer. The equilibrium of the game in which the satisfied state makes the first offer follows immediately from this characterization. Section IV characterizes the equilibrium when the dissatisfied bargainer makes the first offer. This section also shows that as long as the discount factor is close enough to one, then the probabilities of an imposed settlement are approximately the same regardless of which bargainer makes the first offer. Section V uses the equilibrium to examine the relation between the distribution of power and the probability that the bargaining will break down in an imposed settlement.

II. THE MODEL

There are two players, S, and D, and an initial status quo division of benefits. Let $b \leq 1$ represent the total per-period flow of benefits that exists prior to any agreement or to an imposed resolution and take q and $b - q$ to be D's and S's respective status quo flows of benefits. The actors are bargaining about how to divide a total per-period flow of benefits equal to one. If, therefore, b equals one, reaching an agreement will not increase the flow of total benefits. The bargainers, in other words, are already on the Pareto frontier and are negotiating about moving to a new location on the frontier. If $b < 1$, agreement brings joint gains, and the players are bargaining about how to divide these gains. (Analytically, it makes no difference if the bargainers are already on the Pareto frontier ($b = 1$), as would be the case of pretrial or territorial bargaining, or if they are initially inside the Pareto fronter ($b < 1$).)

The players alternate proposing divisions of the flow of benefits. Whenever an actor is considering whether to accept a proposal on the table or to reject it in order to make a counteroffer, it can also force the issue by trying to impose a settlement. The game ends as soon as the players agree on a division or one of them tries to impose a resolution.

To specify the players' payoffs if they reach a mutually acceptable agreement, assume that they agree to (x, y) at time t, where the first component is D's allocation. Then D's utility to having q from the zeroth to the tth period and having x thereafter is the average payoff $(1-\delta^t)U_D(q)+\delta^t U_D(x)$, where δ is the bargainers' common discount factor and $U_D' > 0$ and $U_D'' \leq 0$. Similarly, S's payoff is $(1-\delta^t)U_S(b-q)+\delta^t U_S(y)$, where $U_S' > 0$ and $U_S'' \leq 0$.

To define the payoffs if the players fail to agree and one of them tries to impose a settlement, suppose that someone forces the issue at time t. Then D is assumed to win the entire flow of benefits with probability p and to obtain no benefits with probability $1-p$. The use of power is also costly: D and S pay costs d and s, respectively. D's utility is therefore $(1-\delta^t)U_D(q)+\delta^t(pU_D(1)+(1-p)U_D(0)-d) = (1-\delta^t)U_D(q)+\delta^t(p-d)$, where $U_D(1)$ and $U_D(0)$ have been normalized to be one and zero, respectively. Similarly, S's utility is $(1-\delta^T)U_S(b-q)+\delta^t(1-p-s)$.

Each player has private information about its cost of imposing a settlement. Player D believes that S's costs or types are distributed over the interval $[\underline{s}, \bar{s}]$, where $\underline{s} > 0$ and the distribution function $F_S(s)$ is assumed to have a monotone hazard rate[1] and a bounded and continuous density function $f_S(s)$ such that $f_S(s) > 0$ over $(\underline{s}, \bar{s})$. The smaller s, the lower the cost to S of trying to impose a settlement and, less formally, the "tougher" or more willing S is to use force. Analogously, S believes that D's costs are distributed over $[\underline{d}, \bar{d}]$ according to $F_D(d)$, where $\underline{d} > 0$ and $F_D(d)$ has a monotone hazard rate and a bounded and continuous density function $f_D(d)$ such that $f_D(d) > 0$ over $(\underline{d}, \bar{d})$. The distributions F_D and F_S are common knowledge.

Figure 1 shows the per-period flow of benefits and helps fix ideas. The status quo is Q. If the options of trying to impose a settlement were not present, the game would simply be a complete-information Rubinstein (1982) game. Let R denote the outcome of this underlying game. The bargainers remain at Q until they agree to a different allocation or until one of them forces a settlement. F is the outcome of an imposed settlement if the two toughest types, i.e., $\underline{d}$ and $\underline{s}$, happen to be facing each other. The outcome of an imposed settlement is, however, uncertain because of the bargainers' private information about their costs. Thus, the outcome of a forced resolution is distributed over a rectangle with F as its upper-right corner. As the distribution of power shifts in favor of D, i.e., as the probability that D will win the entire pie, p, increases, F slides upward along the diagonal line in Fig. 1. As the distribution of power shifts against D, F slides downward. In terms of the figure, the problem of characterizing the relation between the distribution of power and the probability of an imposed settlement reduces to seeing how this probability varies as F slides along the diagonal.

Three preliminaries are needed before the game's equilibria can be character-

[1] That is, $d(F_S'(s)/(1-F_S(s)))/ds \geq 0$.

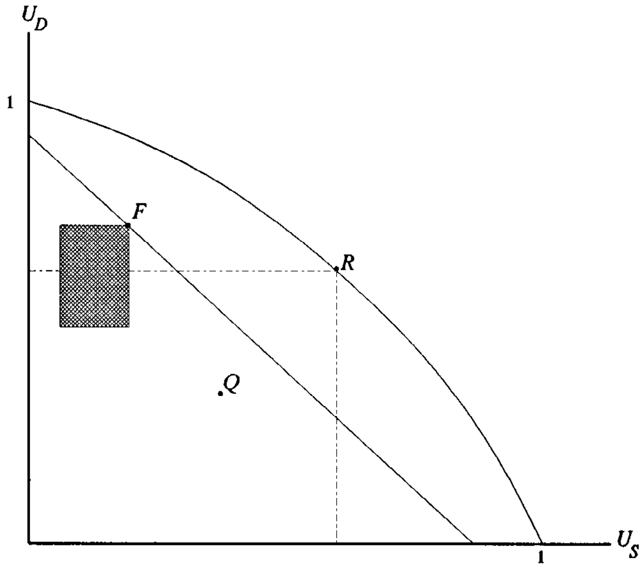

FIG. 1. The bargaining problem.

ized. First, “dissatisfied” player-types must be defined. Second, the equilibrium of the complete-information version of the game must be described. Finally, the bargainers’ strategies and beliefs must be specified for the asymmetric-information game.

Consider the alternating-offer game between D and S if the outside options of imposing a settlement were absent. The alternating-offer game without outside options reduces to a complete-information Rubinstein (1982) game. A player-type is “dissatisfied” if it strictly prefers an imposed settlement to accepting what would be offered to it in the underlying Rubinstein game. If a player-type is not dissatisfied, then it is satisfied. A bargainer is “potentially dissatisfied” if one its types is dissatisfied. Accordingly, D is potentially dissatisfied if the toughest type of D, $\underline{d}$, is dissatisfied, and S is potentially dissatisfied if $\underline{s}$ is dissatisfied. More precisely, in the unique subgame perfect equilibrium of the underlying Rubinstein game, D demands $r_D(\delta)$ for itself and S offers $r_S(\delta)$ to D, where $r_D(\delta)$ and $r_S(\delta)$ satisfy

$$\begin{aligned} U_S(1 - r_D(\delta)) &= (1 - \delta)U_S(b - q) + \delta U_S(1 - r_S(\delta)) \\ U_D(r_S(\delta)) &= (1 - \delta)U_D(q) + \delta U_D(r_D(\delta)). \end{aligned} \tag{1}$$

The first equation ensures that S is indifferent between agreeing to D’s demand now and to D’s agreeing to S’s offer in the next period. Similarly, the second equation means that D is indifferent to agreeing to S's offer now and to obtaining its demand in the next period. Given these demands and offers, the equilibrium outcomes are $(r_D(\delta), 1-r_D(\delta))$ if D makes the initial offer and $(r_S(\delta), 1-r_S(\delta))$ if S makes the initial offer. These offers and outcomes do not depend on D's or S's type, and it will be convenient to refer to them as Rubinstein offers and outcomes. Accordingly, a type d is dissatisfied if its expected payoff to imposing a settlement is strictly greater than its utility for S's Rubinstein offer, i.e., $p-d > U_D(r_S(\delta))$. Similarly, s is dissatisfied if $1-p-s > U_S(1-r_D(\delta))$.

If the discount factor is close enough to one, then at most only one player can be potentially dissatisfied. To see this, suppose that both bargainers are potentially dissatisfied. Then $p-\underline{d} > U_D(r_S(\delta))$ and $1-p-\underline{s} > U_S(1-r_D(\delta))$. Concavity implies $U_D(r_S(\delta)) \geq r_S(\delta)$ and $U_S(1-r_D(\delta)) \geq 1-r_D(\delta)$. Combining the previous inequalities gives $r_D(\delta) - r_S(\delta) > \underline{d} + \underline{s} > 0$. But $r_D(\delta) - r_S(\delta)$ converges to zero as δ goes to one because both $r_D(\delta)$ and $r_S(\delta)$ converge to the Nash bargaining solution. This contradiction implies that both bargainers cannot be potentially dissatisfied.

Without loss of generality, let D denote the potentially dissatisfied bargainer if there is one. Then the equilibria of two games must be characterized. In the first, the potentially dissatisfied bargainer makes the initial offer, while the satisfied bargainer makes the initial offer in the second. Let $\Gamma_D(\delta)$ and $\Gamma_S(\delta)$ denote these two games respectively. (If neither bargainer is potentially dissatisfied, then neither bargainer can credibly threaten to impose a settlement rather than to accept the other's Rubinstein offer. The game effectively degenerates into a complete-information Rubinstein game in which all s offer $r_S(\delta)$ and all d demand $r_D(\delta)$.)

The complete-information versions if $\Gamma_D(\delta)$ and $\Gamma_S(\delta)$ have a simple solution. Suppose that D is dissatisfied. Then in the unique subgame perfect equilibrium S offers D the value of D's outside option. D offers S just enough to leave S indifferent between accepting D's offer and countering it by offering D the value of D's outside option. These offers are always accepted in equilibrium. (See Osborne and Rubinstein, 1990, for a derivation of this equilibrium.)

To specify the strategies in the asymmetric-information game, let h_n be an n-period history which is comprised of a series of offers and rejections and which ends with a rejection. Let H_n be the set of all h_n. Take h'_n to be the history h_n followed by a proposed division and let H'_n be the set of all h'_n. Assuming for notational convenience that the first offer occurs in the zeroth period, then D makes offers in even-numbered periods in $\Gamma_D(\delta)$ and S accepts, rejects, or imposes a resolution. Similarly, S makes offers in odd-numbered periods. Accordingly, a pure strategy for player D in $\Gamma_D(\delta)$ is a family of measurable functions $\{\sigma_D^n\}_{n=0}^{\infty}$, such that if n is even, $\sigma_D^n\colon\ H_n \times [\underline{d}, \bar{d}] \to \{(x, y)\colon\ x + y \leq 1\}$ where the first component of $\sigma_D^n(h_n, d)$ is the share of the total flow of benefits D will receive. If n is odd, $\sigma_D^n\colon\ H'_n \times [\underline{d}, \bar{d}] \to \{Y, N, F\}$, where Y, N, and F respectively denote accepting the offer, rejecting the offer in order to make a counteroffer, and trying to impose a resolution. Pure strategies for S in $\Gamma_D(\delta)$ and for D and S in $\Gamma_S(\delta)$ are defined analogously.

To specify the players' beliefs in $\Gamma_D(\delta)$, let G_D and G_S denote the set of probability distributions over $[\underline{d}, \bar{d}]$ and $[\underline{s}, \bar{s}]$, respectively. Let $\mu_D^n(d)$ for n even and $\mu_S^n(s)$ for n odd denote D's and S's beliefs at the start of the nth round at which point the players are making an offer. Let $\mu_D^n(d)$ for n odd and $\mu_S^n(s)$ for n even denote D's and S's beliefs when deciding how to answer an offer. Then, $\mu_D^n\colon H_n' \to G_S$ for n odd and $\mu_D^n\colon H_n \to G_S$ for n even and similarly for μ_S^n. A player's beliefs will also be assumed to be unaffected by its decision to reject an offer in order to make a counteroffer: $\mu_D^n = \mu_D^{n+1}$ for n odd and $\mu_S^n = \mu_S^{n+1}$ for n even. Beliefs are defined analogously in $\Gamma_S(\delta)$.

A perfect Bayesian equilibrium (PBE) of $\Gamma_D(\delta)$ or $\Gamma_S(\delta)$ is a strategy profile (σ_D, σ_S) which is sequentially rational and a system of beliefs (μ_D, μ_S) which satisfies Bayes' rule whenever possible. That is, a player's beliefs after receiving an offer must be consistent with Bayes' rule applied to that player's beliefs just prior to the offer and the other player's behavioral strategy for making the offer.

III. THE EQUILIBRIA WHEN *S* MAKES THE INITIAL OFFER

The satisfied bargainer S makes the first offer in $\Gamma_S(\delta)$. In equilibrium, S makes its optimal take-it-or-leave-it offer given its prior beliefs F_D and subject to the constraint that this offer is at least as large as its Rubinstein offer $r_S(\delta)$. The potentially dissatisfied bargainer D either accepts this offer or imposes a settlement.

Lemmas 1 and 2 are the keys to this result. Lemma 1 puts bounds on what the satisfied bargainer will offer or accept, starting at any information set at which this bargainer is making an offer. Importantly, these bounds hold in both $\Gamma_S(\delta)$ and $\Gamma_D(\delta)$. Lemma 2 uses these bounds to show that no type that prefers an imposed settlement to accepting S's Rubinstein offer $r_S(\delta)$ will ever reject an offer in order to make a counteroffer. It will either accept the offer on the table or impose a settlement.

The intuition underlying Lemma 1 is straightforward. Suppose that S is at any information set in either $\Gamma_S(\delta)$ or $\Gamma_D(\delta)$ at which it is making an offer to D. Suppose further that d' is the toughest type that S might be facing; i.e., d' is the infimum of the support of S's beliefs at this information set. Then the most pessimistic beliefs that S could have about D are that S is facing d' for sure. Lemma 1 shows that no s will never offer more or accept less than it would in the complete-information game in which s faces d' for sure.

As outlined above, this complete-information game has a simple solution. If d' is satisfied, then the outside options have no effect on the outcome. s will offer $r_S(\delta)$ and reject anything leaving it will less than $1 - r_D(\delta)$. If d' is dissatisfied, s offers d' its certainty equivalent of imposing a settlement, which will be denoted by $\tilde{x}(d')$, and s rejects anything leaving it with less than the present value of countering with d''s certainty equivalent. That is, s would never accept less than $1 - z(\delta)$, where s is indifferent between accepting $1 - z(\delta)$ now and countering with d''s certainty equivalent; i.e., $z(\delta)$ solves $U_S(1 - z(\delta)) = (1 - \delta)U_S(b - q) + \delta U_S(1 - \tilde{x}(d'))$. In sum, s will never offer more than $\max\{r_S(\delta), \tilde{x}(d')\}$ or accept less than $\min\{1 - r_D(\delta), 1 - z(\delta)\}$. Since these bounds do not depend on s, the satisfied bargainer, regardless of type, will never offer more than $\max\{r_S(\delta), \tilde{x}(d')\}$ or accept less than $\min\{1 - r_D(\delta), 1 - z(\delta)\}$. More formally,

LEMMA 1. *Consider any PBE of $\Gamma_S(\delta)$ or $\Gamma_D(\delta)$ and any information set at which the satisfied bargainer is making an offer. Take d' to be the toughest type that S might be facing; i.e., d' is the infimum of the support of S's beliefs at this information set. Then S will never offer more than* $\max\{r_S(\delta), \tilde{x}(d')\}$. *Nor will S ever accept any offer of less than* $\min\{1 - r_D(\delta), 1 - z(\delta)\}$, *where $\tilde{x}(d')$ is the certainty equivalent to d' for imposing a settlement, i.e., $U_D(\tilde{x}(d')) = p - d'$, and $z(\delta)$ solves $U_S(1 - z(\delta)) = (1 - \delta)U_S(b - q) + \delta U_S(1 - \tilde{x}(d'))$.*

Proof. The proof is straightforward adaptation of the argument establishing Lemma 3.1 in Ausbel and Deneckere (1992) and is sketched in the Appendix.

The bounds on S's offers and acceptances imply that no dissatisfied type will reject an offer in order to make a counteroffer. To see this, suppose that a dissatisfied type did make a counteroffer by demanding some x. Let d' be the toughest type that demands x; i.e., d' is the infimum of the set of types demanding x. It is straightforward to show that d' would have done strictly better by imposing a settlement rather than countering with x. This contradiction implies that no dissatisfied type will make a counteroffer.

To obtain this contradiction, note that d' obtains $p - d'$ if it imposes a settlement. If d' forgoes the present opportunity to impose a settlement in order to counter with x, the game can end in only one of three ways following this demand. First, the game might end in an imposed settlement in a future period. Discounting ensures that d' strictly prefers imposing a settlement now to an imposed settlement in the future. The second way the game can end is that d' accepts a future offer. But d' is the toughest type facing S conditional on x. As shown in Lemma 1, S will never offer more than d''s certainty equivalent to imposing a resolution. Discounting again means that d' would strictly prefer imposing a settlement now to accepting the certainty equivalent of imposing a resolution later. Third, the game might end with the satisfied bargainer's accepting d''s demand. Lemma 1 implies that S will never accept a demand that leaves d' with more than $\max\{r_D(\delta), z(\delta)\}$. If, therefore, d' foregoes an opportunity to impose a settlement in order to make an offer that S accepts, d''s payoff is bounded above by $(1 - \delta)U_D(q) + \delta U_D(\max\{r_D(\delta), z(\delta)\})$, which is d''s payoff if it does not impose a settlement, demands $\max\{r_D(\delta), z(\delta)\}$ instead, and S accepts this demand immediately. The Appendix shows that this bound is strictly less than d''s payoff to imposing a settlement now rather than making a counteroffer. Thus, d' strictly prefers not making a counteroffer to any of the ways that the game might end after a counteroffer. Hence, d' can profitably deviate from countering with x by imposing a settlement. This contradiction means that no dissatisfied type will ever make a counteroffer. More formally

LEMMA 2. *Consider any perfect Bayesian equilibrium of* $\Gamma_S(\delta)$ *or* $\Gamma_D(\delta)$. *If* d *is dissatisfied, i.e., if* $p - d > U_D(r_S(\delta))$, *then* d *never makes a counteroffer.*

Proof. See the Appendix.

If a dissatisfied type will never reject an offer in order to make a counteroffer, then a dissatisfied type must either accept the offer on the table or impose a settlement. Lemma 3 formalizes this.

LEMMA 3. *Consider any perfect Bayesian equilibrium of* $\Gamma_S(\delta)$ *or* $\Gamma_S(\delta)$. *If* d *is dissatisfied and the current offer to* d *is* x, *then* d *accepts* x *if* $U_D(x) > p-d$ *and imposes a resolution if* $U_D(x) < p - d$.[2]

Proof. The lemma follows directly from Lemmas 1 and 2, and the proof is omitted.

Lemma 3 implies that a counteroffer unambiguously signals that d's certainty equivalent is no more than S's Rubinstein offer, because only these types might make a counteroffer in equilibrium. Having signaled that $\tilde{x}(d)$ is no more than $r_S(\delta)$, Lemma 1 implies that the best that d can do by making a counteroffer is to counter with its Rubinstein demand $r_D(\delta)$. This demand will leave S with $1-r_D(\delta)$ which is S's minimally acceptable offer. So, d will make a counteroffer only if, first, it is satisfied and, second, if agreeing to the current offer yields less than $U_D(r_S(\delta))$ which is the present value of agreeing to $r_D(\delta)$ in the next round. Lemma 4 formalizes d's decision.

LEMMA 4. *Consider any perfect Bayesian equilibrium of* $\Gamma_S(\delta)$ *or* $\Gamma_D(\delta)$. *If* d *is satisfied and the current offer to* d *is* x, *then* d *accepts any* $x > r_S(\delta)$ *and counters any* $x < r_S(\delta)$ *with* $r_D(\delta)$. *If* $x = r_S(\delta)$ *and there are joint gains, i.e.,* $b < 1$, *then* x *is accepted with probability one.*[3] *If* $x = r_S(\delta)$ *and there are no*

[2] If d is indifferent, then d's actions affect the satisfied player's payoff and the equilibrium outcome only if there is an atom at the value d_0 in the distribution characterizing the satisfied bargainer's beliefs where d_0 satisfies $U_D(x) = p - d_0$. If there is an atom at d_0, then d_0 must accept in equilibrium. For, if d_0 imposes a solution with positive probability, then the satisfied state can always do strictly better by offering slightly more than x which ensures that d_0 will accept.

[3] If $x = r_S(\delta)$, all d for which $\tilde{x}(d) \leq r_D(\delta)$ are indifferent between accepting x now and countering with $r_D(\delta)$ in the next period. Nevertheless, $x = r_S(\delta)$ must be accepted with probability one if there are mutual gains, for if x were rejected with positive probability, the satisfied bargainer would want to offer slightly more than $r_S(\delta)$ and no best reply for the satisfied player would exist.

joint gains so that the bargainers are already on the Pareto frontier, then d's action is irrelevant to the outcome. d may counter and the actors may continue to bargain, because the opportunity cost is zero when there are no joint gains. But the status quo allocation following the rejection of $x = r_S(\delta)$ will never be altered as neither bargainer is willing to impose a settlement.

Proof. The lemma follows directly from the preceding lemmas and the proof is omitted.

The previous lemmas describe the dissatisfied bargainer's behavioral strategy at an arbitrary information set in both $\Gamma_S(\delta)$ and $\Gamma_D(\delta)$ at which this bargainer is considering how to respond to an offer from S. In equilibrium, therefore, S will play a best response to this strategy when making an offer. This best response turns out to be S's optimal take-it-or-leave-it offer, given its beliefs and subject to the constraint that this offer is at least as large as S's Rubinstein offer.

To describe S's best response more precisely, consider a simple ultimatum game in which S makes an offer which D either accepts or rejects by imposing a settlement. Let $\beta_D[a, b]$ denote the satisfied bargainer's beliefs about the dissatisfied bargainer's type, where $\beta_D[a, b]$ is the distribution of d conditional on $d \in [a, b]$, given a prior distribution of F_D. In symbols, $\beta_D[a, b](d)$ equals zero if $d < a$, $((F_D(d) - F_D(a))/(F_D(b) - F_D(a))$ if $d \in [a, b]$, and one if $d > b$. Because d rejects x if $p - d > U_D(x)$ in this ultimatum game, the probability that x will be rejected is $\beta_D[a, b](p - U_D(x))$. Consequently, s's expected utility to offering x, conditional on beliefs $\beta_D[a, b]$ and given that D will either accept x or impose a settlement, is

$$\begin{aligned} T(x, s, \beta_D[a, b]) &= U_S(1 - x)\left(1 - \frac{F_D(p - U_D(x)) - F_D(a)}{F_D(b) - F_D(a)}\right) \\ &\quad + (1 - p - s)\frac{F_D(p - U_D(x)) - F_D(a)}{F_D(b) - F_D(a)}. \end{aligned}$$

Let $t^*(s, \beta_D[a, b])$ denote the optimal of $T(x, s, \beta_D[a, b])$, where the assumptions that F_D has a monotone hazard rate ensures that t^* is unique.

As long as s's optimal take-it-or-leave-it offer t^* is at least as large as $r_S(\delta)$, then Lemmas 1–4 imply that all d will either accept the offer or reject it by imposing a settlement. Thus, t^* is s's best response and is self-confirming as long as $t^* \geq r_S(\delta)$: s is effectively making a take-it-or-leave-it offer whenever it offers at least $r_S(\delta)$; and given that s is making a take-it-or-leave-it offer, t^* is the optimal offer to make. If however, $t^* < r_S(\delta)$, then t^* is no longer the optimal offer. Lemma 4 shows that all satisfied d will reject t^* and counter with a demand of $r_D(\delta)$. If, therefore, s offers less than $r_S(\delta)$, the dissatisfied bargainer no longer responds as if this were a take-it-or-leave-it offer. Accordingly, t^* is no longer optimal. Indeed, the proof of Proposition 1 shows that $r_S(\delta)$ is the optimal offer if $t^* < r_S(\delta)$. In sum, s's optimal offer, given belief $\beta_D[a, b]$ is the bounded take-it-or-leave-it offer of $\max\{t^*(s, \beta_D[a, b]), r_S(\delta)\}$.

The equilibrium outcome of $\Gamma_S(\delta)$ in which S makes the initial offer is now easy to characterize. S makes this offer conditional on its prior beliefs $F_D = \beta_D[\underline{d}, \bar{d}]$. Thus, s offers $\max\{t^*(s, \beta_D[\underline{d}, \bar{d}]), r_S(\delta)\}$ which D either accepts or rejects by imposing a settlement. More formally,

PROPOSITION 1. *In any PBE of $\Gamma_S(\delta)$, the satisfied bargainer S initially proposes its optimal bounded take-it-or-leave-it offer $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) = \max\{r_S(\delta), t^*(s, \beta_D[\underline{d}, \bar{d}])\}$. Dissatisfied types accept x^* if $U_D(x^*) > p - d$ and impose a settlement if $U_D(x^*) < p - d$. Satisfied types, accept x^* with probability one.*

Proof. See the Appendix.

Lemmas 1–4, which hold regardless of which bargainer moves first, show that the equilibria in these games exhibit a kind of bargaining shutdown. This shutdown accounts for the simplicity of the games' equilibria and is reminiscent of the equilibria of models in which the players have outside options the values of which are common knowledge and in which bargainers pay fixed per-period bargaining costs rather than discount (Fudenberg and Tirole, 1983, p. 246; Fudenberg, Levine, and Tirole, 1985, pp. 87–89; Perry, 1986). This shutdown also occurs in models in which the players do discount but also have to pay a known entry fee before beginning to bargain (Fudenberg and Tirole, 1991, pp. 414, 429). Bargaining collapses in all of these models for the same basic reason. If bargaining is to continue in equilibrium, there must be a toughest type that continues (or, more precisely, an infimum of the set of continuing types). But in these models one of the bargainers is unable to commit itself to giving the toughest type of the other bargainer that continues enough of the surplus to make continuing worthwhile to this toughest type. Bargaining shuts down because this type always strictly prefers not to continue.

IV. THE EQUILIBRIA WHEN *D* MAKES THE INITIAL OFFER

This section characterizes the equilibria of $\Gamma_D(\delta)$ in which the dissatisfied bargainer makes the initial offer. The section also shows that which bargainer makes the first offer has no effect on the probability that bargaining will break down in an imposed settlement. The probabilities of an imposed settlement in $\Gamma_D(\delta)$ and $\Gamma_S(\delta)$ are equal in the limit as the discount factor goes to one.

D makes the first offer in $\Gamma_D(\delta)$, and Lemmas 1–4 imply that S will either accept this offer or reject it by countering with its optimal bounded take-it-or-leave-it offer given its updated beliefs. D will then either accept this offer or impose a settlement. The problem of characterizing the equilibria of $\Gamma_D(\delta)$ thus reduces to describing D's initial offer.

There are three cases to be considered depending on S's prior beliefs, and two further assumptions are needed to facilitate the analysis of these cases. The first assumption is that D's demands are connected in type. That is, if two types, say d_1 and d_2, make the same demand after some history, then all types between d_1 and d_2 make the same demand after the same history. The second assumption imposes a restriction on a bargainer's beliefs following an equilibrium demand made by a nonempty but measure-zero set of types. Suppose that a set of types of D makes a common demand x in equilibrium and that this set, although nonempty, has probability zero. Then the support of S's beliefs conditional on x will be assumed to be a subset of the set of types that made this demand and that were in the support of S's beliefs when this demand was made. That is, let $\mathcal{D}_x$ be the set of types that demand x and suppose that this set is nonempty but has measure zero. Then, the support of S's beliefs following x must be in the intersection of $\mathcal{D}_x$ and the support of S's beliefs just prior to receiving x. D's beliefs are similarly restricted. Finally, note that this assumption places no restriction on the bargainers' beliefs following a demand that no type would make in equilibrium.

The first case that needs to be examined is at one extreme. Suppose that S is so confident that D is dissatisfied that all s offer D the certainty equivalent of D's toughest type. In symbols, $t^*(s, \beta_D[\underline{d}, \bar{d}]) = \tilde{x}(\underline{d})$ for all s. This offer is large enough to ensure that all d accept. In the context of $\Gamma_S(\delta)$, these beliefs imply that all s initially offer $\tilde{x}(\underline{d})$ and that the probability of breakdown is zero. In the context of $\Gamma_D(\delta)$, these beliefs mean that if δ is sufficiently close to one, then essentially all d pool on a common demand in any PBE of $\Gamma_D(\delta)$. This pooling leaves S's initial beliefs substantially unchanged. If S makes a counteroffer, it continues to offer the certainty equivalent of the toughest type it might be facing and the probability of breakdown is also zero in $\Gamma_D(\delta)$.

At the other extreme, S is so confident that D will not impose a settlement that S's optimal take-it-or-leave-it offer for all types s is its Rubinstein offer $r_S(\delta)$, which is what it would offer if the option of imposing a settlement were absent.[4] In the context of $\Gamma_S(\delta)$, S's initial offer would be $r_S(\delta)$. All d that prefer an imposed settlement to this offer impose a settlement, and all other d accept the offer. Thus, the probability of breakdown is the probability that $p - d > r_S(\delta)$ or, in other words, that D is actually dissatisfied. In $\Gamma_D(\delta)$, all $d \in [\underline{d}, \hat{d})$ pool

[4] Models of pretrial bargaining often exclude this case by assuming that the plaintiff (i.e., the potentially dissatisfied bargainer) always prefers going to court if the defendant refuses to pay any damages. (See, for example, Reinganum and Wilde, 1986, and Spier, 1992. Nalebuff, 1987, is, however, an exception.) This case is likely to arise when the dissatisfied bargainer is weak, i.e., p is small, and needs to be considered if the relation between the probability of breakdown and p is to be characterized for p small.

on a nonserious demand, i.e., a demand that is rejected with probability one, and all $d \in (\hat{d}, \bar{d}]$ demand $r_D(\delta)$. $\hat{d}$ is the unique type that is just indifferent between obtaining $r_D(\delta)$ immediately or obtaining S's counteroffer conditional on S's beliefs $\beta_D[\underline{d}, \hat{d}]$. In the limit as the discount factor δ goes to one, the probability of breakdown equals the probability that D is dissatisfied.

In the third and intermediate case, S makes counteroffers between the bounds $r_S(\delta)$ and $\tilde{x}(\underline{d})$ given its prior of F_D. Here, all d pool on a common, high demand in any PBE of $\Gamma_D(\delta)$ if δ is sufficiently close to one. Because the demand is high, S generally rejects it and makes a counteroffer. Because of pooling, updating does not change S's priors, and S's counter in $\Gamma_D(\delta)$ is the same as its initial offer in $\Gamma_S(\delta)$. The probabilities of breakdown in these two games are the same in the limit as the discount factor goes to one.

All types pool in the intermediate case above, and there are substantial amounts of pooling in the extreme cases. Figure 2 helps make the reason for the complete pooling in the intermediate case clear and helps develop the intuition underlying the proof of this case in Proposition 2. Suppose that there are at least two distinct demands. The assumptions that the set of types making the same demand is connected means that the set $[\underline{d}, d^0)$ makes one demand, say x, and $(d', \bar{d}]$ makes another demand y, where $x \neq y$ and $d^0 \leq d'$. Conditional on an initial demand x, S would never counter by offering D less than d^0's certainty equivalent of imposing a settlement. Such an offer would be rejected for sure and is always dominated by offering d^0 its certainty equivalent. Indeed in the intermediate case, S's counter, if it makes one, is strictly bounded below by d^0's certainty equivalent, which is denoted by $\tilde{x}(d^0)$. Accordingly, d^0's payoff to demanding x is strictly bounded below by $(1-\delta)U_D(q) + \delta U_D(\tilde{x}(d^0))$ which is d^0's payoff if S counters x with $\tilde{x}(d^0)$.

Because d^0 is offered more than its certainty equivalent of imposing a settlement, d^0 accepts S's counter. Because d^0 accepts, any other type can costlessly mimic d^0. In particular, any d demanding y can costlessly mimic d^0 by demanding x instead and thereby obtain a payoff strictly bounded below by $(1-\delta)U_D(q) + \delta U_D(\tilde{x}(d^0))$.

For discount factors close to one, the types demanding y strictly prefer to deviate by demanding x. To see this, observe that demanding y signals that D's payoff to an imposed settlement is no more than d''s certainty equivalent of $\tilde{x}(d')$. Lemma 1 then implies that S will never offer more than the maximum of its Rubinstein offer $r_S(\delta)$ and the certainty equivalent of d', the toughest type that S might be facing conditional on y. In the intermediate case, the certainty equivalent is larger than the Rubinstein offer, so S never offers more than $\tilde{x}(d')$. This maximum offer implies that the best that any d can do, given that it has signaled that its payoff to an imposed settlement is no more than $\tilde{x}(d')$, is to make a demand that leaves S indifferent between accepting this demand and countering with $\tilde{x}(d')$. Thus, d''s payoff to demanding y is bounded above by

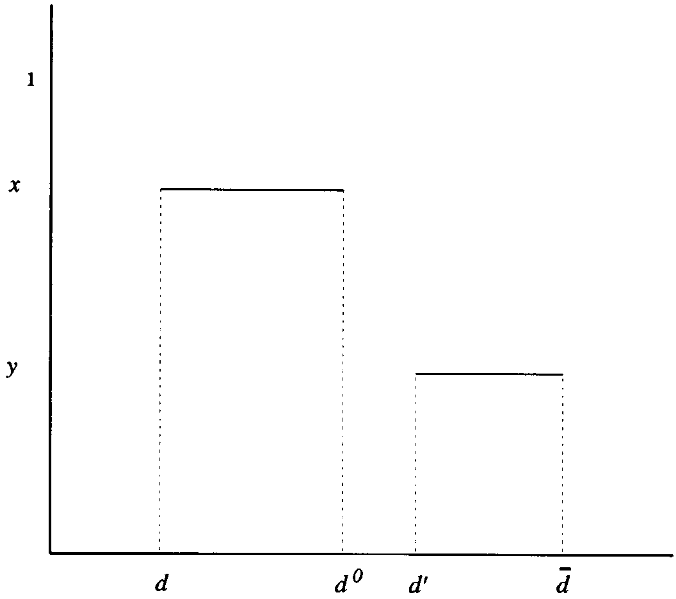

FIG. 2. D's initial demand.

$U_D(z(\delta))$, where $z(\delta)$ solves $U_S(1-z(\delta)) = (1-\delta)U_S(b-q)+\delta U_S(1-\tilde{x}(d'))$. Incentive compatibility requires that the lower bound to $\bar{d}$'s payoff to deviating to x be at least as large as the upper bound to demanding y. But d''s payoff to deviating to x is strictly bounded below by $(1-\delta)U_D(q) + \delta U_D(\tilde{x}(d^0))$, and $d^0 \leq d'$ implies $\tilde{x}(d^0) \geq \tilde{x}(d')$. Consequently, the incentive-compatibility requirement cannot hold in the limit as δ goes to one. Thus, d' would have a positive incentive to deviate from y to x if there were two distinct demands.[5]

To characterize the equilibrium of $\Gamma_D(\delta)$ more formally, recall that the limit of the Rubinstein offer $r_S(\delta)$ as δ goes to one is the Nash bargaining solution which is denoted by r. Then,

PROPOSITION 2. *For any* $\epsilon > 0$, *there exists a* $\underline{\delta} < 1$ *such that*:

(*i*) *If* $t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}]) = \tilde{x}(\underline{d}) > r$, *then all* $d \in (\underline{d}+\epsilon, \bar{d}]$ *pool on a common demand in any PBE of* $\Gamma_D(\delta)$ *whenever* $\delta > \underline{\delta}$.

(*ii*) *If* $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) < r$, *then all* $d \in [\underline{d}, \hat{d}(\delta))$ *pool on a nonserious demand and all* $d \in (\hat{d}(\delta), \bar{d}]$ *pool on* $r_D(\delta)$ *in any PBE of* $\Gamma_D(\delta)$ *whenever* $\delta > \underline{\delta}$,

[5] This argument needs to be amended in the two extreme cases. In these cases, S's counter to y may not be strictly bounded above d^0's certainty equivalent. Or, S may counter x with $r_S(\delta)$ which is larger than d''s certainty equivalent. These possibilities make for somewhat less pooling.

where $\hat{d}(\delta)$ *is the type that is indifferent between obtaining* $r_D(\delta)$ *immediately or waiting for S's optimal counteroffer conditional on S's beliefs* $\beta_D[\underline{d}, \hat{d}(\delta)]$; *i.e.,* $\hat{d}(\delta)$ *solves* $U_D(r_D(\delta)) = (1-\delta)U_D(q)+\delta \int_{\underline{s}}^{\bar{s}} U_D(x^*(s, \delta, \beta_D[\underline{d}, \hat{d}(\delta)]))dF_S(s)$.

(*iii*) *If* $\tilde{x}(\underline{d}) > t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}])$ *and* $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) > r$, *then all* $d \in [\underline{d}, \bar{d}]$ *pool on a common demand in any PBE of* $\Gamma_D(\delta)$ *whenever* $\delta > \underline{\delta}$.[6]

Proof. See the Appendix.

Two remarks about Proposition 2 are in order. The first focuses on the restriction put on beliefs at information sets following an offer made by a nonempty but zero-measure set of types. Recall that if a nonempty set of types in the support of S's beliefs at some information set makes a demand x, then the support of S's beliefs conditional on x must be contained in the nonempty set that might have demanded x even if this nonempty set has probability zero. Without this restriction, the equilibrium outcomes of $\Gamma_S(\delta)$ and $\Gamma_D(\delta)$ may be quite different even in the limit as δ goes to zero.

To illustrate this possibility, suppose, as in the first case above in Proposition 2, that S's prior beliefs, F_D, are such that s's optimal take-it-or-leave-it offer is to ensure that a settlement will not be imposed by offering the certainty equivalent of the toughest type; i.e., all s offer $\tilde{x}(\underline{d})$. Assume further that all types of D are dissatisfied. Proposition 1 then implies that all s offer $\tilde{x}(\underline{d})$ in $\Gamma_S(\delta)$. This offer is accepted with probability one, leaving the probability of breakdown equal to zero.

Without some restriction on what S can believe after receiving an offer from a nonempty, measure-zero set of types, it is easy to construct an equilibrium of $\Gamma_D(\delta)$ in which the probability of breakdown is one. Suppose each d makes the largest possible demand that s would accept conditional on being sure of facing d. That is, d demands $z(d)$, where $z(d)$ solves $U_S(1 - z(d)) = (1 - \delta)U_S(b - q) + \delta U_S(1 - \tilde{x}(d))$. Although each type makes a distinct demand, the probability that any particular demand will be made is zero. If no restriction is put on these beliefs, s may be assumed to put probability one on facing the weakest type, $\bar{d}$. These beliefs imply that s will agree to any demand that leaves it with at least $1 - z(\bar{d})$ and will counter any greater demand by offering $\tilde{x}(\bar{d})$. These strategies form an equilibrium in which the probability of breakdown is one. The restriction put on beliefs above eliminates equilibria of this type by requiring S to infer from a demand of $z(d)$ that it is facing a type that might have made this demand, namely d.

The second remark focuses on D's participation constraint in $\Gamma_D(\delta)$. The extensive form of $\Gamma_D(\delta)$ does not permit a player to try to impose a settlement

[6] The cases in which $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) = r$ or $\tilde{x}(\underline{d}) = r$ are excluded as being nongeneric.

in the same round in which it is making an offer. When making an offer, a player's next opportunity to impose a resolution will come in the next round if its offer is rejected. Consequently, D cannot impose a settlement in the first round of $\Gamma_D(\delta)$, and $\underline{d}$ generally obtains less in the pooling equilibria of $\Gamma_D(\delta)$ described in Proposition 2 than $U_D(\tilde{x}(\underline{d}))$ which is the payoff $\underline{d}$ would receive if it could impose a settlement in the first round or prior to entering the bargaining. Depending on the substantive interpretation underlying the model, this would seem to violate $\underline{d}$'s participation constraint. The idea here is that there is some positive but arbitrarily small cost to not entering the negotiations. In international politics, for example, it may be costly in terms of public support to be seen as unwilling to sit down at the negotiating table. If there is a positive but arbitrarily small cost, then as long as the discount factor is close enough to one, $\underline{d}$ will strictly prefer making a demand to forcing the issue without having been at the table.

Propositions 1 and 2, respectively, characterize the equilibria when the satisfied bargainer moves first or second. Proposition 3 shows that the order of play has no significant effect on the probability of breakdown if the discount factor is sufficiently close to one.

PROPOSITION 3. *For any distribution of power p and any $\epsilon > 0$, there exists a $\underline{\delta} < 1$ such that the probabilities of breakdown in any PBE of $\Gamma_S(\delta, p)$ and $\Gamma_D(\delta, p)$ differ by less than ϵ whenever $\delta > \underline{\delta}$.*

Proof. See the Appendix.

Proposition 3 means that the relation between the probability of breakdown and the distribution of power can be examined solely in terms of the much simpler equilibrium of $\Gamma_S(\delta)$, where the probability of breakdown is just the probability that S's bounded take-it-or-leave-it offer will be rejected.

V. THE PROBABILITY OF BREAKDOWN

The disparity between the Nash bargaining solution and the allocation expected to result from an imposed settlement is crucial to the relation between the distribution of power and the probability of breakdown. Figure 1 provides some intuition. The option of imposing a settlement is incredible whenever F is Pareto-dominated by the Rubinstein outcome R. Bargaining never breaks down in these circumstances. R, moreover, dominates F if the expected imposed settlement $(p, 1-p)$ is not too different from the Nash bargaining solution $(r, 1-r)$. To see this, recall that in the limit as the discount factor goes to one, the Rubinstein outcome R converges to the player's utilities for the Nash bargaining solution, i.e., $R = (U_D(r), U_S(1-r))$ as the discount factor goes to one.Thus, R Pareto-dominates F in the limit if $U_D(r) > p - \underline{d}$ and $U_S(1-r) > 1 - p - \underline{s}$.

Concavity implies $U_D(r) \geq r$ and $U_S(1-r) \geq 1-r$, so R Pareto-dominates F as long as $r > p - \underline{d}$ and $1 - r > 1 - p - \underline{s}$. The latter two inequalities hold whenever $|p - r| < \min\{\underline{d}, \underline{s}\}$. Hence, the probability of an imposed settlement is zero as long as the difference between the expected allocation of an imposed settlement and the Nash bargaining solution is not too large.

A natural conjecture about the relationship between the probability of breakdown and the distribution of power is that this probability is nondecreasing in $|p - r|$. (It cannot be strictly increasing, because it is zero whenever $|p - r|$ is small.) As shown in the Appendix, this conjecture holds if the bargainers are risk neutral. It also holds if the bargaining problem is sufficiently symmetric in that the bargainers have identical utility functions and the Nash bargaining solution is an even distribution of the benefits ($r = \frac{1}{2}$). A counterexample, however, shows that if the actors are risk averse and the bargaining problem is sufficiently asymmetric, then the probability of breakdown may not be monotonic in the size of the disparity.

In the context of pretrial bargaining, these results imply that the probability that a dispute will go to trial is nondecreasing in the strength of the plaintiff's case. If the issue in dispute is whether the defendant will pay damages, then the plaintiff is the potentially dissatisfied bargainer as only the plaintiff could gain by an imposed settlement. If, moreover, the plaintiff and the defendant are risk neutral as is generally assumed (e.g., Reinganum and Wilde, 1986; Nalebuff, 1987; Schweizer, 1989; and Spier, 1992), then the probability that the dispute will go to trial is nondecreasing in $|p - r|$, where p is the probability that the plaintiff will win and r is the plaintiff's share in the Nash bargaining solution of the game if the outside option of imposing a settlement were not present. If the option of going to court were not present, the defendant would not pay anything to the plaintiff. The plaintiff and the defendant are already on the Pareto frontier, so the Nash bargaining solution is simply the existing allocation. Thus the plaintiff's share at the Nash bargaining solution is $r = 0$, and the probability that the bargaining will break down is nondecreasing in p.

In the context of international relations theory, the results derived here conflict with both the balance-of-power school, which holds that an even distribution of power is more stable (e.g., Morganthau, 1966; Wright, 1965), and the proponderance-of-power school, which holds that a preponderance of power is more stable (Blainey, 1973; Organski and Kugler, 1980; Levy, 1989). If a balance of power were more stable, the probability of breakdown should reach a minimum at $p = \frac{1}{2}$ and generally increase as the distribution of power becomes more uneven or as $|p - \frac{1}{2}|$ increases. If a preponderance of power were more stable, then the probability of breakdown should be smallest at the extremes of $p = 0$ or $p = 1$ and generally increase as p approaches $\frac{1}{2}$. In the bargaining model analyzed here, the probability of breakdown is smallest at $p = r$, and r is, in general, not equal to 0, $\frac{1}{2}$, or 1.

CONCLUSION

In many bargaining contexts, a bargainer can use some form of power—be it legal, military, or political—to try to impose a settlement. Whether it chooses to exercise this "outside" option depends on the prospects of reaching a more favorable agreement. Bargaining in the shadow of power raises at least two questions: How does the shadow of power affect the equilibrium distribution of benefits, and how does the probability of breakdown vary with the distribution of power? In the model studied here, the equilibrium distribution is given by the satisfied bargainer's constrained optimal take-it-or-leave-it offer. And, the probability of breakdown is zero if the allocation of benefits expected from an imposed settlement is the same as the Nash solution. If, moreover, the bargainers are risk neutral or if the bargaining is symmetric, the probability of breakdown is nondecreasing in the disparity between these two allocations.

APPENDIX

Preliminaries. It will be convenient in what follows to introduce some notation and basic relations. Define $A(x, \delta)$ and $B(x, \delta)$ as follows:

$$\begin{aligned} A(x, \delta) &= (1-\delta)U_D(q) + \delta U_D(1 - U_S^{-1}((1-\delta)U_S(b-q) + \delta U_S(1-x))) \\ B(x, \delta) &= (1-\delta)U_S(b-q) + \delta U_S(1 - U_D^{-1}((1-\delta)U_D(q) + \delta U_D(x))). \end{aligned}$$

The equation $U_D(x) = A(x, \delta)$ is equivalent to eliminating $r_D(\delta)$ from Eq. (1). Thus $r_S(\delta)$ is the unique solution of $U_D(x) = A(x, \delta)$, where uniqueness follows from the assumptions that U_S and U_D are increasing and concave. Moreover, $U_D(x) > A(x, \delta)$ if $x > r_S(\delta)$ and $U_D(x) < A(x, \delta)$ if $x < r_S(\delta)$. Similarly, $r_D(\delta)$ is the unique solution of $U_S(1-x) = B(x, \delta)$ with $B(x, \delta) > U_S(1-x)$ if $x > r_D(\delta)$ and $B(x, \delta) < U_S(1-x)$ if $x < r_D(\delta)$.

Proof of Lemma 1. The proof is an adaptation of the argument establishing Lemma 3.1 in Ausubel and Deneckere (1992, p. 606) and so it will only be sketched here. Let h be any information set at which s is making an offer and let d' denote the infinum of the support of s's beliefs about the dissatisfied bargainer. Less formally, d' is the toughest type t hat s believes it might be facing at h. Assume further that z satisfies $U_S(1-z) = (1-\delta)U_S(b-q) + \delta U_S(1-\tilde{x}(d'))$, where $\tilde{x}(d')$ is the certainty equivalent of d' for imposing a settlement; i.e., $U_D(\tilde{x}(d'))p - d'$.

To see that s will never offer more than $\max\{r_S(\delta), \tilde{x}(d')\}$ or agree to give the dissatisfied bargainer more than $\max\{r_D(\delta), z\}$ in a perfect Bayesian equilibrium given that h has been reached, let $\bar{x}$ be the supremum of s's offers or acceptances. Now consider d's decision at any information set along the equilibrium path, given that h has been reached. d's payoff to rejecting s's offer or forcing the issue is bounded above by $\max\{(1-\delta)U_D(q)+\delta U_D(\bar{x}), U_D(\tilde{x}(d'))\}$. The first element is the best that potentially dissatisfied bargainer can do if it rejects s's offer and the bargainers subsequently reach an agreement. The second element is the upper bound on d's payoff if S or D ultimately compels a resolution. This bound means that d would accept any offer x such that $x > \max\{y, \tilde{x}(d'))\}$, where y solves $U_D(y) = (1-\delta)U_D(q)+\delta U_D(\bar{x})$. But, s will never offer more than the minimal amount required to induce the potentially dissatisfied power to accept. That is, s will only make offers x such that $x \le \max\{y, \tilde{x}(d')\}$.

In sum, s never offers more than $\max\{y, \tilde{x}(d')\}$. So, Lemma 1 will hold if $\max\{y, \tilde{x}(d')\} \le \max\{r_S(\delta), \tilde{x}(d')\}$. To establish this inequality, it will suffice to show that $y > \tilde{x}(d')$ implies $y \le r_S(\delta)$. Assume $y > \tilde{x}(d')$. Then, by construction, s either accepts or proposes a $v \in (\bar{x}-\epsilon, \bar{x}]$ for any $\epsilon > 0$. Because $\bar{x} > y$, ϵ can be taken small enough to ensure that $v > y$. Because $v > y = \max\{y, \tilde{x}(d')\}$, d must have proposed v and s must have accepted.

Now suppose $\bar{x} > r_D(\delta)$, then s's acceptance leads to a contradiction. To see this, note that in equilibrium s cannot improve its payoff by rejecting v. But suppose s rejects v and counters with some $w > y > \tilde{x}(d')$. Because $w > \max\{y, \tilde{x}(d')\}$, the dissatisfied bargainer can never expect a more favorable agreement in equilibrium. Thus, d accepts, leaving s with a payoff of $(1-\delta)U_S(b-q)+\delta U_S(1-w)$ from countering with w; s therefore has an incentive to deviate if there exists a $w > y$ such that $U_S(1-v) < (1-\delta)U_S(b-q)+\delta U_S(1-w)$ for v close enough to $\bar{x}$. But the assumption that $\bar{x} > r_D(\delta)$ implies $U_S(1-\bar{x}) < B(\bar{x},\delta)$, where B is defined in the preliminaries above. Continuity and the fact that the previous inequality is strict then imply that there exists an $\bar{\epsilon} > 0$ such that $U_S(1-(\bar{x}-\epsilon')) < B(\bar{x}-\epsilon,\delta)$ for any ϵ and ϵ' in $[0,\bar{\epsilon}]$. But, $B(\bar{x}-\epsilon,\delta) = (1-\delta)U_S(b-q)+\delta U_S(1-(y+\mu))$ for a $\mu > 0$. Finally, recall that $v \in (\bar{x}-\bar{\epsilon}, \bar{x}]$. Taking $\epsilon' = \bar{x}-v$ then gives $U_S(1-v) = U_S(1-(\bar{x}-\epsilon')) < B(\bar{x}-\epsilon,\delta) = (1-\delta)U_S(b-q)+\delta U_S(1-(y+\mu))$; s, therefore, strictly prefers to counter v with $y+\mu$.

This contradiction leaves $r_D(\delta) \ge \bar{x}$. This inequality in turn implies $U_D(y) = (1-\delta)U_D(q)+\delta U_D(\bar{x}) \le (1-\delta)U_D(q)+\delta U_D(r_D(\delta)) = U_D(r_S(\delta))$. Thus, $r_S(\delta) \ge y$, and s will never offer more than $\max\{r_S(\delta), \tilde{x}(d')\}$. It immediately follows that s will never agree to more than $\max\{r_D(\delta), z\}$. ■

Proof of Lemma 2. The discussion preceding the formal statement of Lemma 2 shows that it will suffice to demonstrate that d''s payoff to imposing a settlement is strictly greater than d''s payoff if it does not impose a settlement, it demands the upper bound of what S might accept, and S immediately accepts this maximal demand. That is, Lemma 2 holds if $p-d' > (1-\delta)U_D(q)+\delta U_D(\max\{r_D(\delta), z(\delta)\})$, where $z(\delta)$ solves $U_S(1-z(\delta)) = (1-\delta)U_S(b-q)+\delta U_S(1-\tilde{x}(d'))$. Because d' is dissatisfied, $\tilde{x}(d') > r_S(\delta)$ which implies $\max\{r_D(\delta), z(\delta)\} = z(\delta)$. Further, $\tilde{x}(d')$ is d''s certainty equivalent to imposing a settlement, so $U_D(\tilde{x}(d')) = p-d'$. Thus, it suffices to show

$U_D(\tilde{x}(d')) > (1-\delta)U_D(q)+\delta U_D(1-U_S^{-1}((1-\delta)U_S(b-q)+\delta U_S(1-\tilde{x}(d'))))$. In terms of the preliminaries above, this inequality is equivalent to $U_D(\tilde{x}(d')) > A(\tilde{x}(d'), \delta)$ which holds as long as $\tilde{x}(d') > r_S(\delta)$. But $\tilde{x}(d') > r_S(\delta)$ because d' is dissatisfied. ■

Proof of Proposition 1. Lemmas 1 through 4 characterize D's reaction for all types d to an offer at any stage of the game and, in particular, to S's initial offer in $\Gamma_S(\delta)$. To see that s's best reply is $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) = \max\{r_S(\delta), t^*(s, \beta_D[\underline{d}, \bar{d}])\}$, suppose that some s did offer some $x < r_S(\delta)$. Lemma 3 implies that all dissatisfied types impose a settlement. Lemma 4 shows that all satisfied types reject x and counter with $r_D(\delta)$ which S accepts with probability one. Accordingly, s's expected payoff is $(1-p-s)F_D(p-U_D(r_s(\delta)))+((1-\delta)U_S(b-q)+\delta U_S(1-r_D(\delta))))(1-F_D(p-U_D(r_s(\delta))))$. But, s's expected payoff to offering $r_S(\delta)$ is $(1-p-s)F_D(p-U_D(r_S(\delta)))+U_S(1-r_S(\delta))(1-F_D(p-U_D(r_S(\delta))))$. The latter payoff is strictly greater than the payoff to offering x if $F_D(p-U_D(r_S(\delta)) < 1$ because s strictly prefers to settle on $r_S(\delta)$ now to settling on $r_D(\delta)$ in the next period. If $F_D(p-U_D(r_S(\delta))) = 1$, then the certainty equivalent of the weakest type is at least $r_S(\delta)$, i.e., $\tilde{x}(\bar{d}) \geq r_S(\delta)$, and an offer of $r_S(\delta)$ or less will be rejected with probability one. But it is easy to show (see Lemma 1A below) that s would strictly prefer to offer slightly more than the certainty equivalent of the weakest type in order to create at least some chance that its offer will be accepted. Again, an offer of $x < r_S(\delta)$ is strictly dominated.

s's best reply is then to maximize $T(x, s, \beta_D[\underline{d}, \bar{d}])$ subject to $x \in [r_S(\delta), 1]$. If $t^*(s, \beta_D[\underline{d}, \bar{d}]) \geq r_S(\delta)$, then the requirement that $x \geq r_S(\delta)$ is irrelevant and $x^* = t^*$. If $t^* < r_S(\delta)$, then the concavity of T over $[\tilde{x}(\bar{d}), \tilde{x}(\underline{d})]$ ensures that T is decreasing for $x \geq r_S(\delta)$, so T takes on its maximum at $r_S(\delta)$ if $x \in [r_S(\delta), 1]$. Accordingly, $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) = \max\{r_S(\delta), t^*(s, \beta_D[\underline{d}, \bar{d}])\}$.

Constructing PBEs that satisfy the requirements of Lemma 1 through 4 and Proposition 1 is straightforward. ■

Proof of Proposition 2. The proofs of cases (i) and (iii) are given here. The proof of case (ii) is tedious and much of it parallels the proof of case (iii). Accordingly, the intuition underlying case (ii) will be sketched, but the proof will be omitted.

Before considering cases (i) and (iii), it will be useful to introduce some notation and establish a lemma. Consider any perfect Bayesian equilibrium $(\sigma(\delta), \mu(\delta))$ of $\Gamma_D(\delta)$ and let $x(\underline{d}, \delta)$ be $\underline{d}$'s equilibrium demand at the start of the game and take $d^0(\delta)$ to be the weakest type demanding $x(\underline{d}, \delta)$. More formally, $d^0(\delta) = \sup\{d\colon x(d, \delta) = x(\underline{d}, \delta)\}$. Then all types $d \in [\underline{d}, d^0(\delta))$ propose $x(\underline{d}, \delta)$. (Type $d^0(\delta)$ may or may not demand $x(\underline{d}, \delta)$.) Similarly, $x(\bar{d}, \delta)$ is $\bar{d}$'s equilibrium demand at the beginning of $\Gamma_D(\delta)$ in $(\sigma(\delta), \mu(\delta))$, and $d'(\delta)$ equals $\inf\{d\colon x(d, \delta) = x(\bar{d}, \delta)\}$. Accordingly, all types in $(d'(\delta), \bar{d}]$ demand $x(\bar{d}, \delta)$. Table 1A summarizes these definitions, as well as some additional helpful notation which will be introduced as needed. ■

TABLE IA
Notation

$x(d,\delta)$	=	d's initial demand in the equilibrium $(\sigma(\delta), \mu(\delta))$
$d'(\delta)$	=	$\inf\{d\colon x(d,\delta) = x(\bar{d},\delta)\}$
$d^0(\delta)$	=	$\sup\{d\colon x(d,\delta) = x(\underline{d},\delta)\}$
$\underline{d}^0$	=	a lower bound on $d^0(\delta)$ for δ in a neighborhood of one
$\tilde{x}(d)$	=	d's certainty equivalent for fighting, i.e., $p - d = U_D(\tilde{x}(d))$
$\beta_D[a,b]$	=	s's updated beliefs if all d in $[a,b]$ make the same initial offer
$T(x,s,\beta_D[a,b])$	=	s's expected payoff to making the take-it-or-leave-offer x given beliefs $\beta_D[a,b]$
$t^*(s,\beta_D[a,b])$	=	the offer that maximizes $T(x,s,\beta_D[a,b])$
$x^*(s,\delta,\beta_D[a,b])$	=	$\max\{r_S(\delta), t^*(s,\beta_D[a,b])\}$

The lemma shows that if s is uncertain of the type it is facing, then its optimal take-it-or-leave-it offer t^* is always strictly greater than the certainty equivalent of the weakest type it might be facing.

LEMMA 1A. $t^*(s,\beta_D[a,b]) > \tilde{x}(b)$ *for all* s *whenever* $b > a$.

Proof. Suppose s offers $\tilde{x}(b)+\epsilon$. Letting $\rho > 0$ denote the probability that this offer is rejected, then s's payoff will be $(1-p-s)\rho + (1-\rho)U_S(1-(\tilde{x}(b)+\epsilon))$. The payoff to offering $\tilde{x}(b)$ is $1-p-s$. Because $f_D > 0$ over $(\underline{d},\bar{d})$, $\rho < 1$. Accordingly, s will strictly prefer to offer $\tilde{x}(b)+\epsilon$ whenever $U_S(1-(\tilde{x}(b)+\epsilon)) > 1-p-s$. But, $U_S(1-\tilde{x}(b)) > U_S(1-\tilde{x}(a)) \geq U_S(1-\tilde{x}(\underline{d}))$ because $b > a \geq \underline{d}$. And, $U_S(1-\tilde{x}(\underline{d})) \geq 1-p-s$ because S is satisfied. Consequently, $U_S(1-\tilde{x}(b)) > 1-p-s$. So there exists an $\epsilon > 0$ such that $U_S(1-(\tilde{x}(b)+\epsilon)) > 1-p-s$. ∎

Case (i). The satisfied bargainer is so confident that the dissatisfied bargainer is tough that all s offer the certainty equivalent of the toughest type conditional on their prior beliefs, i.e., $t^*(\underline{s},\beta_D[\underline{d},\bar{d}]) = \tilde{x}(\underline{d}) > r$.

Assume the claim made in the proposition does not hold. Then it is possible to construct a sequence of PBEs $\{\mu(\delta_n),\sigma(\delta_n)\}_{n=0}^{\infty}$ of $\Gamma_D(\delta_n)$ such that δ_n converges to one and the $d'(\delta_n)$ are bounded away from $\underline{d}$. In the limit as δ_n goes to one, $\bar{d}$ will prefer to deviate from $x(\bar{d},\delta_n)$ to $x(\underline{d},\delta_n)$, and this contradiction will establish the claim.

Three observations ensure that any d that mimics $\underline{d}$ by demanding $x(\underline{d},\delta_n)$ will receive a payoff of at least $(1-\delta_n)U_D(q) + \delta_n U_D(\tilde{x}(\underline{d}))$, which is the payoff $\underline{d}$ would receive from making a nonserious demand and then being offered its certainty equivalent. First, s's optimal counter to $x(\underline{d},\delta_n)$ is the certainty equivalent of the toughest type that might make this demand; i.e., $x^*(s,\delta,\beta_D[\underline{d},d^0(\delta_n)]) = \tilde{x}(\underline{d})$ for all s. This follows from the envelope theorem which ensures $t^*(s,\beta_D[\underline{d},e])$ is nonincreasing in e and nondecreasing in s. So, $x^*(s,\delta,\beta_D[\underline{d},e])$ is nonincreasing in e and nondecreasing in s which implies $\tilde{x}(\underline{d}) \geq x^*(s,\delta,\beta_D[\underline{d},d^0(\delta_n)]) \geq x^*(\underline{s},\delta,\beta_D[\underline{d},\bar{d}]) = \tilde{x}(\underline{d})$ for all s, where the first inequality simply reflects the fact that no s would offer more than the certainty equivalent of the toughest type.

Second, any d that demands $x(\underline{d}, \delta_n)$ will obtain what $\underline{d}$ obtains from making this demand. If this demand is accepted, both d and $\underline{d}$ receive $U_D(x(\underline{d}, \delta_n))$. If this demand is rejected, then all s counter with $\tilde{x}(\underline{d})$ as was just shown. But the payoffs to both d and $\underline{d}$ of receiving this counter are $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$. Thus any d can obtain the same payoff as $\underline{d}$ does by demanding just what $\underline{d}$ demands.

The third observation is that $\underline{d}$'s payoff to demanding $x(\underline{d}, \delta_n)$ is bounded below by $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$ and, therefore, any d can obtain at least $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$ by demanding $x(\underline{d}, \delta_n)$. To see this, assume $x(\underline{d}, \delta_n)$ is rejected with probability one. The first observation shows that all s will counter with $x^*(s, \delta, \beta_D[\underline{d}, d^0(\delta)]) = \tilde{x}(\underline{d})$, leaving $\underline{d}$ with $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$. Now suppose $x(\underline{d}, \delta_n)$ is accepted with positive probability. Individual rationality requires that $\underline{d}$'s payoff to demanding $x(\underline{d}, \delta_n)$ be at least as large as $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$, which is what $\underline{d}$ could obtain by making a nonserious offer and then imposing a resolution. To establish this requirement, note that $\underline{d}$'s payoff conditional on $x(\underline{d}, \delta_n)$'s being rejected is $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$; $\underline{d}$'s payoff conditional on $x(\underline{d}, \delta_n)$'s acceptance is $U_D(x(\underline{d}, \delta_n))$. If $\rho(x(\underline{d}, \delta_n))$ denotes the probability that $x(\underline{d}, \delta_n)$ is rejected, individual rationality then means $U_D(x(\underline{d}, \delta_n))[1-\rho(x(\underline{d}, \delta_n))] + [(1-\delta_n)U_D(q)+\delta_n(U_D(\tilde{x}(\underline{d}))]\rho(x(\underline{d}, \delta_n)) \geq (1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$ which leaves $U_D(x(\underline{d}, \delta_n)) \geq (1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$. Thus, the three observations imply that any d that demands $x(\underline{d}, \delta_n)$ will receive a payoff of at least $(1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$.

Turning to $\underline{d}$'s payoff from making its purported equilibrium demand of $x(\underline{d}, \delta_n)$, Lemma 1 implies those demanding $x(\bar{d}, \delta_n)$ can do no better than $U_D(\max\{r_D(\delta_n), z(\delta_n)\})$, where $z(\delta_n)$ solves $U_S(1-z(\delta_n)) = (1-\delta_n)U_S(1-q)+\delta_n U_S(\tilde{x}(d'(\delta_n)))$. Incentive compatibility then requires that $\bar{d}$ cannot benefit by demanding $x(\underline{d}, \delta_n)$ instead of $x(\bar{d}, \delta_n)$. Accordingly, $U_D(\max\{r_D(\delta_n), z(\delta_n)\}) \geq (1-\delta_n)U_D(q)+\delta_n U_D(\tilde{x}(\underline{d}))$. Since $\{d'(\delta_n)\}_{n=0}^{\infty}$ is bounded away from $\underline{d}$, it is a subset of $[\underline{d}+\epsilon, \bar{d}]$ for some $\epsilon > 0$. Taking the limit of the previous inequality along a convergent subsequence yields the contradiction $U_D(\max\{r, \tilde{x}(d^0)\}) \geq U_D(\tilde{x}(\underline{d}))$ for some $d^0 > \underline{d}$. ∎

Case (*iii*). $t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}]) < \tilde{x}(\underline{d})$ and $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) > r$. The proof takes four steps. Assuming that there are two distinct demands in a PBE for δ arbitrarily close to one, the first step shows that the probability that any $d > d^0(\delta)$ imposes a settlement is zero. The second step establishes that $d^0(\delta)$ is bounded away from $\underline{d}$. With $d^0(\delta)$ bounded away from $\underline{d}$, the third step uses Lemma 1A to demonstrate that $d^0(\delta)$'s payoff to demanding $x(\underline{d}, \delta)$ is strictly greater than and bounded away from the payoff to $d^0(\delta)$'s certainty equivalent for δ close to one. The final step then shows that $\bar{d}$ prefers to deviate to $x(\underline{d}, \delta)$ instead of demanding $x(\bar{d}, \delta)$.

Step 1. *If $d > d^0(\delta)$, then the probability that d imposes a settlement is zero.* Suppose the contrary. Then there exists an $e > d^0$ such that e imposes a settlement with positive probability. (The δ in, for example, $d^0(\delta)$ will be suppressed in order to simplify the notation whenever this can be done unambiguously.) Because $e > d^0$, d^0's cost for trying to impose a resolution is strictly less than e's. Thus, the payoff to d^0 of mimicking e's initial demand of $x(e)$ is strictly greater than e's payoff because e imposes a settlement with positive probability. Letting $\mathcal{U}_D(d, x)$ denote d's expected payoff to demanding x and thereafter playing according to Lemmas 1–3 then gives $\mathcal{U}_D(d^0, x(e)) > \mathcal{U}_D(e, x(e)) + \epsilon$, where ϵ is some small positive number.

Because $e > d^0$, e's equilibrium demand of $x(e)$ is distinct from $\underline{d}$'s demand of $x(\underline{d})$. Consequently, e can have no positive incentive to deviate from $x(e)$ by demanding $x(\underline{d})$ instead. That is, $\mathcal{U}_D(e, x(e)) \geq \mathcal{U}_D(e, x(\underline{d}))$. Lemma 1A, however, implies that any s rejecting $x(\underline{d})$ will offer strictly more than $\tilde{x}(d^0)$. Thus, the probability that d^0 will subsequently impose a settlement if it demands $x(\underline{d})$ is zero. Hence, e can demand $x(\underline{d})$ and obtain the same payoff that d^0 obtains from demanding $x(\underline{d})$. So, $\mathcal{U}_D(e, x(\underline{d})) = \mathcal{U}_D(d^0, x(\underline{d}))$. Finally, $\mathcal{U}_D(d^0, x(\underline{d})) \geq \mathcal{U}_D(d^0, x(e))$. Otherwise, continuity would ensure that some d less than d^0 but arbitrarily close to d^0 would have an incentive to deviate from their equilibrium demand of $x(\underline{d})$ to $x(e)$. These relations yield the contradiction $\mathcal{U}_D(e, x(e)) \geq \mathcal{U}_D(e, x(\underline{d})) = \mathcal{U}_D(d^0, x(\underline{d})) \geq \mathcal{U}_D(d^0, x(e)) > \mathcal{U}_D(e, x(e)) + \epsilon$.

Step 2. *d^0 is bounded away from $\underline{d}$; i.e., there exists a $\underline{d}^0 > \underline{d}$ and a $\underline{\delta} < 1$ such that $d^0(\delta) > \underline{d}^0$ whenever $\delta > \underline{\delta}$.* Suppose the contrary. Then there is a sequence of PBEs of $\Gamma_D(\delta_n)$ such that δ_n converges to one, $d^0(\delta_n)$ converges to $\underline{d}$, and, looking along a subsequence if necessary, $d'(\delta_n)$ also converges. This assumption leads to a contradiction.

$d'(\delta_n)$ must converge to $\underline{d}$. This follows from Lemma 1 which implies that $\bar{d}$'s payoff to demanding $x(\bar{d}, \delta_n)$ is bounded above by $U_D(\max\{r_D(\delta_n), z(\delta_n)\})$, where $z(\delta_n)$ solves $U_S(1 - z(\delta_n)) = 1 - \delta_n)U_S(b - q) + \delta_n U_S(1 - \tilde{x}(d'(\delta_n)))$. $d'(\delta_n)$'s payoff to demanding $x(\underline{d}, \delta_n)$ is bounded below by what it obtains if s rejects this demand and counters with the certainty equivalent of the weakest type s might be facing. This payoff is $(1 - \delta_n)U_D(q) + \delta_n U_D(\tilde{x}(d^0(\delta_n)))$. Incentive compatibility then requires that $\bar{d}$ cannot benefit by demanding $x(\underline{d}, \delta_n)$: $U_D(\max\{r_D(\delta_n), z(\delta_n)\}) \geq (1 - \delta_n)U_D(q) + \delta_n U_D(\tilde{x}(d^0(\delta_n)))$. Taking the limit gives $U_D(\max\{r, \tilde{x}(d')\}) \geq U_D(\tilde{x}(\underline{d}))$. But $\tilde{x}(\bar{d}) > r$ by assumption. So, d' must equal $\underline{d}$.

The convergence of $d'(\delta_n)$ to $\underline{d}$ implies that if δ is sufficiently close to one, then $x(\bar{d}, \delta_n)$ must be accepted with positive probability. Assume the opposite. Then there would exist a subsequence $\{\delta_m\}$ converging to one such that $x(\bar{d}, \delta_m)$ is rejected with probability one. Step 1, however, shows that no $d > d'(\delta_m) \geq d^0(\delta_m)$ can impose a settlement with positive probability. Thus, all s must counter $x(\bar{d}, \delta_m)$ with at least $\tilde{x}(d'(\delta_m))$; otherwise the fact that S's offers are continuous in type would imply that some d slightly larger than $d'(\delta_m)$ would reject the counter and impose a settlement with positive probability. In particular, the toughest type of S, $\underline{s}$, must counter $x(\bar{d}, \delta_m)$ with $x^*(\underline{s}, \delta_m, \beta_D[d'(\delta_m), \bar{d}]) \geq \tilde{x}(d'(\delta_m))$. Taking the limit along this subsequence then gives $x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}]) \geq \tilde{x}(\underline{d})$. This, however, is a contradiction, because the condition defining case (iii) implies

$$x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}]) = \max\{r, t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}])\} < \tilde{x}(\underline{d}).$$

If δ_n is close enough to one, then the payoff to $\underline{s}$ of accepting $x(\bar{d}, \delta_n)$ must be at least as large as its payoff to countering. Suppose that this is not so. Then there would exist a subsequence $\{\delta_m\}$ converging to one such that $\underline{s}$ strictly prefers to counter $x(\bar{d}, \delta_m)$. This strict preference implies that a neighborhood around $\underline{s}$ which has positive measure would reject $x(\bar{d}, \delta_m)$. If, therefore, $\underline{s}$ counters with less than $\tilde{x}(d'(\delta_m))$, then the fact that S's offers are continuous in type means that some $d > d'(\delta_m)$ will reject this counter and impose a settlement with positive probability in equilibrium. This contradicts step 1. Consequently, $\underline{s}$ must counter with at least $\tilde{x}(d'(\delta_m))$, i.e. $x^*(\underline{s}, \delta_m, \beta_D[d'(\delta_m), \bar{d}]) \geq \tilde{x}(d'(\delta_m))$. The argument in the previous paragraph now yields a contradiction.

The sequence $x(\bar{d}, \delta_n)$ contains a subsequence which converges to an x^0. The fact that $\underline{s}$'s payoff to accepting $x(\bar{d}, \delta_n)$ is at least as large as its payoff to countering implies $\tilde{x}(\underline{d}) > x^0$. In symbols, $U_S(1 - x(\bar{d}, \delta_n)) \geq (1 - \delta_n)U_S(b - q) + \delta_n T(x^*(\underline{s}, \delta_n, \beta_D[d'(\delta_n), \bar{d}]), \underline{s}, \beta_D[d'(\delta_n), \bar{d}])$ for δ_n close enough to one implies $\tilde{x}(\underline{d}) > x^0$. To see this, take the limit as δ_n goes to one. This gives $U_S(1 - x^0) \geq T(x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}]), \underline{s}, \beta_D[\underline{d}, \bar{d}])$. But $x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}])) = \max\{r, t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}])\} < \tilde{x}(\underline{d})$, where the inequality follows from the condition defining case (iii). Moreover, $x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}])$ is the unique x that maximizes $T(x, \underline{s}, \beta_D[\underline{d}, \bar{d}])$ for all $x \in [r, 1]$. So, $T(x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}]), \underline{s}, \beta_D[\underline{d}, \bar{d}]) > T(\tilde{x}(\underline{d}), \underline{s}, \beta_D[\underline{d}, \bar{d}])$, where the inequality is strict because $x^*(\underline{s}, 1, \beta_D[\underline{d}, \bar{d}]) \neq \tilde{x}(\underline{d})$. Finally, $T(\tilde{x}(\underline{d}), \underline{s}, \beta_D[\underline{d}, \bar{d}]) = U_S(1 - \tilde{x}(\underline{d}))$ because an offer of $\tilde{x}(\underline{d})$ is accepted. Putting these inequalities together gives $U_S(1 - x^0) > U_S(1 - \tilde{x}(\underline{d}))$ or $\tilde{x}(\underline{d}) > x^0$.

The previous inequality and the fact that $x(\bar{d}, \delta_n)$ is accepted with positive probability leads to the contradiction that $\bar{d}$ prefers deviating to $x(\underline{d}, \delta_n)$ for δ_n sufficiently close to one. The positive probability of acceptance implies that $d'(\delta_n)$'s payoff if this demand is accepted must be at least as large as its payoff to making a nonserious demand and then imposing a settlement: $U_D(x(\bar{d}, \delta_n)) \geq (1 - \delta_n)U_D(q) + \delta_n U_D(\tilde{x}(d'(\delta_n)))$. Taking the limit yields $U_D(x^0) \geq U_D(\tilde{x}(\underline{d}))$ or $x^0 \geq \tilde{x}(\underline{d})$ which contradicts the inequality $\tilde{x}(\underline{d}) > x^0$ and, accordingly, implies that $d^0(\delta)$ is bounded away from $\underline{d}$.

Step 3. $d^0(\delta)$'s payoff to demanding $x(\underline{d}, \delta)$ is strictly greater than and bounded away from the payoff to obtaining the larger of its certainty equivalent or its

Rubinstein share, i.e., there exists a $\underline{\delta} < 1$ *and an* $\epsilon > 0$ *such that* $d^0(\delta)$*'s payoff to demanding* $x(\underline{d}, \delta)$ *is at least* $U_D(\max\{r_D(\delta), \tilde{x}(d^0(\delta))\}) + \epsilon$ *whenever* $\delta > \underline{\delta}$.

The payoff to $d^0(\delta_n)$ of demanding $x(\underline{d}, \delta_n)$ is bounded below by $(1-\delta_n)U_D(q) + \delta_n \int_{\underline{s}}^{\bar{s}} U_D(x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)]))dF_S$. This is clearly so if this demand is rejected with probability one, for s will counter with $x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)])$ which $d^0(\delta_n)$ accepts because it is greater than $d^0(\delta_n)$'s certainty equivalent. If $x(\underline{d}, \delta_n)$ is accepted with positive probability, then, as shown in case (i), $\underline{d}$'s payoff if this demand is accepted must be at least as large as its payoff to making a nonserious demand and then imposing a settlement. That is, $U_D(x(\underline{d}, \delta_n)) \geq (1 - \delta_n)U_D(q) + \delta_n U_D(\tilde{x}(\underline{d}))$. But no s ever offers more than $\tilde{x}(\underline{d})$, so $U_D(\tilde{x}(\underline{d})) \geq U_D(x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)])$. So, $d^0(\delta_n)$'s payoff to demanding $x(\underline{d}, \delta_n)$ conditional on facing an s that accepts is bounded below by $(1 - \delta_n)U_D(q) + \delta_n U_D(x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)])$. $d^0(\delta_n)$'s payoff is exactly $(1 - \delta_n)U_D(q) + \delta_n U_D(x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)])$ conditional on facing an s that rejects. Thus, $d^0(\delta_n)$'s payoff is bounded below by $(1 - \delta_n)U_D(q) + \delta_n \int_{\underline{s}}^{\bar{s}} U_D(x^*(s, \delta_n, \beta_D[\underline{d}, d^0(\delta_n)]))dF_S$ if $x(\underline{d}, \delta_n)$ is accepted with positive probability.

To establish the claim made in this step, it will suffice to show that the previous expression is strictly greater than and bounded away from $U_D(\max\{r_D(\delta), \tilde{x}(d^0(\delta))\})$ for δ close enough to one. Suppose the contrary. Then there must exist a sequence of PBEs of $\Gamma_D(\delta_n)$ such that δ_n converges to one, $d^0(\delta_n)$ converges to some $d^0 > \underline{d}$, and $\lim_{n\to\infty}[(1 - \delta_n)U_D(q) + \delta_n \int_{\underline{s}}^{\bar{s}} U_D(\max\{r_S(\delta_n), t^*(s, \beta_D[\underline{d}, d^0(\delta_n)])\})dF_S] \leq U_D(\max\{r, \tilde{x}(d^0)\})$. The functions $\{U_D(\max\{r_S(\delta_n), t^*(s, \beta_D[\underline{d}, d^0(\delta_n)])\})\}_{n=0}^{\infty}$ are bounded, measurable, and converge to $U_D(\max\{r, t^*(s, \beta_D[\underline{d}, d^0])\})$. So, it must be that

$$\int_{\underline{s}}^{\bar{s}} U_D(\max\{r, t^*(s, \beta_D[\underline{d}, d^0])\})dF_S] \leq U_D(\max\{r, \bar{x}(d^0)\}).$$

To see that this in fact cannot be the case, choose a $\underline{d} < 1$ and a $\underline{d}^0 > 0$ such that $d^0(\delta) > \underline{d}^0$ in any PBE of $\Gamma_D(\delta)$ for $\delta \geq \underline{\delta}$. Because t^* is continuous and $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) > r$, there exists a $\xi > 0$ such that $t^*(s, \beta_D[\underline{d}, d^0]) \geq t^*(s, \beta_D[\underline{d}, \bar{d}]) \geq t^*(\bar{s} - \xi, \beta_D[\underline{d}, \bar{d}]) > r$ for all $s \geq \bar{s} - \xi$, where the first and second inequalities reflect the facts that t^* is nonincreasing in d^0 and nondecreasing in s. Lemma 1A implies further $t^*(s, \beta_D[\underline{d}, d^0]) > \tilde{x}(d^0)$. Thus, $\max\{r, t^*(s, \beta_D[\underline{d}, d^0])\} \geq \max\{r, \tilde{x}(d^0)\}$ with the inequality being strict for $s \in [\bar{s} - \xi, \bar{s}]$ as long as $\delta > \underline{\delta}$. So

$$\int_{\underline{s}}^{\bar{s}} U_D(\max\{r, t^*(s, \beta_D[\underline{d}, d^0])\})dF_S > U_D(\max\{r, \tilde{x}(d^0)\}).$$

This contradiction establishes the claim.

Step 4. *If there are two distinct demands, i.e., if $x(\underline{d}, \delta) \neq x(\bar{d}, \delta)$, then $\bar{d}$ strictly prefers to deviate to $x(\underline{d}, \delta)$ for δ close enough to one.* Suppose the contrary. Then there exists a sequence of PBEs of $\Gamma_D(\delta_n)$ such that δ_n converges to one and $x(\underline{d}, \delta_n) \neq x(\bar{d}, \delta_n)$.

Step 2 implies $d^0(\delta_n)$ converges to some $d^0 > \underline{d}$. Step 3 ensures that there exists a $\underline{\delta} < 1$ and an $\epsilon > 0$ such that $d^0(\delta_n)$'s payoff to demanding $x(\underline{d}, \delta_n)$ is at least $U_D(\max\{r_D(\delta_n), \tilde{x}(d^0(\delta_n))\}) + \epsilon$. But $d^0(\delta_n)$ never imposes a settlement if it demands $x(\underline{d}, \delta_n)$, so $\bar{d}$ can costlessly mimic $d^0(\delta_n)$. Thus $\bar{d}$'s payoff to demanding $x(\underline{d}, \delta_n)$ is at least $U_D(\max\{r_D(\delta_n), \tilde{x}(d^0(\delta_n))\}) + \epsilon$.

Lemma 1 implies that the best that $\bar{d}$ can do by demanding $x(\bar{d}, \delta_n)$ is bounded above by $U_D(\max\{r_D(\delta_n), z(\delta_n)\})$, where $z(\delta_n)$ solves $U_S(1 - z(\delta_n)) = (1 - \delta_n)U_D(q) + \delta_n U_D(\tilde{x}(d^0(\delta_n)))$. Incentive compatibility then necessitates $U_D(\max\{r_D(\delta_n), z(\delta_n)\}) \geq U_D(\max\{r_D(\delta_n), \tilde{x}(d^0(\delta_n))\}) + \epsilon$. As δ_n goes to one, the left side becomes arbitrarily close to $U_D(\max\{r, \tilde{x}(d^0)\})$ and the right side approaches $U_D(\max\{r, \tilde{x}(d^0)\}) + \epsilon$. The previous inequality must therefore breakdown for δ_n close enough to one. This contradiction means $x(\underline{d}, \delta) = x(\bar{d}, \delta)$ and establishes case (iii). ∎

Case (*ii*). $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}] < r$. To sketch the intuition underlying the result, note that any d can obtain at least $U_D(r_D(\delta))$ by demanding its Rubinstein share $r_D(\delta)$ which all s are sure to accept as Lemma 1 shows. Thus, d's equilibrium payoff must be at least $U_D(r_D(\delta))$. But this implies that all d cannot pool on a common demand. If they tried to pool on a nonserious demand, all s counter with $r_S(\delta)$. Satisfied types accept this counter and are left with $(1 - \delta)U_D(q) + \delta U_D(r_S(\delta))$. But these satisfied types could have done better by initially demanding $r_D(\delta)$ instead. This contradiction implies that if all of the d pool on a common demand, this demand must be accepted with a positive probability. This, however, also leads to a contradiction. If the initial demand is serious, then its acceptance must leave $\underline{d}$ with at least as much as it could have attained by making a nonserious demand and then imposing a settlement. If, therefore, δ is close enough to one and the initial demand is serious, this demand must be approximately equal to $\tilde{x}(\underline{d})$, the certainty equivalent of the toughest type. S, however, is confident that D is weak in case (ii) and all the s prefer to reject this large demand in order to counter with $r_S(\delta)$. The fact that this demand is sure to be rejected contradicts the assumption that it was serious.

In equilibrium the complete pooling breaks down into partial pooling. The high-cost types demand $r_D(\delta)$, and the other d pool on a common, nonserious demand $x(\underline{d}, \delta)$. This upper tail is just large enough to ensure that the toughest type to demand $r_D(\delta)$ is indifferent between demanding $r_D(\delta)$ and $x(\underline{d}, \delta)$. A unique d satisfies this requirement and pins down the equilibrium.

Proof of Proposition 3. Let $\pi_S(\delta)$ and $\pi_D(\delta)$ denote the probabilities of breakdown in any PBE of $\Gamma_S(\delta)$ and $\Gamma_D(\delta)$, respectively. Recalling that the probability that D will reject an offer of x is $F_D(p - U_D(x))$. Then, $\pi_S(\delta) = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])))dF_S$.

In case (i) of Proposition 2, all s offer the certainty equivalent of the toughest type in $\Gamma_S(\delta)$ and the probability of breakdown is zero. That is, $t^*(s, \beta_D[\underline{d}, \bar{d}]) \geq t^*(\underline{s}, \beta_D[\underline{d}, \bar{d}]) = \tilde{x}(\underline{d}) > r$. So, $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) = \tilde{x}(\underline{d})$ and $\pi_S(\delta) = 0$ for δ close enough to one.

In $\Gamma_D(\delta)$, δ can be chosen close enough to one to ensure that for any arbitrarily small η, $t^*(\underline{s}, \beta_D[\underline{d} + \eta, \bar{d}] > r_S(\delta)$ and all $d \in (\underline{d} + \eta, \bar{d}]$ pool on a common demand. Then $x^*(s, \delta, \beta_D[\underline{d} + \eta, \bar{d}]) = t^*(s, \beta_D[\underline{d} + \eta, \bar{d}])$. Accordingly, the probability of breakdown is $\Gamma_D(\delta)$ is bounded above by $\bar{\pi}_D(\eta) = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(t^*(s, \beta_D[\underline{d} + \eta, \bar{d}])))dF_S + F_D(\underline{d} + \eta)$, where the last term is the probability that all d in $[\underline{d}, \underline{d} + \eta]$ impose a settlement and the first term is the probability of breakdown given that the common demand of the $[\underline{d}, +\eta, \bar{d}]$ is rejected for sure.

To establish the claim, it will suffice to show that the upper bound $\bar{\pi}_D(\eta)$ goes to zero as η goes to zero. Taking this limit gives $\lim_{\eta \to 0} \bar{\pi}_D(\eta) = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(t^*(s, \beta_D[\underline{d}, \bar{d}])))dF_S$. But, $t^*(s, \beta_D[\underline{d}, \bar{d}])$ is $\underline{d}$'s certainty equivalent, so $F_D(p - U_D(t^*(s, \beta_D[\underline{d}, \bar{d}]))) = 0$ for all s.

In case (ii), all s offer $r_S(\delta)$ in $\Gamma_S(\delta)$ if δ is sufficiently close to one, so $\pi_S(\delta) = F_D(p - U_D(r_S(\delta)))$. To establish this, observe that $t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}]) < r$ implies that there exists a $\underline{\delta} < 1$ such that $r_S(\delta) \leq x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) \leq x^*(\bar{s}, \delta, \beta_D[\underline{d}, \bar{d}]) = \max\{r_S(\delta), t^*(\bar{s}, \beta_D[\underline{d}, \bar{d}])\} = r_S(\delta)$ whenever $\delta > \underline{\delta}$. Thus, $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}]) = r_S(\delta)$ for all s, and the probability of breakdown is $F_D(p - U_D(r_S(\delta)))$.

In $\Gamma_D(\delta)$, $[\underline{d}, \hat{d}(\delta))$ pool on a nonserious demand, and $(\hat{d}(\delta), \bar{d}]$ demand $r_D(\delta)$ which is accepted with probability one. So,

$$\pi_D(\delta) = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \hat{d}(\delta)])))dF_S.$$

Taking the limit as δ goes to one gives

$$\pi_D(1) = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \hat{d}(1)])))dF_S.$$

But, $\hat{d}(\delta)$ solves

$$U_D(r_D(\delta)) = (1 - \delta)U_D(q) + \delta \int_{\underline{s}}^{\bar{s}} U_D(x^*(s, \delta, \beta_D[\underline{d}, \hat{d}(\delta)])dF_S(s).$$

Taking the limit leaves $U_D(r) = \int_{\underline{s}}^{\bar{s}} U_D(x^*(s, 1, \beta_D[\underline{d}, \hat{d}(1)])dF_S(s)$. Given that $x^*(s, 1, \beta_D[\underline{d}, \hat{d}(1)]) \geq r$, x^* is continuous, and that $f_S > 0$ for $\underline{s} < s < \bar{s}$, the previous equality can hold only if $x^*(s, 1, \beta_D[\underline{d}, \hat{d}(1)]) = r$ for all s. Thus, $\pi_D(1) = F_D(p - U_D(r)) = \pi_S(1)$, implying that the difference between $\pi_S(\delta)$ and $\pi_D(\delta)$ can be made arbitrarily small.

Turning to case (iii), if the common demand is not serious, then all s reject $x(\underline{d}, \delta)$ and counter with $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])$. $\pi_S(\delta)$ and $\pi_D(\delta)$ are identical.

Suppose $x(\underline{d}, \delta)$ is serious. Let $\tilde{s}$ be the type whose optimal take-it-or-leave-it offer t^* is $\underline{d}$'s certainty equivalent: $t^*(\tilde{s}, \beta_D[\underline{d}, \bar{d}]) = \tilde{x}(\underline{d})$. (If no such type exists, define $\tilde{s}$ to be $\bar{s}$.) Because t^* is nondecreasing in s, all $s > \tilde{s}$ offer $\tilde{x}(\underline{d})$ in $\Gamma_S(\delta)$ and reach agreement with probability one. This leaves $\pi_S(\delta) = \int_{\underline{s}}^{\tilde{s}} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])))dF_S$.

Now take $\hat{s}(\delta)$ to be the infimum of the set of s that accept $x(\underline{d}, \delta)$ in $\Gamma_D(\delta)$. Then all $s > \hat{s}(\delta)$ accept, leaving

$$\pi_D(\delta) = \int_{s}^{\hat{s}(\delta)} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])))dF_S.$$

To show that $|\pi_S(\delta) - \pi_D(\delta)|$ can be made arbitrarily small by taking δ close enough to one, it suffices to show that $\hat{s}(\delta)$ can be made arbitrarily close to $\tilde{s}$ for δ sufficiently close to one.

Suppose the contrary. Then there is a sequence of PBEs of $\Gamma_D(\delta_n)$ with δ_n converging to one such that $\hat{s}(\delta_n)$ converges to a $\hat{s} < \tilde{s}$ and $x(\underline{d}, \delta_n)$ converges to some x^0. ($\hat{s}$ cannot be greater than $\tilde{s}$ because any $s > \tilde{s}$ will avoid an imposed settlement in both $\Gamma_S(\delta)$ and $\Gamma_D(\delta)$). Since $x(\underline{d}, \delta_n)$ is serious, $\underline{d}$'s payoff if $x(\underline{d}, \delta_n)$ is accepted must be at least as large as its payoff to imposing a settlement in the second period: $U_D(x(\underline{d}, \delta_n)) \geq (1-\delta_n)U_D(q) + \delta_n U_D(\tilde{x}(\underline{d}))$. This leaves $x^0 \geq \tilde{x}(\underline{d})$ and, accordingly, $U_S(1 - \tilde{x}(\underline{d})) \geq U_S(1 - x^0)$. Furthermore, s's acceptance for $s > \hat{s}(\delta)$ implies $U_S(1 - x(\underline{d}, \delta_n)) \geq (1 - \delta_n)U_S(b - q) + \delta_n T(x^*(\hat{s}(\delta_n), \delta_n, \beta_D[\underline{d}, \bar{d}]), \hat{s}(\delta_n), \beta_D[\underline{d}, \bar{d}])$. Letting δ_n go to one leaves $U_S(1 - x^0) \geq T(x^*(\hat{s}, 1, \beta_D[\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}])$.

Because $\hat{s} < \tilde{s}$, these types' optimal offers differ. To see this, observe that the definition of $\tilde{s}$ means that $t^*(\hat{s}, \beta_D[\underline{d}, \bar{d}])$ is less than the certainty equivalent of $\underline{d}$. Consequently, an offer of $t^*(\hat{s}, \beta_D[\underline{d}, \bar{d}])$ entails some risk of breakdown, and the envelope theorem shows that t^* is strictly increasing in s in these circumstances. Thus, $t^*(\hat{s}, \beta_D[\underline{d}, \bar{d}]) < t^*(\tilde{s}, \beta_D[\underline{d}, \bar{d}])$. The conditions defining case (iii) also ensure $t^*(\hat{s}, \beta_D[\underline{d}, \bar{d}]) > r$. Accordingly, $x^*(\hat{s}, 1, \beta_D[\underline{d}, \bar{d}]) = \max\{r, t^*(\hat{s}, \beta_D[\underline{d}, \bar{d}])\} < \max\{r, t^*(\tilde{s}, \beta_D[\underline{d}, \bar{d}])\} = x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}])$.

Since x^* is the unique offer that maximizes T and $\hat{s}$ could offer $\tilde{s}$'s optimal offer, the fact that $\hat{s}$'s and $\tilde{s}$'s optimal offers differ implies that $\hat{s}$ must strictly prefer to offer $x^*(\hat{s}, 1, \beta_D[\underline{d}, \bar{d}])$: $T(x^*(\hat{s}, 1, \beta_D[\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}]) > T(x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}])$. Thus, $U_S(1 - x^0) > T(x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}])$.

The fact that $\hat{s} < \tilde{s}$ also means that $\hat{s}$'s cost of imposing a settlement is strictly less than $\tilde{s}$'s cost and, therefore, $\hat{s}$'s payoff to making any offer is at least as large as

$\tilde{s}$'s payoff to making this offer. Accordingly, $T(x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}]) \geq T(x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}]), \tilde{s}, \beta_D[\underline{d}, \bar{d}])$. Moreover, $\tilde{s}$'s maximum payoff is at least $U_S(1 - \tilde{x}(\underline{d}))$ which $\tilde{s}$ could obtain by offering $\tilde{x}(\underline{d})$. Combining the bounds on $U_S(1 - x^0)$ leaves $U_S(1 - \tilde{x}(\underline{d})) \geq U_S(1 - x^0) > T(x^*(\tilde{s}, 1, \beta_D, [\underline{d}, \bar{d}]), \hat{s}, \beta_D[\underline{d}, \bar{d}]) \geq T(x^*(\tilde{s}, 1, \beta_D[\underline{d}, \bar{d}]), \tilde{s}, \beta_D[\underline{d}, \bar{d}]) \geq U_S(1 - \tilde{x}(\underline{d}))$. This contradiction ensures $|\pi_S(\delta) - \pi_D(\delta)|$ can be made arbitrarily small. ■

Proof that the probability of breakdown is nondecreasing in $|p - r|$ if the bargainers are risk neutral, or if they have identical utility functions and the Nash bargaining solution, r, equals $\frac{1}{2}$. Type d rejects s's initial offer $x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])$ if $p - d > U_D(x^*)$ so the probability that this offer is rejected is $F_D(p - U_D(x^*(s))$. Thus the probability that S's initial offer is rejected and that bargaining breaks down is $\pi = \int_{\underline{s}}^{\bar{s}} F_D(p - U_D(x^*(s)))dF_S$. To see if π is nondecreasing in $|p - r|$, assume $p \geq r$. Then it will suffice to show that the probability that any s's offer is rejected is nondecreasing in p. There are three cases to consider.

First, suppose that s's optimal offer at p is the corner solution $x^*(s) = \tilde{x}(\underline{d})$. This offer is sure to be accepted, because it is the certainty equivalent of the toughest type s might be facing. Consequently, the probability that s's offer is rejected is zero and cannot decrease as p increases.

Assume now that s's optimal, unbounded take-it-or-leave-it offer $t^*(s)$ at p is strictly less than its Rubinstein share $r_S(\delta)$. Continuity ensures that if p increases slightly to some p', s's optimal unbounded offer will still be less than $r_S(\delta)$. Thus, s offers $x^*(s) = \max\{r_S(\delta), t^*(s)\} = r_S(\delta)$ at both p and p'. The probability that this offer is rejected is $F_D(p - U_D(r_S(\delta)))$ at p and $F_D(p' - U_D(r_S(\delta)))$ at p'. The latter probability is at least as large as the former, so the probability that s's offer is rejected is nondecreasing in p.

Finally, assume $r_S(\delta) \leq t^*(s) < \tilde{x}(\underline{d})$. Then s's optimal bounded take-it-or-leave-it offer $x^*(s)$ equals $t^*(s)$. Further, $x^*(s) = t^*(s)$ maximizes $T(s)$ and so satisfies the first-order conditions:

$$\frac{F_D'(p - U_D(x^*))}{1 - F_D(x^*)} = \frac{U_S'(1 - x^*)}{[U_S(1 - x^*) - (1 - p - s)]U_D'(x^*)}.$$

The probability that s's offer $x^*(s)$ is rejected is $F_D(p - U_D(x^*(s)))$ and this probability is nondecreasing in p if $p - U_D(x^*(s))$ is nondecreasing in p. Differentiating implicitly with respect to p, solving for $d(p - U_D(x^*(s)))/dp$, and recalling F_D has a monotone hazard rate show the sign of $d(p - U_D(x^*(s)))/dp$ to be the same as the sign of

$$-\frac{U_S''(1 - x)}{U_S'(1 - x)} - \frac{U_D''(x)}{U_D'(x)} + \frac{U_S'(1 - x) - U_D'(x)}{U_S(1 - x) - (1 - p - s)}\bigg|_{x = x^*}. \quad (2)$$

This expression is easy to sign in two circumstances. First, if S and D are risk neutral, then this expression equals zero. (The normalization of U_S and U_D

implies $U_D' = U_S'$ if S and D are risk neutral.) Accordingly, the probability that s's optimal offer will be rejected is independent of p. Thus, the probability that any s's offer will be rejected is a nondecreasing function of p in all three cases as long as the bargainers are risk neutral and $p \geq r$.

The expression in (2) is also easy to sign if the bargainers are risk averse and the bargaining problem is sufficiently symmetric. Suppose that the bargainers have identical utility functions U and the Nash bargaining solution is to divide the total benefits in half: $r = \frac{1}{2}$. Then the expression in (2) is nonnegative if the discount factor is close enough to one. To see this, note that at any $x^* \geq \frac{1}{2}$, $U_S'(1-x^*)/U_D'(x^*) = U'(1-x^*)/U'(x^*) \geq 1$. Accordingly, (2) is strictly greater than zero if the bargainers are risk averse and $x^* \geq \frac{1}{2}$. Continuity then ensures that there is an $\epsilon > 0$ such that (2) is positive if $x^* > \frac{1}{2} - \epsilon$. Now choose δ close enough to one to ensure $r_S(\delta) > r - \epsilon = \frac{1}{2} - \epsilon$. Then, $x^* > \frac{1}{2} - \epsilon$ because x^* is bounded below by $r_S(\delta)$. Hence, $p - U_D(x^*)$ is increasing if $t^* < \tilde{x}(\underline{d})$, and the probability of breakdown is nondecreasing in p in this symmetric case.

The preceding has been based on the assumption that $p \geq r$. If $p < r$ the probability of breakdown is still nondecreasing in $|p-r|$ if the bargainers are risk neutral or if the problem is symmetric. With $p < r$, D is satisfied. That is, $p < r$ implies $p - \underline{d} < p < r \leq U_D(r)$, where the last inequality follows from the concavity of U_D. Thus, $p - \underline{d} < U_D(r_S(\delta))$ for δ close enough to one. If p is sufficiently small, S becomes the potentially dissatisfied bargainer if there is one, and the probability of breakdown becomes $\pi = \int_{\underline{s}}^{\bar{s}} F_S(p' - U_S(x^*(d, \delta, \beta_S[\underline{s}, \bar{s}])))dF_D$, where $x^*(d, \delta, \beta_S[\underline{s}, \bar{s}])$ is the share d offers to s and $p' = 1 - p$ is the probability that S will win all of the benefits in the event of an imposed settlement. With $p < r$, showing π to be nondecreasing in $|p - r|$ is equivalent to demonstrating that it is nonincreasing in p or nondecreasing in p'. So, it will suffice to show that the integrand is nondecreasing in p'. But this follows analogously from the argument showing that $\int_{\underline{s}}^{\bar{s}} F_D(p - U_D(x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])))dF_S$ is nondecreasing in p.

Unfortunately, it is not always the case that the probability of breakdown is nondecreasing in p and consequently in the disparity. To sketch an example in which the probability of breakdown is decreasing over a range of values of p, suppose D and S have a constant level of risk aversion α and let $U_D(x) = (1 - e^{-\alpha x})/(1 - e^{-\alpha})$ and $U_S(1 - x) = (1 - e^{-\alpha(1-x)})/(1 - e^{-\alpha})$. Substituting these functions into (2) and taking the limit as x^* goes to zero give $2\alpha - \alpha/(p+s)$ which is negative if $p + s < \frac{1}{2}$. This suggests that the probability of breakdown may decrease as p increases if D is weak, i.e., p is small, and the optimal offer, x^*, is also small. Indeed this turns out to be the case. For example, let $\alpha = 1$, $s = 0.1$, $\underline{d} = 0.05$, $\bar{d} = 0.01$, and $f_D(d) = 2(d - \underline{d})/(\bar{d} - \underline{d})^2$. Then, $d(F_D(p - x^*(s, \delta, \beta_D[\underline{d}, \bar{d}])))/dp < 0$ at $p = 0.2$. So, the probability of breakdown is decreasing at $p = 0.2$ if the types S are distributed in a small neighborhood of 0.1. ∎

REFERENCES

Ausubel, L., and Deneckere, R. (1992). “Bargaining and the Right to Remain Silent,” *Econometrica* **60**, 627–650.

Blainey, G. (1973). *The Causes of War*. New York: Free Press.

Bueno de Mesquita, B., and Lalman, D. (1992). *Reason and War*. New Haven, CT: Yale Univ. Press.

Cooter, R., Marks, S., and Mnookin, R. (1982). “Bargaining in the Shadow of the Law,” *J. Legal Stud*. **11**, 225–251.

Cooter, R., and Rubinfeld, D. (1989). “Economic Analysis of Legal Disputes and Their Resolution,” *J. Econ. Lit*. **27**, 1067–1097.

Crawford, V. (1982). “A Theory of Disagreement in Bargaining,” *Econometrica* **50**, 607–638.

Fearon, J. (1992). “War, Relative Power, and Private Information,” in *the Annual Meeting of the International Studies Association, March* 31–*April* 4, 1992, *Atlanta*, *GA*.

Fearon, J. (1993). “Ethnic Conflict as a Commitment Problem.” Unpublished manuscript, Department of Political Science, University of Chicago.

Fudenberg, D., and Tirole, J. (1983). “Sequential Bargaining with Incomplete Information,” *Rev. Econ. Stud*. **50**, 221–247.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. Cambridge, MA: MIT Press.

Fudenberg, D., Levine, D., and Tirole, J. (1985). “Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information,” in Alvin Rothe, *Game Theoretic Models of Bargaining* (A. Rothe, Ed.). Cambridge: Cambridge Univ. Press.

Fudenberg, D., Levine, D., and Tirole, J. (1987). “Incomplete Information Bargaining with Outside Opportunities,” *Quart. J. Econ*. **102**, 37–50.

Gerber, E. (1995). “Legislative Response to the Threat of Popular Initiative.” Unpublished manuscript, Department of Political Science, University of California, San Diego.

Houantchekon, L. (1994). “Electoral competition with Outside Options.” Unpublished manuscript, Department of Economics, Northwestern University.

Kennan, J., and Wilson, R. (1993). “Bargaining with Private Information.” *J. Econ. Lit*. **31**, 45–104.

Levy, J. (1989). “The Causes of War: A Review of Theories and Evidence.” In *Behavior, Society, and Nuclear War*, Vol. 1 (P. Tetlock, J. Husbands, R. Jervis, P. Stern, and C. Tilly, Eds.). New York: Oxford Univ. Press.

Lupia, A., and Strom, K. (1995). “Coalition Termination and the Strategic Timing of Parliamentary Elections.” Unpublished manuscript, Department of Political Science, University of California, San Diego.

Maoz, Z. (1983). “Resolve, Capabilities, and the Outcomes of Interstate Disputes, 1815–1976.” *J. Conflict Resolution* **27**, 195–229.

Morgenthau, H. (1966). *Politics Among Nations*, 4th ed. New York: Alfred Knopf.

Moul, W. (1988). “Balances of Power and the Escalation to War of Serious International Disputes Among the European Great Powers, 1815–1939.” *Amer. J. Polit. Sci*. **32**, 241–275.

Nalebuff, B. (1987). “Credible Pretrial Negotiation.” *Rand J. Econ*. **18**, 198–210.

Organski, A. F. K., and Kugler, J. (1980). *The War Ledger*. Chicago: Univ. of Chicago Press.

Osborne, M., and Rubinstein, A. (1990). *Bargaining and Markets*. New York: Academic Press.

Perry, M. (1986). “An Example of Price Formation in Bilateral Situations.” *Econometrica* **54**, 313–321.

Priest, G., and Klein, B. (1984). “The Selection of Disputes for Litigation.” *J. Legal Stud*. **13**, 1–55.

Reinganum, J., and Wilde, L. (1986). “Settlement, Litigation, and the Allocation of Litigation Costs.” *Rand J. Econ*. **17**, 557–566.

Rubinstein, A. (1982). “Perfect Equilibrium in a Bargaining Model.” *Econometrica* **50**, 97–109.

Schweizer, U. (1989). “Litigation and Settlement Under Two-Sided Incomplete Information.” *Rev. Econ. Stud.* **18**, 163–178.

Siverson, R., and Tennefoss, M. (1984). “Power, Alliance, and the Escalation of International Conflict, 1815–1965.” *Amer. Polit. Sci. Rev.* **78**, 1057–1059.

Spier, K. (1992). “The Dynamics of Pretrial Negotiation.” *Rev. Econ. Stud.* **59**, 93–108.

Wright, Q. (1965). *A Study of War*. Chicago: Univ. of Chicago Press.