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Does Private Information Lead to Delay or War in Crisis Bargaining?*

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Many game-theoretic models of crisis bargaining find that under incomplete information, an initial offer is either accepted, or war occurs. However, this finding is odd in two ways: (a) empirically, there are many cases of an agreement being peacefully reached after a number of offers and counteroffers and (b) theoretically, it is not clear why a state would ever leave the bargaining table and opt for inefficient war. We analyze a model in which, as long as the dissatisfied state is not too impatient, equilibria exist in which an agreement is peacefully reached through the offer–counteroffer process. Our results suggest that private information only leads to war in conjunction with other factors that are correlated with impatience, such as domestic political vulnerability, exogenous obstacles to the ability to make counteroffers rapidly, and bargaining tactics that create incentives to strike quickly or that lock the actors into war.

The idea that costly war could occur between two more-or-less rationally led states due to some kind of incomplete information, uncertainty, or misperception between them has a long history among students of international relations (e.g., Blainey 1988; Jervis 1976; Van Evera 1999). For example, Blainey (1988) argues that wars often occur when both sides are very optimistic about their chances of victory.

A large number of game-theoretic works have formally examined the process by which uncertainty can lead to inefficient war. For example, Fearon (1995) uses a formal model of crisis bargaining to show that even in a scenario where war is costly and there thus exists negotiated settlements that both sides strictly prefer to go to war, war could be a rational outcome between two states when there is private information about military capabilities or resolve and incentives

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to misrepresent it. Fearon uses a model in which only one take-it-or-leave-it (henceforth TILI) settlement offer is allowed. However, Powell (1996a, 1996b, 1999) considers a full-fledged bargaining model in which a potentially infinite number of offers and counteroffers are allowed, and also shows that inefficient war can rationally occur under uncertainty.[1]

Fearon and Powell's models treat war as a game-ending costly lottery. More recently, a number of models of crisis bargaining have arisen that allow the bargaining process to continue after a war begins (Filson and Werner 2002; Slantchev 2003b; Powell 2004; Smith and Stam 2004; Wagner 2000; Wittman 1979). These models also find that inefficient war can rationally occur under uncertainty.

A striking feature of virtually all of these models is that no prewar bargaining (in the sense of offers and counteroffers) takes places in equilibrium—instead, either the first offer is accepted, or war occurs. In most of these models, this is structurally built into the game-tree—after state 1 makes its initial offer, state 2's only options are to accept it or go to war.[2] Hence, there is no *opportunity* for an agreement to be reached after a number of offers and counteroffers, without war occuring in the meantime. In Powell's (1996a, 1996b, 1999) costly lottery model, as well as his (2004) bargaining-while-fighting model, there is an *opportunity* for an agreement to be peacefully reached after some offers and counteroffers, but this never happens *in equilibrium*: in all perfect-Bayesian equilibria (henceforth PBE), if the satisfied state's initial offer is too small, the dissatisfied state goes to war rather than peacefully making a counteroffer.

A large number of formal models of crisis bargaining thus imply that under incomplete information, state 1 faces a hair-trigger decision: either its initial offer is accepted, or it is rejected and war occurs. The absence of prewar bargaining in these models suggests that under incomplete information, state 1 inevitably faces a ''risk-return tradeoff,'' in which it has to assume that if its initial offer is not accepted, war will break out. However, this result is puzzling in at least two ways. First, it does not seem to make much logical sense: given that war is costly and it is common knowledge that there exists agreements that both sides strictly prefer over war, why would a state ever leave the bargaining table and opt for inefficient war instead?

Second, there seem to be many empirical cases of an agreement being peacefully reached after a number of offers and counteroffers. For example, in the Agadir Crisis of 1911 (also commonly called the Second Moroccan Crisis), sparked by the deployment of the German gunboat *Panther* off the coast of French-controlled Morocco, a number of offers and counteroffers were made before an agreement was finally reached. The German foreign minister initially demanded the entire French Congo as compensation for German acceptance of a French protectorate over Morocco—in the final agreement reached, however, Germany accepted considerably less, and no conflict occurred in the meantime (Barraclough 1982; Lowe 1994, 174–83). Beginning in 2003, the ongoing six-nation talks (involving China, Japan, North Korea, Russia, South Korea, and the United States) to resolve the North Korean nuclear crisis involved a number of offers and counteroffers before an agreement was reached in February 2007 (Sang-Hun 2007). These episodes, and many others like them, suggest that agreements are often peacefully reached after some offers and counteroffers.

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[1] Earlier models of how incomplete information can lead to war include Bueno de Mesquita and Lalman (1992) and Morrow (1989).

[2] In Fearon's take-it-or-leave-it-offer model, that describes the entire game. In the bargaining-while-fighting models of Filson and Werner (2002), Slantchev (2003b), and Smith and Stam (2004), if war occurs in the first period, then a new offer can be made in the second period (if neither state collapses before then). However, there is no way to *peacefully* reach the second period.

In this article, we analyze a costly lottery model of crisis bargaining in which an agreement can be reached in equilibrium after some delay. We first point out that the reason that the risk-return tradeoff (in which the satisfied state's initial offer is accepted or war occurs) is the *unique* PBE outcome of Powell's (1996a, 1996b, 1999) costly lottery model because that model gives *all* of the bargaining leverage to the satisfied state (even though both sides can make proposals), and this rules out any incentive for the dissatisfied state to make a counteroffer. Thus, although Powell's model does not *structurally* build in the risk-return tradeoff (as other models do), it is *effectively* built in by giving all of the bargaining leverage to the satisfied state. We show that the reason that the satisfied state has all of the bargaining leverage in Powell's model is because a state can only go to war in periods in which the *other* side makes an offer. Thus, in a period in which the dissatisfied state makes an offer, the satisfied state knows that the dissatisfied state cannot go to war until the next period, and hence the satisfied state can demand a big offer even in periods in which the dissatisfied state makes a proposal.

We then modify Powell's model to allow states to go to war in *any* period. In the modified model, it turns out that both sides have bargaining leverage in equilibrium, as we would expect from a model in which both sides can make proposals. We find that risk-return equilibria emerge, in which the dissatisfied state goes to war if the satisfied state's initial offer is too small. These equilibria always exist: in particular, they exist regardless of whether the dissatisfied state's discount factor (the extent to which it values future payoffs relative to current payoffs) is low, medium, or high. However, because the dissatisfied state has some bargaining leverage in our model, we also find "non-risk" equilibria, in which the dissatisfied state simply makes a counteroffer (which is accepted) if the satisfied state's initial offer is too small, and in which the probability of war is therefore zero. These equilibria exist as long as the dissatisfied state's discount factor is medium or high, and are novel to our model.[3]

Our results thus suggest that under incomplete information, if the satisfied state's initial beliefs cause her to make a small initial offer and the dissatisfied state ends up being highly resolved, bargaining need not break down in war (as previous models have implied). Instead, the offer–counteroffer process can lead to an agreement being peacefully reached, which is consistent with many historical cases. Indeed, we will argue that the nonrisk equilibria are more plausible than the risk-return equilibria, when both exist (namely, when the dissatisfied state's discount factor is medium or high). It is only when the dissatisfied leader attaches little value to future payoffs relative to current payoffs (i.e., his discount factor is low) that the risk-return tradeoff is the *unique* equilibrium outcome. This is because the dissatisfied leader values future payoffs too little to *wait* until an agreement is reached, and so goes to war immediately if he gets a small initial offer.

Thus, our results suggest that private information is not *sufficient* to cause war. As long as the dissatisfied state's discount factor is not too low, there exists peaceful equilibria in which a negotiated settlement is reached. Under uncertainty, it may take a number of offers and counteroffers to reach an agreement that both sides prefer over war, but as long as the dissatisfied state values future payoffs enough, it is willing to wait until then and does not leave the bargaining table if it gets a low initial offer.

This suggests then that private information only leads to war in conjunction with other factors. There are a number of such factors suggested by the analysis.

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[3] Referring to the finding that his model has a unique PBE outcome, Powell (1996b, fn. 30) writes "The fact that there is a unique outcome is surprising. Typically in bargaining games in which an informed bargainer (i.e., a bargainer with private information) can make offers, there is a multiplicity of equilibrium outcomes." We show that when Powell's model is modified to allow states to go to war in any period, multiple equilibrium outcomes do indeed emerge.

For example, how can we substantively interpret the discount factor being low (in which case the risk-return tradeoff is the *unique* equilibrium outcome)? One way would be if the dissatisfied leader is domestically vulnerable and does not expect to remain in power for long, and hence does not attach much value to future payoffs. Our results suggest that for such leaders, private information is very conducive to war breaking out.[4]

Second, as the time between offers and counteroffers shrinks, the actors' discount factors become larger. This suggests that exogenous obstacles to the ability to make counteroffers rapidly are conducive to war breaking out (Powell 2004 makes a similar point). This might be related to factors such as the technology of communication, geographical barriers to communication, norms against harming emissaries, etc.

Third, if the other side is mobilizing and/or finding new allies as the bargaining proceeds, there might be incentives to attack if an agreement is not reached quickly. Although these factors are not explicitly incorporated in our model (this is a promising area for future research), they are clear examples of why a leader might be impatient in crisis bargaining, which our analysis suggests is a crucial factor in whether private information leads to war or merely delays reaching an agreement.

Finally, as is well recognized in the credible signaling literature, an important mechanism by which private information can lead to war is through the tactics by which leaders try to credibly signal their private information. Examples of such credible signaling tactics include audience costs (Fearon 1994), military mobilization (Fearon 1997; Slantchev 2005), opposition party endorsement of the government's threats (Schultz 1998), private diplomatic signals (Kurizaki 2007; Sartori 2002), and generating an autonomous risk of war (the ''threat that leaves something to chance;'' Schelling 1960).[5] This literature often emphasizes how the credible signaling tactics that leaders use can further *increase* the risk of war under incomplete information. Indeed, this is one of the main reasons why they are *credible* signals of resolve, as opposed to bluffs. For example, generating audience costs can lead to ''lock-in,'' whereby a leader now prefers war to backing down (Fearon 1994, 1997).

The rest of the article is organized as follows. In the next section, we briefly present Fearon's (1995) and Powell's (1996a, 1996b, 1999) costly lottery models of how private information leads to war. We then present our model and establish the main results, under complete information, and then under incomplete information. Finally, we offer some concluding remarks.

War as a Bargaining Breakdown

Figure 1, drawn from Fearon (1995), graphically illustrates the war-as-a-bargaining-breakdown approach to crisis bargaining. Two countries (labeled *D*, henceforth a ''he,'' and *S*, henceforth a ''she'') are involved in a dispute over a divisible good (e.g., territory), whose value to both sides is normalized to 1. The two sides can either peacefully reach an agreement on a division of the good or they can go to war, in which case the side that wins obtains the entire good and the side that loses receives none of it. Moreover, war is costly, with side *D*'s and side *S*'s cost of war being c_D, $c_S > 0$, respectively. Assume that if war occurs, side *D* wins with probability $1 > p > 0$ and side *S* wins with probability $1-p$ (thus, p

[4] Note that our prediction about the greater conflict-propensity of domestically vulnerable leaders is observationally similar to the diversionary theory of war. However, Smith (1996) argues that other leaders might strategically avoid disputes with leaders with diversionary incentives, and Tarar (2006) shows that in a bargaining setting, diversionary incentives can lead to a crisis that ends in a negotiated settlement rather than war.

[5] Jervis (1970) and Schelling (1960) are foundational works on signaling in international relations.

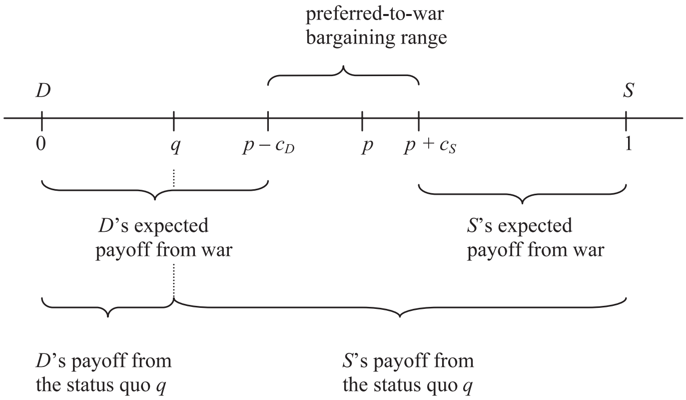

Fig. 1. War as a Bargaining Breakdown

measures the extent to which the military balance favors D). Then, country D's expected utility from war is $EU_D(war) = (p)(1) + (1-p)(0) - c_D = p - c_D$. Similarly, country S's expected utility from war is $EU_S(war) = (p)(0) + (1-p)(1) - c_S = 1 - p - c_S = 1-(p + c_S)$. Thus, as seen in Figure 1, the costliness of war opens up a bargaining range of agreements $[p-c_D, p + c_S]$ such that for all agreements in this range, both sides prefer the agreement over war.[6]

There is some status quo division of the disputed good $(q, 1-q)$, where $1 \geq q \geq 0$ is D's share and $1-q$ is S's share. A state is ''satisfied'' if the status quo division of the good provides it with at least as much utility as going to war (Powell 1996a, 1996b, 1999). In contrast, a state is ''dissatisfied'' if it strictly prefers to go to war rather than live with the status quo. Thus, D is satisfied if $q \geq p-c_D$ and dissatisfied if $q < p-c_D$ (this is the case shown in Figure 1). S is satisfied if $1-q \geq 1-p-c_S$ or $q \leq p + c_S$. S is dissatisfied if $q > p + c_S$. Both sides are satisfied if $p + c_S \geq q \geq p-c_D$ (i.e., if the status quo lies within the preferred-to-war bargaining range). Only D is dissatisfied if $q < p-c_D$, and only S is dissatisfied if $q > p + c_S$. If the two sides agree on the probability that each prevails in war, then at most one state can be dissatisfied.

Fearon's Model

Fearon (1995) game-theoretically analyzes this expected-utility framework using a TILI offer bargaining model in which one side, say S, can propose some division of the disputed good, say $(x, 1-x)$, where $1 \geq x \geq 0$ is D's share and $1-x$ is S's share. D can either accept this offer, in which case each side's utility is simply its proposed share (we assume risk neutrality throughout this paper), or it can reject it, in which case war occurs and each side gets its payoff from war. Under complete information, this TILI offer bargaining model gives a unique subgame-perfect equilibrium (SPE), in which D accepts all offers that give it at least its utility from war (i.e., it accepts all offers such that $x \geq p-c_D$), and S offers D exactly its utility from war. That is, agreement is reached on $[p-c_D, 1-(p-c_D)]$, and war is avoided. Because S can make a TILI offer, it gets all of the gains from avoiding war; thus it gets its most preferred outcome in the preferred-to-war bargaining range (Romer and Rosenthal 1978).

Fearon then supposes that S is uncertain about D's cost of war, c_D. Suppose that c_D lies in the range $[c_{D_l}, c_{D_h}]$. That is, c_{D_l} is the lowest cost ''type'' of D that S

[6] As Powell (2002) points out, the interpretation that the war is total and the victorious side wins everything while the losing side gains nothing is not necessary for this argument. Simply interpret p to be the expected division of the good resulting from war.

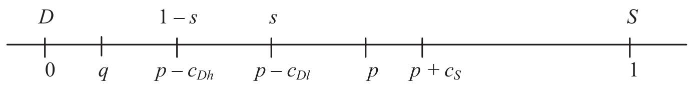

FIG. 2. Uncertainty About *D*'s Costs of War

might be facing (or most resolved, because its expected payoff from war is the highest; see Figure 2), and c_{D_h} is the highest cost type (or least resolved). In equilibrium, each type of *D* accepts all agreements that give it at least its expected payoff from war. Thus, *S* faces a tradeoff: it can either make the ''big'' offer $p - c_{D_l}$ which *all* types of *D* accept and thus avoid war with certainty, or it can make a lower offer which only less-resolved types of *D* accept, but which leaves *S* with a bigger share of the pie *if* it is accepted. If *S*'s initial belief about *D*'s type puts sufficient weight on *D* being a less-resolved type, then *S*'s optimal offer in equilibrium is to offer less than $p - c_{D_l}$, and thus war occurs if *D* ends up being a highly resolved type. Powell (1999) calls this the ''risk-return tradeoff,'' because *S* accepts a positive risk of war in exchange for a greater return at the negotiating table if war does *not* occur. That is, making a limited offer can be an optimal gamble under uncertainty, and this is how private information leads to war.

Powell's Model

The major limitation of Fearon's TILI offer model is that only one side can make an offer, and rejection of that offer automatically results in war. That is, the other side cannot make a counteroffer, and hence the risk-return tradeoff is built into the game-tree. However, in most actual bargaining situations, there is no reason why the other side cannot make a counteroffer and why rejection of the initial offer must automatically result in war. Thus, Fearon's model is not ideal for studying the decision to deliberately launch a war, because in his model that decision is not separated from the decision to reject an offer.

Powell (1996a, 1996b, 1999) generalizes Fearon's model to allow each side to make a counteroffer if it rejects an offer. More specifically, Powell's model is shown in Figure 3 (only three periods are shown in the figure, but this is actually an infinite horizon game). The two sides take turns making offers and counteroffers (Figure 3 shows *D* making the first proposal, but this is not necessary), and war only occurs if one side opts out of the bargaining process because it decides that war is preferable to further bargaining. In general, if an agreement is reached on some division of the good $(z, 1-z)$ in period t $(t = 0,1,2,\ldots)$, then *D*'s payoff is $\sum_{i=0}^{t-1} \delta^i q + \sum_{i=t}^{\infty} \delta^i z$ and *S*'s payoff is $\sum_{i=0}^{t-1} \delta^i (1-q) + \sum_{i=t}^{\infty} \delta^i (1-z)$, where $1 > \delta > 0$ is the players' common discount factor. If they go to war in some period t $(t = 0,1,2,\ldots)$, then *D*'s payoff is $\sum_{i=0}^{t-1} \delta^i q + \sum_{i=t}^{\infty} \delta^i (p - c_D)$ and *S*'s payoff is $\sum_{i=0}^{t-1} \delta^i (1-q) + \sum_{i=t}^{\infty} \delta^i (1 - p - c_S)$.

When both sides are satisfied, then neither can credibly threaten to use force to try to change the status quo, and hence war does not occur in equilibrium,

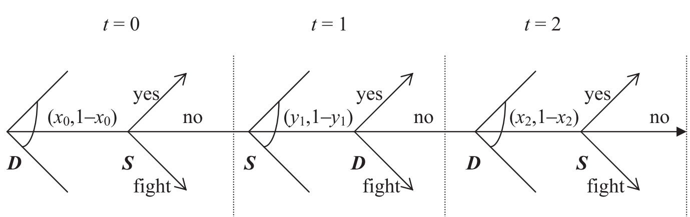

FIG. 3. Powell's Model

nor does any revision of the status quo take place. Now suppose that one side is dissatisfied; in particular, suppose that D is dissatisfied (hence the labels D and S), for example, suppose $q < p - c_D$. Then, as Powell points out, in the subgame perfect equilibria of this game, war is avoided, and the status quo is peacefully revised in D's favor.[7] Whenever S makes an offer, it offers D just its utility from war, for example, it offers $p - c_D$, and keeps the rest of the pie for itself, and this offer is accepted. This is the same outcome in Fearon's model, where S gets to make a TILI offer. Note that S gets all of the gains from avoiding war whenever it makes an offer. However, a rather odd agreement is reached when D makes an offer. In any period in which D makes an offer, it proposes for itself the share of the pie $q(1-\delta) + \delta(p - c_D)$, and the rest of the pie for S, and S accepts this offer. The strange thing is that D's offer for itself is *less than its utility from war*: $q(1-\delta) + \delta(p - c_D) < p - c_D$ for $\delta < 1$. This can be seen from Figure 4, which graphs the equilibrium shares of the pie that D (solid line) and S (dashed line) propose for D, as δ ranges from 0 to 1.[8]

As seen in Figure 4, there are three somewhat odd findings in Powell's complete information results. First, there is a first mover *disadvantage*, which is atypical for a Rubinstein (1982) bargaining model with complete information. In particular, each player would rather have the other make the first offer (which is accepted). Second, the satisfied state has *all* of the bargaining leverage, in that it gets all of the gains from avoiding war (and gains even more than when D makes the first offer). In equilibrium, S has the same bargaining power that it would if it could make a TILI offer (even more, when D makes the first offer), even though *both sides have proposal power* (proposal power typically confers bargaining

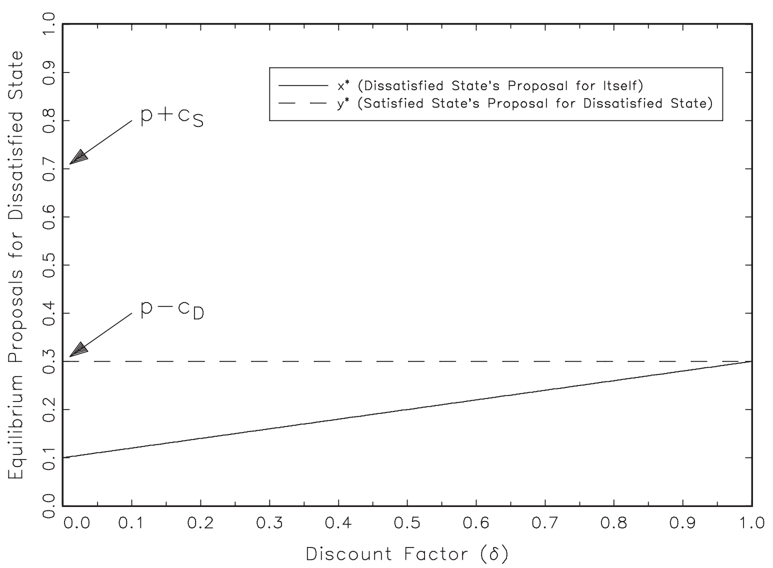

Fig. 4. Equilibrium Proposals in Powell's Model

[7] Powell (1996a, 263) identifies one (stationary) SPE and states that it is the unique SPE. However, because of an indifference condition, there are in fact an infinite number of SPE. However, the average per-period payoffs to the players are the same in all of these equilibria, which is what is important. We characterize the subgame perfect equilibria in Proposition 10 in the Appendix, while generalizing the model to allow the players to have different discount factors.

[8] Notice that D's offer for itself is a weighted average (convex combination) of its status quo payoff and its utility from war. As its status quo payoff is strictly less than its utility from war, this weighted average is strictly less than its payoff from war. Note that Figure 4 is drawn for $p = 0.5$, $c_D = c_S = 0.2$, and $q = 0.1$. It has the same general shape for all values of these parameters.

leverage). And third, the oddest of all from a substantive viewpoint, D actually proposes for itself less than its payoff from war, knowing that this proposal will be accepted. (Note from Figure 4 that as $\delta \to 1$, the first and third ''problems'' disappear in the limit. In the limit, the outcome approaches one in which S can make a TILI offer.)

Under complete information, war does not occur in equilibrium. Instead, the status quo is peacefully revised in D's favor, with S getting (at least, depending on who makes the first offer) all of the gains from avoiding war.[9]

Powell then analyzes a case where the two sides are uncertain about each other's cost of war. He shows that this game has a *unique* PBE outcome, in which the same risk-return tradeoff as in Fearon's (1995) model emerges. In particular, a dissatisfied type of D never rejects S's initial offer to make a counteroffer (even though it can in principle). Instead, it goes to war if it gets a proposal that gives it less than its utility from war. Therefore, S is essentially in the position of making its optimal TILI offer, knowing that if it is rejected, war will result rather than a counteroffer. If S's initial belief puts sufficient weight on D being a low-resolve type, then S's optimal offer in equilibrium is low enough that war occurs with positive probability. Moreover, this risk-return tradeoff is the *unique* perfect Bayesian equilibrium outcome. Thus, even Powell's rather general model of crisis bargaining suggests that the offer–counteroffer process cannot lead to a negotiated settlement being peacefully reached under uncertainty, and that the risk-return tradeoff is the mechanism for how uncertainty leads to bargaining breakdown, and thus war.

The main result that Powell uses to establish the uniqueness of this PBE outcome is that a dissatisfied type of D would never reject S's initial offer in order to make a counteroffer. The formal proof of this result is given by Powell (1999, 248), and is not repeated here. We simply note that the core reason for this result is that S has *all* of the bargaining leverage, and hence the best that D can get in the next period (if it chooses to make a counteroffer) is its utility from war. It is better to go to war now rather than live with the (worse) status quo in the current period and get (at best) its utility from war in the next period. Hence, because S has *all* of the bargaining leverage, D never makes a counteroffer in equilibrium.

A Modification of Powell's Model

We now present a modification of Powell's model that ''corrects'' all three of the odd features of Powell's complete information results. Most importantly, in the modified model, D also has some bargaining leverage in equilibrium, and hence we will characterize incomplete-information equilibria in which D chooses to make a counteroffer (which is accepted) if S's initial offer is too small. Thus, the risk-return tradeoff is not the unique equilibrium outcome in the modified model, and a negotiated settlement can be peacefully reached through the offer–counteroffer process.

The most problematic feature of Powell's complete information results is that D proposes for itself less than its utility from war, knowing that this offer will be accepted. This can only happen in equilibrium because D cannot go to war in periods in which it itself makes an offer (see Figure 3). If it could, it would never choose to make such an offer, because it would rather go to war instead. To capture this, we modify Powell's model by allowing actors to go to war in *any* period, rather than just in periods in which the other side makes an offer. The game-tree is shown in Figure 5.

[9] See Langlois and Langlois (2006) and Slantchev (2003a) for crisis bargaining models in which costly war is an equilibrium outcome even under complete information.

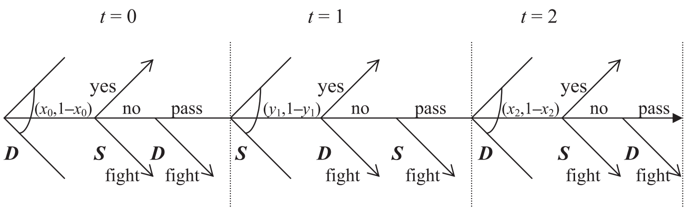

Fig. 5. The Modified Model

Complete Information Results

Propositions 1–4 in the Appendix describe the SPE of this model for different values of the actors' discount factors, δ_S and δ_D, when D is dissatisfied (when both sides are satisfied, then, as in Powell's model, no revision of the status quo takes place in equilibrium).

The results are illustrated graphically in Figure 6 for the case where $\delta_D = \delta_S = \delta$, thus, the two sides have the same discount factor. The figure shows the equilibrium proposals for D, x^* and y^*, when D and S make proposals, respectively.[10] These proposals are accepted in equilibrium, and hence these are D's actual average per-period payoffs in the model, depending on who gets to make the first proposal. When δ is low, each side's proposal just offers the other side its payoff from war. So whoever gets to make the first proposal gets *all* of the gains from avoiding war. When δ gets in the medium range, then S has to start compromising when making a proposal, so, y^* starts increasing. When δ gets large, then both sides' proposals for D start decreasing.

The intuition behind Figure 6 is given in the Appendix. The important thing to note from the figure is that the three odd results that emerge under complete information in Powell's model do not emerge in the modified model. First, there

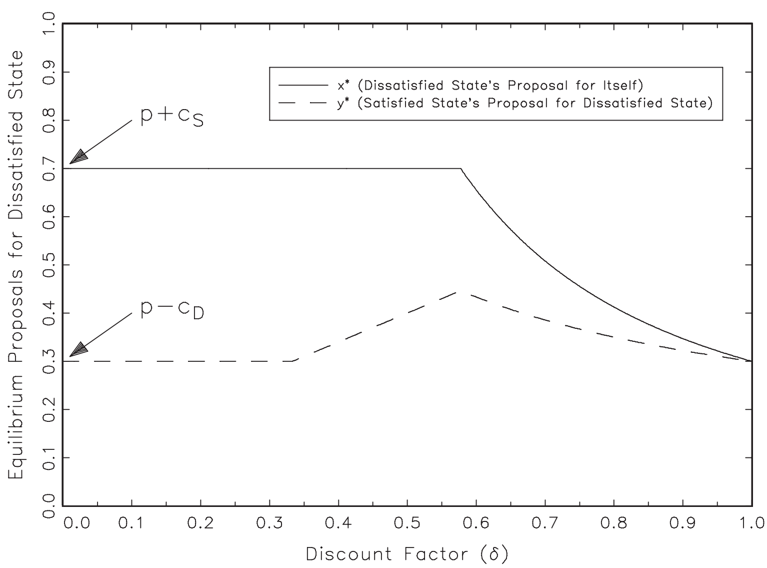

Fig. 6. Stationary Equilibrium Proposals in the Modified Model

[10] Figure 6 is drawn for $p = 0.5$, $c_D = c_S = 0.2$, and $q = 0.1$. It has the same general shape for all values of these parameters.

is a first-mover advantage, which is typical in Rubinstein (1982) bargaining models with complete information. Each side strictly prefers to make the first proposal. Second, each side has some bargaining leverage in equilibrium, which we would expect in a model in which both sides can make proposals. Finally, and most importantly from a substantive viewpoint, any agreement reached gives each side at least its utility from war (i.e., lies within the preferred-to-war bargaining range), which we would substantively expect given that under anarchy, a state can launch a war at any time (Waltz 1979).

Figure 6 shows the *stationary* SPE proposals, SPE in which an actor uses the same strategy in structurally identical subgames (see Baron and Ferejohn 1989 for more discussion of stationarity in bargaining games). These SPE are characterized in Propositions 1–3. Proposition 4 shows that when δ is high (the region in Figure 6 where x^* and y^* are both decreasing), a broad range of agreements can be reached in *nonstationary* SPE. In particular, in the model where D makes the first offer, any agreement that gives D between $p-c_D$ and $p + c_S$ can be reached in the first period of a nonstationary SPE, and in the model where S makes the first offer, any agreement that gives D between $p-c_D$ and $q(1-\delta_D) + \delta_D(p + c_S)$ (this is the linearly increasing portion of the dashed line in Figure 6) can be reached in the first period of a nonstationary SPE.

In the Appendix, we make an equilibrium selection argument that the equilibrium most likely to be played when δ is in the high range is one in which the trend in Figure 6 when δ is in the medium range continues even when δ becomes high, thus D continues to get all of the gains from avoiding war when he makes the first proposal, and S compromises more and more (the linearly increasing dashed line) as δ becomes larger and larger when she makes the first proposal. The core of the argument is that when δ is in the high range, there is a decision-node at which D is indifferent (given the equilibrium strategies for the rest of the game) between fighting and passing. This indifference is what allows for the multiplicity of equilibria, as D can choose to pass, fight, or mix (randomize) between passing and fighting. D's payoff is highest in the SPE in which he chooses to fight (which is the SPE described earlier in this paragraph), and since both players realize this, it seems reasonable that this is where their expectations will converge. This equilibrium selection argument will also be referred to in the incomplete information results.

Incomplete Information Results

We now turn to crisis bargaining under incomplete information. Recall that in Powell's (1996a, 1996b, 1999) model, there is a *unique* perfect Bayesian equilibrium outcome in which, if S's initial belief causes her to make a small initial offer, D goes to war rather than makes a counteroffer if he turns out to be highly resolved. In this section, we want to examine whether this is also the case in the modified model, in which not all of the bargaining leverage lies with S.

To examine this, we consider a case of one-sided uncertainty, in which S is uncertain about D's cost of war c_D (this means that S is uncertain about D's utility from war, or resolve). We assume that D's cost of war takes on one of two values, meaning, there are two possible types of D: S believes that D's cost is c_{D_l} with probability $1 > s > 0$ and c_{D_h} with probability $1-s$, with $c_{D_l} < c_{D_h}$. That is, c_{D_l} is the more resolved (low-cost) type, because its expected utility from war is higher (see Figure 2). We assume that both types of D are dissatisfied: $q < p-c_{D_h}$. Finally, we consider the model in which S makes the first offer.

We first show that, as in Powell's model, there exists risk-return tradeoff equilibria in which war occurs if S's initial belief causes her to make a small initial offer and D ends up being the highly resolved type. These equilibria exist when δ_D is low, medium, or high. We discuss the intuition behind them, and point out how they differ from Powell's equilibria. We then show that, unlike in Powell's model, when δ_D is medium or high, there also exists peaceful, ''non-risk'' equilibria in which D simply makes a counteroffer (which is accepted) if S makes too small an initial offer, and in which the probability of war is therefore zero.

Risk-Return Tradeoff Equilibria

Proposition 5: *If* $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, *there is a PBE in which, in the first period, type* c_{D_l} *accepts all offers* $(y,1-y)$ *such that* $y \geq p-c_{D_l}$ *and goes to war for any lower offer, and type* c_{D_h} *accepts all offers* $(y,1-y)$ *such that* $y \geq p-c_{D_h}$ *and goes to war for any lower offer. If* $s \geq s_{critical}$, *where* $s_{critical} = \frac{c_{D_h}-c_{D_l}}{c_{D_h}+c_S}$, *then* S *makes the large initial offer* $y^* = p-c_{D_l}$, *which both types accept, and war is avoided. If* $s \leq s_{critical}$, *then* S *makes the small initial offer* $y^* = p-c_{D_h}$, *which only type* c_{D_h} *accepts. Type* c_{D_l} *rejects it and goes to war instead. If the second period is reached in this equilibrium (this is off-the-equilibrium-path behavior), agreement would be reached on* $x^* = p + c_S$.

Proposition 6: *If* $\frac{(p-c_{D_h})-q}{(p+c_S)-q} \leq \delta_D \leq min\{\frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}, \frac{(p-c_{D_l})-q}{(p+c_S)-q}\}$, *there is a PBE in which, in the first period, type* c_{D_l} *accepts all offers* $(y,1-y)$ *such that* $y \geq p-c_{D_l}$, *and goes to war for any lower offer, and type* c_{D_h} *accepts all offers* $(y,1-y)$ *such that* $y \geq q(1-\delta_D) + \delta_D(p + c_S)$ *and says no (rather than fight) for any lower offer. If s is sufficiently high, then* S *makes the large initial offer* $y^* = p-c_{D_l}$, *which both types accept, and war is avoided. If s is not sufficiently high, then* S *makes a low initial offer of* $y^* = q(1-\delta_D) + \delta_D(p + c_S)$ if $\delta_S \geq \delta_D$ *and some even lower offer if* $\delta_S \leq \delta_D$. *Type* c_{D_l} *rejects these low offers and goes to war. If the second period is reached, agreement is reached on* $x^* = p + c_S$.[11]

Proposition 7: *If* $\delta_D \geq max\{\frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}, \frac{(p-c_{D_h})-q}{(p-c_{D_l})-q}\}$, *there is a PBE in which, in the first period, type* c_{D_l} *accepts all offers* $(y,1-y)$ *such that* $y \geq p-c_{D_l}$, *and goes to war for any lower offer, and type* c_{D_h} *accepts all offers* $(y,1-y)$ *such that* $y \geq \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$ *and says no (rather than fight) for any lower offer. If s is sufficiently high, then* S *makes the large initial offer* $y^* = p-c_{D_l}$, *which both types accept, and war is avoided. If s is not sufficiently high, then* S *makes a low initial offer of* $y^* = \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$ if $\delta_S \geq \delta_D$ *and some even lower offer if* $\delta_S \leq \delta_D$. *Type* c_{D_l} *rejects these low offers and goes to war. If the second period is reached (by type* c_{D_h}*), agreement is reached on* $x^* = \frac{(p-c_{D_h})-q(1-\delta_D^2)}{\delta_D^2}$.[12]

We have thus constructed PBE in which the risk-return tradeoff emerges, whether δ_D is low, medium, or high. What is interesting is that the substantive dynamics behind these risk-return equilibria are quite different from Powell's, and they differ depending on whether δ_D is low, medium, or high. Recall that in Powell's model, *no* dissatisfied type of D rejects an offer in order to make a counteroffer. It either accepts the initial offer (if it is as least as great as its utility from war), or goes to war. When δ_D is low (Proposition 5), this is what happens in our model as well. However, the reason is quite different. In Powell's model, because S has all the bargaining leverage in equilibrium, the best that D can get in the second period is its utility from war, and this is the reason why it never

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[11] If $\delta_S \geq \delta_D$, then $s_{critical} = \frac{(p-c_{D_l})-[q(1-\delta_D)+\delta_D(p+c_S)]}{(p+c_S)-[q(1-\delta_D)+\delta_D(p+c_S)]} \in [0,1]$.
If $\delta_S \leq \delta_D$, then $s_{critical} = \frac{(p-c_{D_l})-[q(1-\delta_S)+\delta_S(p+c_S)]}{(p+c_S)-[q(1-\delta_S)+\delta_S(p+c_S)]} \in [0,1]$.

[12] If $\delta_S \geq \delta_D$, then $s_{critical} = \frac{[q(1-\delta_D)+\delta_D(p-c_{D_l})]-(p-c_{D_h})}{[q(1-\delta_D)+\delta_D(p+c_S)]-(p-c_{D_h})} \in [0,1]$.
If $\delta_S \leq \delta_D$, then $s_{critical} = \frac{\delta_D^2[(p-c_{D_l})-q]-\delta_S[(p-c_{D_h})-q]}{\delta_D^2[(p+c_S)-q]-\delta_S[(p-c_{D_h})-q]} \in [0,1]$.

makes a counteroffer (D's discount factor plays no role in this). In the equilibrium we have constructed, however, agreement would be reached on $x^* = p + c_S$ in the second period; D would get *all* of the gains from avoiding war. This is why δ_D has to be sufficiently low in this equilibrium. For D to go to war instead of waiting until then (if he gets too small of an initial offer), he has to value future payoffs so little that he would prefer to get the war payoff in the current period rather than the (worse) status quo, even though that means that he forsakes getting all of the gains from avoiding war (the agreement $x^* = p + c_S$) from the next period onward.

Now consider when δ_D is in the medium range (Proposition 6). In this equilibrium, only the highly resolved (low cost) type of D goes to war if he gets a low initial offer. The less-resolved (higher cost) type makes a counteroffer in the next period (which is accepted) if he gets a low initial offer. This equilibrium shows that in the modified model (in which D has some bargaining leverage), and unlike in Powell's model, *it is possible for a dissatisfied type of D to reject an offer in order to make a counteroffer*, rather than go immediately to war.

What is the intuition behind δ_D having to be medium? Again, notice that in this equilibrium, the agreement $x^* = p + c_S$ would be reached in the second period; D would get all of the gains from avoiding war. Thus, D would only go to war in the first period if he gets a low initial offer, rather than make a counteroffer (in which he gets a very favorable agreement), if he discounts future payoffs sufficiently. Because the two types of D have different payoffs from going to war, they have different thresholds for δ_D below which they would rather go to war than wait and get all of the gains from avoiding war from the next period onward. In this equilibrium, δ_D is below the highly resolved type's threshold (who thus would rather go to war), but above the less-resolved type's threshold (who thus would rather move to the next period if he gets a low initial offer).

Finally, consider the equilibrium in which δ_D is high (Proposition 7). This is similar to the previous equilibrium in that only the highly resolved type goes to war if he gets a low initial offer—the less-resolved type makes a counteroffer (which is accepted) if he gets too low an offer. However, unlike the previous two equilibria, D does not get all of the gains from avoiding war in the second period. In constructing this equilibrium, we stipulate that if the second period is reached by the low-resolve type, then the actors use the strategies of Proposition 3. Recall from Figure 6 that when δ is in the high range, both sides' proposals for D (in the stationary SPE of Proposition 3) start decreasing. In the incomplete information equilibrium when δ_D is high (Proposition 7), what is happening is that δ_D is so high (we are in the far right range of Figure 6, where the proposals for D are very low) that the most favorable (for D) agreement that could be reached in the second period is actually less than the highly resolved type's payoff from war, but exceeds the low-resolve type's payoff from war. Thus, even though δ_D is high, the highly resolved type prefers to go to war rather than move to the next period (in which he could, at best, get a worse agreement than war), whereas the low-resolve type prefers to move to the next period and get some of the gains from avoiding war therein. We now turn to the ''non-risk'' equilibria, which are novel to our model.

Peaceful (Non-Risk) Equilibria

Proposition 8: *If* $\frac{(p-c_{D_l})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, *then for any value of s, there is a PBE in which, in the first period, both types of* D *accept all offers* $(y,1-y)$ *such that* $y \geq q(1-\delta_D) + \delta_D(p + c_S)$ *and say no (rather than fight) for any lower offer. If* $\delta_S \geq \delta_D$, *then* S *offers* $y^* = q(1-\delta_D) + \delta_D(p + c_S)$ *in the first period, which both types accept. If*

$\delta_S \leq \delta_D$, then S *offers some $y < y^*$, and agreement is reached on $x^* = p + c_S$ in the second period.*

Proposition 9: *If $\delta_D \geq max\{\frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}, \frac{(p-c_{D_l})-q}{(p+c_S)-q}\}$, then for any value of s, there is a PBE in which, in the first period, both types of* D *accept all offers $(y, 1-y)$ such that $y \geq q(1-\delta_D) + \delta_D(p + c_S)$ and say no (rather than fight) for any lower offer. If $\delta_S \geq \delta_D$, then* S *offers $y^* = q(1-\delta_D) + \delta_D(p + c_S)$ in the first period, which both types accept. If $\delta_S \leq \delta_D$, then* S *offers some $y < y^*$, and agreement is reached on $x^* = p + c_S$ in the second period.*

These are equilibria under incomplete information, when δ_D is medium or large, in which the risk-return tradeoff does *not* emerge. That is, these are equilibria in which even the highly resolved (low cost) type simply makes a counteroffer (which is accepted) if he gets a low initial offer, rather than going to war. What is the intuition behind these results? We have constructed equilibria in which agreement would be reached on $x^* = p + c_S$ in the second period, that is, *D* would get all of the gains from avoiding war. Thus, as long as *D* attaches sufficient value to future payoffs, he prefers to make a counteroffer rather than go to war if he gets a low initial offer. The risk-return tradeoff is the *unique* PBE outcome in Powell's model only because that model gives *all* of the bargaining leverage to *S*. When *D* also has some bargaining leverage, as in the modified model, there exist equilibria when δ_D is medium or large in which *D* finds it worthwhile to make a counteroffer rather than go to war, and in which the probability of war is therefore zero (*regardless* of *S*'s initial beliefs and *D*'s actual type).

Equilibrium Selection

We have identified risk-return as well as non-risk equilibria, for various ranges of δ_D. When δ_D is low, then *only* risk-return tradeoff equilibria exist. This is because, even if *D* gets *all* of the gains from avoiding war in the second period (the best he can possibly do), he would rather go to war in the first period than wait until then (and live with the worse-than-war status quo in the current period), for δ_D sufficiently low.

However, when δ_D is medium or high, both types of equilibria exist, and a natural question to ask is whether the risk-return equilibria Pareto-dominate the non-risk equilibria, or vice-versa, when they both exist. If so, there would be good reasons for expecting the Pareto-dominant equilibrium to be played. However, it turns out that neither equilibrium Pareto-dominates the other, in general. When δ_D is sufficiently high, the equilibria of Proposition 7 (a risk-return equilibrium) as well as Proposition 9 (a non-risk one) both exist. In Proposition 7, when *s* is low, *S* makes a low initial offer that is only accepted by the less-resolved type—the high-resolve type rejects it and goes to war. Thus, war occurs with probability *s*, and an agreement is reached with probability $1-s$. As δ_D converges to 1, this agreement that is reached converges to $p-c_{D_h}$; that is, *S* gets all of the gains from avoiding war against the less-resolved type. In Proposition 9, *S* makes an offer that both types accept. As δ_D converges to 1, this agreement converges to $p + c_S$; that is, *D* (both types) gets all of the gains from avoiding war. Thus, as δ_D converges to 1, both types of *D* strictly prefer the non-risk equilibrium of Proposition 9, in which *S* compromises a lot, where *s* is sufficiently low, *S* strictly prefers the risk-return equilibrium of Proposition 7, in which she is very likely to get all of the gains from avoiding war (against the less-resolved type), and not likely to have to go to war. Neither equilibrium Pareto-dominates the other in general.

However, there are other reasons for expecting the non-risk equilibrium to be played when δ_D is high, rather than the risk-return equilibrium. We saw from

Proposition 4 that when δ_D is high, a broad continuum of agreements can be reached in nonstationary SPE (under complete information). In the *stationary* SPE of Proposition 3, as δ_D approaches 1, *D*'s share of the pie approaches $p-c_D$ (i.e., in the limit *D* does not get *any* of the gains from avoiding war), regardless of who makes the first proposal (see Figure 6). In constructing the risk-return equilibrium (under incomplete information) when δ_D is high (Proposition 7), we have to use (the strategies of) this stationary SPE in the second period. That is, when δ_D is high, the only way we can get the highly resolved type of *D* to go to war rather than make a counteroffer if *S*'s initial offer is too small, is by constructing an equilibrium in which, if the second period is reached, the highly resolved type of *D* does not get *any* of the gains from avoiding war. As long as the highly resolved type of *D* gets *some* (no matter how little) of the gains from avoiding war in the second period, the risk-return tradeoff cannot exist high enough for δ_D, because *D* would prefer to move to the second period and get those gains (no matter how small) rather than go to war in the first period. To put it another way, constructing a risk-return equilibrium when δ_D is high requires us to use a SPE (more particularly, a set of continuation strategies) in which the highly resolved type of *D* does not get *any* of the gains from avoiding war in the second period, and there are many SPE (and hence continuation strategies) in which he *does* get some of the gains from avoiding war (Proposition 4). Moreover, we made an equilibrium selection argument in the complete information section that the SPE most likely to be played when δ_D is high is one in which *D* in fact gets a substantial amount of the gains from avoiding war. When δ_D is high, the risk-return equilibrium is not particularly compelling.

More Complicated Uncertainty

One potential limitation of our analysis is that we only consider one-sided uncertainty, in which an agreement is reached in the second period, because *D* knows *S*'s type, and hence exactly how large an offer to make. However, when the uncertainty is about the costs of war, it seems that the same overall results would hold regardless of how complicated the uncertainty is (e.g., two-sided uncertainty with a continuum of types). Suppose, for example, that *D*'s cost of war comes from the interval $[c_{D_l}, c_{D_h}]$, and *S*'s cost of war comes from the interval $[c_{S_l}, c_{S_h}]$, with $c_{D_l}, c_{S_l} > 0$; so war is costly for all types. This is shown graphically in Figure 7. Although each side is uncertain of the other side's exact cost of war, it is common knowledge that *all* types of *both* players strictly prefer any agreement in the $(p-c_{D_l}, p + c_{S_l})$ interval to war. The risk-return tradeoff is essentially the idea that a side's attempt to guess the other side's *exact* reservation value (and hence achieve *maximal* bargaining gains) will lead to bargaining breakdown and hence war. Our argument is that this might lead to delay in reaching an agreement, but eventually their expectations should converge to the $(p-c_{D_l}, p + c_{S_l})$ interval, and the agreement $(p, 1-p)$ stands out as a focal point (Schelling 1960) within this interval (simply divide the disputed good in the ratio of the military capabilities). If the dissatisfied state's discount factor is low, then *D* is not willing to *wait* until then, and so goes to war immediately if he gets a small initial offer. However, as long as he expects to get at least some (no matter how little) of the gains from avoiding war in any future agreement reached (which is certainly the case for any agreement in the $[p-c_{D_l}, p + c_{S_l}]$ interval), then for a δ_D large enough, he

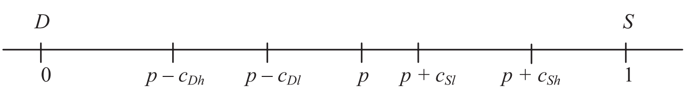

FIG. 7. Two-Sided Uncertainty About Costs of War

strictly prefers to wait until then rather than go to war immediately upon getting a small initial offer. When there is uncertainty about the costs of war, the analysis here suggests that the dissatisfied state's discount factor is a crucial determinant of whether the uncertainty merely leads to *delay* in reaching a preferred-to-war agreement, or whether it leads to a take-it-or-leave-it dynamic and hence a risk of *war*.

Conclusion

A strikingly large number of formal models of crisis bargaining give the result that under uncertainty, state 1's initial offer is accepted or war occurs. The absence of prewar bargaining in these models suggests that under uncertainty, state 1 inevitably faces a risk-return tradeoff in making its initial offer.[13] In this article, we analyze a crisis bargaining model in which the greater array of choices available to the actors leads to *both* sides having some bargaining leverage in equilibrium, and as a result, we derive equilibria in which agreements can be peacefully reached through the offer–counteroffer process.[14] Our results suggest that under uncertainty about the costs of war, the dissatisfied state's time preferences are a crucial factor in whether the uncertainty leads to a take-it-or-leave-it dynamic and hence a risk of *war*, or whether it merely leads to *delay* in reaching a negotiated settlement.

The substantive implications of these results are discussed in detail in the introduction and are not repeated here. We merely note that in addition to their implications for the mechanisms by which uncertainty leads to war, our results also have potential policy implications for conflict resolution/prevention strategies. There is a growing literature that examines how third party actors, such as international organizations or foreign mediators, can help resolve or prevent civil or international conflict. Based in part on Fearon's (1995) work on rationalist explanations for war, a lot of this work examines how third-party actors can help disputants overcome informational asymmetries or make credible commitments, Fearon's two primary rationalist explanations (e.g., Kydd 2003; Walter 2002).

Our analysis suggests additional methods, if perfect information revelation cannot be achieved. First, third-party actors can help the disputants recognize the possibility of making counteroffers, if previous offers are not acceptable (or, if possible, to actually structure the bargaining process so that it has an offer–counteroffer format). In a TILI offer bargaining setting, or in a setting in which the satisfied side makes all of the offers, the risk-return tradeoff is the unique equilibrium outcome under incomplete information. As we have shown here, however, in an offer–counteroffer bargaining setting, non-risk equilibria also exist as long as the dissatisfied side attaches sufficient value to future payoffs.

Second, third-party actors can help the disputants to have a longer time horizon (i.e., be less impatient). Perhaps the most practical way of doing this is to structure the bargaining so as to reduce the time between counteroffers, or between negotiating rounds (which effectively increases the actors' discount factors).[15]

[13] There are other types of risk-return tradeoffs that can arise in crisis bargaining (we thank an anonymous reviewer for pointing this out). For example, there is a risk-return tradeoff in choosing whether to generate audience costs to signal resolve: on the one hand, it might get the other side to concede; on the other hand, it might lock one or both sides into war (Fearon 1997; Kurizaki 2007). In this article, we focus on the risk-return tradeoff of making a limited initial offer.

[14] In Powell's (2004) bargaining-while-fighting model, which is the other model (besides his costly lottery model, which we have modified here) in which the uniqueness of the risk-return tradeoff emerges endogenously rather than being built into the game-tree, the satisfied state has all of the bargaining leverage because only it is allowed to make offers.

[15] Our finding that high discount factors help lead to cooperative outcomes is consistent with the literature on the repeated prisoner's dilemma (Axelrod 1984).

Finally, third-party actors should emphasize agreements that give *both* sides some of the gains from avoiding war. In an interesting article, Kydd (2003) uses a game-theoretic model to show that if conflict occurs because of private information and incentives to misrepresent it, then biased mediators reduce the probability of conflict more than unbiased mediators. Kydd builds on Fearon's (1995) take-it-or-leave-it-offer bargaining model to establish his argument. We have shown here that in a more general bargaining setting, if impatience is not a factor, then incomplete information only leads to war if the dissatisfied side does not expect to get *any* of the gains from avoiding war in any future agreement reached. By placing emphasis on agreements that give both sides some of the gains from avoiding war, the likelihood of conflict is reduced, and it seems that unbiased mediators would be better willing/able to play this role.[16] Exploring these issues in more detail is an obvious area for future research.

Appendix

Stationary SPE in the Modified Model

Propositions 1–3 characterize SPE when D is dissatisfied (i.e., $q < p - c_D$) and δ_D is low, medium, or high. These SPE are stationary up to the point that, at certain decision nodes, an actor may be indifferent among different courses of actions, and hence may be choosing different actions at histories which are different but which lead to structurally identical subgames. When δ_D is low or medium, as in Propositions 1 and 2, we believe, but have not been able to prove, that these are all of the SPE. When δ_D is large, as in Proposition 3, then there exists genuinely nonstationary SPE, which are described in Proposition 4.

Proposition 1: *If* $\delta_D \leq \frac{(p - c_D) - q}{(p + c_S) - q}$, *then the following are SPE:*

(a) D *always proposes* $(x^*, 1 - x^*)$, *where* $x^* = p + c_S$. *He always accepts any offer* $(y, 1 - y)$ *such that* $y \geq p - c_D$. *In any period in which he gets a lower offer than this, he fights (does not say no). In any period in which* S *rejects his offer, he fights rather than passes.*

(b) S *always proposes* $(y^*, 1 - y^*)$, *where* $y^* = p - c_D$. *She always accepts any offer* $(x, 1 - x)$ *such that* $x \leq p + c_S$. *In any period in which she gets a worse offer, she is indifferent between fighting and saying no (as in the latter case* D *fights anyway), and hence can be choosing either (or mixing). In any period in which* D *rejects her offer, she passes rather than fights.*

Proposition 2: *If* $\frac{(p - c_D) - q}{(p + c_S) - q} \leq \delta_D \leq \frac{(p - c_D) - q}{\delta_D[(p + c_S) - q]}$, *then the following are SPE:*

(a) D *always proposes* $(x^*, 1 - x^*)$, *where* $x^* = p + c_S$. *He always accepts any offer* $(y, 1 - y)$ *such that* $y \geq q(1 - \delta_D) + \delta_D(p + c_S)$. *In any period in which he gets a lower offer than this, he says no (does not fight). In any period in which* S *rejects his offer, he fights rather than passes.*

(b) *In any period in which* S *makes a proposal, she* (i) *proposes* $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ if $\delta_S > \delta_D$, (ii) *proposes some* $y < y^*$ *if* $\delta_S < \delta_D$ *(this proposal is rejected and agreement is reached on* x^* *in the next period), and (iii) is indifferent between proposing* y^* *and proposing some* $y < y^*$ *if* $\delta_S = \delta_D$ *(this latter proposal is rejected and agreement is reached on* x^* *in the next*

[16] Rauchhaus (2006) modifies Kydd's analysis to argue that unbiased mediators will be more effective, and finds some empirical support for this. Kydd (2006) provides further analysis of the tradeoffs between biased and unbiased mediators.

period), and hence can be choosing either (or mixing). She always accepts any offer $(x, 1-x)$ such that $x \leq p + c_S$. In any period in which she gets a worse offer, she is indifferent between fighting and saying no (as in the latter case D *fights anyway), and hence can be choosing either (or mixing). In any period in which* D *rejects her offer, she passes rather than fights.*

Proposition 3: *If* $\frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]} \leq \delta_D$, *then the following are SPE:*

(a) D *always proposes* $(x^*, 1-x^*)$, *where* $x^* = \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$. *He always accepts any offer* $(y, 1-y)$ *such that* $y \geq \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$. *In any period in which he gets a lower offer than this, he says no (does not fight). In any period in which* S *rejects his offer, he fights with probability* $\beta \in (0,1)$ *and passes with probability* $1-\beta$, *where (i)* $\beta = \frac{(1-\delta_S\delta_D)[(p-c_D)-q]}{\delta_D\{\delta_D[(p+c_S)-q]-\delta_S[(p-c_D)-q]\}}$ *if* $\delta_S > \delta_D$, *(ii)* $\beta = \frac{(1-\delta_S^2)[(p-c_D)-q]}{\delta_D^2[(p+c_S)-q]-\delta_S^2[(p-c_D)-q]}$ *if* $\delta_S < \delta_D$, *and (iii)* $\beta = \frac{(1-\delta^2)[(p-c_D)-q]}{\delta^2(c_D+c_S)}$ *if* $\delta_D = \delta_S = \delta$. *(Note that* $\beta \to 1$ *from below as* $\delta_D \to \frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]}$ *from above and that* $\beta \to 0$ *from above as* $\delta_D, \delta_S \to 1$ *from below.)*

(b) *In any period in which* S *makes a proposal, she (i) proposes* $y^* = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$ if $\delta_S > \delta_D$, *(ii) proposes some* $y < y^*$ *if* $\delta_S < \delta_D$ *(this proposal is rejected and agreement is reached on* x^* *in the next period), and (iii) is indifferent between proposing* y^* *and proposing some* $y < y^*$ *if* $\delta_S = \delta_D$ *(this latter proposal is rejected and agreement is reached on* x^* *in the next period), and hence can be choosing either (or mixing). She always accepts any offer* $(x, 1-x)$ *such that* $x \leq \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$. *In any period in which she gets a worse offer, she says no (does not fight). In any period in which* D *rejects her offer, she passes rather than fights.*

Intuition Behind Figure 6

The intuition behind Figure 6, which is based on Propositions 1–3, is as follows. When δ is low, each side's proposal just offers the other side its payoff from war; whoever gets to make the first proposal gets *all* of the gains from avoiding war. Note that in this range of δ, *D* finds it optimal to fight rather than move to the next period, if his proposal is rejected or if *S* makes a small offer. Because *D* optimally chooses to fight if given a low offer, he cannot credibly demand more than his payoff from war when *S* is making a proposal, and hence *S* just needs to offer *D* his utility-equivalent from war. Similarly, because *D* chooses to fight if *S* rejects his offer, *S* cannot credibly be demanding more than her payoff from war when *D* is making a proposal, and so *D* just offers *S* her utility-equivalent from war.

When δ gets in the medium range, however, then *D* cares enough about future payoffs that he prefers to move to the next period (and get all of the gains from avoiding war therein) if *S* makes a low offer, rather than go to war. Therefore, in periods where *S* makes an offer, *D* is now demanding his average payoff for getting the status quo in the current period and all of the gains from avoiding war in the next period. As this average payoff is now (with δ high enough) greater than his payoff from war, *D* is demanding more than his payoff-equivalent from war, and so *S* now has to compromise when she makes a proposal. Thus, as seen in the figure, y^* (*S*'s proposal for *D*) is increasing in this range. However, *S* still gets some of the gains from avoiding war when she makes a proposal, for example, $y^* < p + c_S$. In this range, δ is still low enough that D prefers to go to war if S rejects his offer (because D gets the relatively small payoff of y^* in the next period, and with sufficient discounting war is preferred to living with the status quo in the current period and not getting some of the gains from avoiding war y^* until the next period), and so S cannot credibly demand more than her utility-equivalent from war. Therefore, D is still getting all of the gains from avoiding war when he makes a proposal (i.e., $x^* = p + c_S$).

When δ gets even larger, however, then the credibility of D's threat to go to war if S rejects D's offer starts diminishing. This is because D will get at least some of the gains from avoiding war in the next period (recall that $y^* > p - c_D$), and hence for δ large enough, D prefers to live with the status quo in the current period and get some of the gains from avoiding war y^* in the next period rather than going to war in the current period, if S rejects his offer. This diminished credibility means that S can be demanding more than just her payoff-equivalent from war, and hence D starts compromising when he makes a proposal (i.e., x^* starts decreasing). And because D gets a smaller payoff in periods in which he makes a proposal, S can offer less to D in periods where she makes a proposal (and, hence, y^* also starts decreasing). In this (high) range of δ, D is probabilistically choosing between fighting and passing if S rejects his proposal, and as δ increases, the probability with which D chooses to fight (β in Proposition 3) decreases. As δ approaches 1, the probability of fighting approaches 0.

Delay Under Complete Information

Figure 6 shows the stationary equilibrium proposals when the two sides have the same discount factor, for example, when $\delta_D = \delta_S = \delta$. However, now suppose that the discount factors can differ. Figure 6 then shows D's equilibrium proposal for itself. x^* (which is accepted), and y^* in the figure is now the minimal demand made by D in periods in which S makes an offer, but not necessarily the *actual* offer that S makes (this depends on the value of δ_S relative to δ_D, as explained below). δ on the horizontal axis should now read δ_D. When δ_D is low, then in equilibrium, D goes to war if S makes a low offer, and hence S always makes the minimally acceptable offer y^* (regardless of the value of δ_S; this is all established in Proposition 1). However, once δ_D gets in the medium or high range, then D says no if S makes a low offer, rather than going to war, for the reasons discussed above. In the medium or high range, if $\delta_S > \delta_D$, then S makes the minimally acceptable offer y^* whenever she makes a proposal. However, if $\delta_S < \delta_D$, thus, if S discounts future payoffs sufficiently, then S proposes some $y < y^*$, which is peacefully rejected, and agreement is reached on x^* in the next period.[17] What is the intuition behind this delaying behavior by S?[18] S prefers the agreement y^* to x^* (see Figure 6), but she prefers the status quo even more. If S is sufficiently myopic, then she cares primarily about her payoff in the current period, and hence she would rather get the very favorable status quo for one more period, even though that means that in the long run, she lives with a less favorable agreement (x^*) than the one that she could have gotten (y^*). Even in complete information crisis bargaining, a myopic satisfied state may delay reaching an agreement and ultimately accepts a less favorable agreement in order to enjoy the benefits of the very favorable status quo for a little longer. However, this can only occur if the dissatisfied state attaches enough value to

[17] If $\delta_S = \delta_D$, then S is indifferent between offering y^* and some $y < y^*$, and hence can be choosing either, or mixing. This is all established in Propositions 2 and 3.

[18] Langlois and Langlois (2006) also find that a delayed agreement can be reached in equilibrium under complete information.

future payoffs that it accepts that delay rather than going to war immediately. Thus, if S makes a low offer, δ_D has to be in the medium or high range.

Nonstationary SPE when δ_D is High

Proposition 4: *If* $\frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]} \leq \delta_D$ *(the same condition for Proposition 3) and* $\delta_S = \delta_D$, *then, in the model where* D *makes the first offer, any agreement that gives* D *between* $p-c_D$ *and* $p + c_S$ *can be reached in the first period of a nonstationary SPE, and in the model where* S *makes the first offer, any agreement that gives* D *between* $p-c_D$ *and* $q(1-\delta_D) + \delta_D(p + c_S)$ *(this is the linearly increasing portion of the dashed line in Figure 6) can be reached in the first period of a nonstationary SPE.*[19]

To understand how these nonstationary SPE are generated, note that, when δ_D is high, then in the stationary SPE of Proposition 3, D is mixing (probabilistically choosing) between fighting and passing, at decision nodes at which S has said no to D's offer. D can be mixing because he is indifferent (given the equilibrium strategies for the rest of the game), and hence can be choosing either option. In the stationary SPE of Proposition 3, he is randomizing with the same probability (fight with probability β, pass with probability $1-\beta$) at *each* such decision node. However, suppose that at the *first* such decision node, he chooses to fight with certainty (which is fine, because he is indifferent), and then uses the stationary strategy (of Proposition 3) in subsequent periods. Because D chooses to fight with certainty if his first offer is rejected, S can credibly demand no more than her payoff from war in that period, and hence D gets all of the gains from avoiding war. This is the upper bound of what D can achieve in a nonstationary SPE when δ_D is in the high range. On the other hand, suppose that D chooses to *pass* with certainty at the first such decision node (again, this is fine, as he is indifferent), and then uses the stationary strategy in subsequent periods. This allows S to demand a lot more in that period, and hence D's proposal for itself is lower. This establishes the lower bound of what D can achieve in a nonstationary SPE when δ_D is in the high range. Any payoff between these two extreme values can also be achieved in a nonstationary SPE, by supposing that at the first such decision node, D chooses to fight with the appropriate probability, and then reverts to the stationary strategy for the rest of the game.

SPE in Powell's Model

The following proposition characterizes SPE in a generalization of Powell's (1996a, 1996b, 1999) model in which D and S are allowed to have different discount factors, δ_D and δ_S. The SPE that Powell (1996a, 263) characterizes is the one in which $\delta_S = \delta_D$, and D chooses to always satisfy S's minimal demand when making a proposal.

Proposition 10: *The following are SPE when* D *is dissatisfied, when* $q < p-c_D$.

(a) *In any period in which* D *makes a proposal, he (i) proposes* $x^* = q(1-\delta_S) + \delta_S(p-c_D)$ if $\delta_D < \delta_S$, *(ii) proposes some* $x > x^*$ *if* $\delta_D > \delta_S$ *(this proposal is rejected and agreement is reached on* y^* *in the next period), and (iii) is indifferent between proposing* x^* *and proposing some* $x > x^*$ *if* $\delta_D = \delta_S$ *(this*

[19] In the technical supplement to this article, we show that nonstationary SPE exist regardless of the value of δ_S, and in Proposition 4 we stipulate that $\delta_S = \delta_D$ simply so that we can give precise upper and lower bounds to the payoffs that can be achieved in the nonstationary SPE.

latter proposal is rejected and agreement is reached on y^* in the next period), and hence can be choosing either (or mixing). He always accepts any offer $(y, 1-y)$ such that $y \geq p-c_D$. In any period in which he gets a lower offer than this, he fights (does not continue to the next period).

(b) S *always proposes $(y^*, 1-y^*)$ where $y^* = p-c_D$, and always accepts any offer $(x, 1-x)$ such that $x \leq q(1-\delta_S) + \delta_S(p-c_D)$. In any period in which she gets a worse offer than this, she continues to the next period (does not fight).*

References

Axelrod, Robert. 1984. *The Evolution of Cooperation.* New York: Basic Books.

Baron, David P., and John A. Ferejohn. 1989. ''Bargaining in Legislatures.'' *American Political Science Review* 83 (4): 1181–1206.

Barraclough, Geoffrey. 1982. *From Agadir to Armageddon: Anatomy of a Crisis.* London: Weidenfeld and Nicolson.

Blainey, Geoffrey. 1988. *The Causes of War*, 3rd ed. New York: Free Press.

Bueno de Mesquita, Bruce, and David Lalman. 1992. *War and Reason.* New Haven, CT: Yale University Press.

Fearon, James D. 1994. ''Domestic Political Audiences and the Escalation of International Disputes.'' *American Political Science Review* 88 (3): 577–592.

Fearon, James D. 1995. ''Rationalist Explanations for War.'' *International Organization* 49 (3): 379–414.

Fearon, James D. 1997. ''Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs.'' *Journal of Conflict Resolution* 41 (1): 68–90.

Filson, Darren, and Suzanne Werner. 2002. ''A Bargaining Model of War and Peace.'' *American Journal of Political Science* 46 (4): 819–838.

Jervis, Robert. 1970. *The Logic of Images in International Relations.* Princeton, NJ: Princeton University Press.

Jervis, Robert. 1976. *Perception and Misperception in International Politics.* Princeton, NJ: Princeton University Press.

Kurizaki, Shuhei. 2007. ''Efficient Secrecy: Public versus Private Threats in Crisis Diplomacy.'' *American Political Science Review* 101 (3): 543–558.

Kydd, Andrew. 2003. ''Which Side Are You On? Bias, Credibility, and Mediation.'' *American Journal of Political Science* 47: 597–611.

Kydd, Andrew. 2006. ''When Can Mediators Build Trust?'' *American Political Science Review* 100 (3): 449–462.

Langlois, Catherine C., and Jean-Pierre P. Langlois. 2006. ''When Fully Informed States Make Good the Threat of War.'' *British Journal of Political Science* 36: 645–669.

Lowe, John. 1994. *The Great Powers, Imperialism and the German Problem, 1865–1925.* London and New York: Routledge.

Morrow, James D. 1989. ''Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining.'' *American Journal of Political Science* 33 (4): 941–972.

Powell, Robert. 1996a. ''Bargaining in the Shadow of Power.'' *Games and Economic Behavior* 15 (August): 255–289.

Powell, Robert. 1996b. ''Stability and the Distribution of Power.'' *World Politics* 48 (January): 239–267.

Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics.* Princeton, NJ: Princeton University Press.

Powell, Robert. 2002. ''Bargaining Theory and International Conflict.'' *Annual Review of Political Science* 5 (June): 1–30.

Powell, Robert. 2004. ''Bargaining and Learning While Fighting.'' *American Journal of Political Science* 48 (2): 344–361.

Rauchhaus, Robert. 2006. ''Asymmetric Information, Mediation, and Conflict Management.'' *World Politics* 58 (1): 207–241.

Romer, Thomas, and Howard Rosenthal. 1978. ''Political Resource Allocation, Controlled Agendas, and the Status-quo.'' *Public Choice* 33: 27–43.

Rubinstein, Ariel. 1982. ''Perfect Equilibrium in a Bargaining Model.'' *Econometrica* 50 (1): 79–109.

Sang-Hun, Choe. 2007. ''North Korea Offers Nuclear Concessions.'' *New York Times,* July 18.

SARTORI, ANNE E. 2002. ''The Might of the Pen: A Reputational Theory of Communication in International Disputes.'' *International Organization* 56 (1): 121–149.

SCHELLING, THOMAS C. 1960. *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

SCHULTZ, KENNETH A. 1998. ''Domestic Opposition and Signaling in International Crises.'' *American Political Science Review* 92 (4): 829–844.

SLANTCHEV, BRANISLAV. 2003a. ''The Power to Hurt: Costly Conflict with Completely Informed States.'' *American Political Science Review* 97 (1): 123–133.

SLANTCHEV, BRANISLAV. 2003b. ''The Principle of Convergence in Wartime Negotiations.'' *American Political Science Review* 97 (4): 621–632.

SLANTCHEV, BRANISLAV. 2005. ''Military Coercion in Interstate Crises.'' *American Political Science Review* 99 (4): 533–547.

SMITH, ALASTAIR. 1996. ''Diversionary Foreign Policy in Democratic Systems.'' *International Studies Quarterly* 40 (1): 133–153.

SMITH, ALASTAIR, AND ALLAN C. STAM. 2004. ''Bargaining and the Nature of War.'' *Journal of Conflict Resolution* 48 (6): 783–813.

TARAR, AHMER. 2006. ''Diversionary Incentives and the Bargaining Approach to War.'' *International Studies Quarterly* 50 (1): 169–188.

VAN EVERA, STEPHEN. 1999. *Causes of War: Power and the Roots of Conflict*. Ithaca, NY: Cornell University Press.

WAGNER, R. HARRISON. 2000. ''Bargaining and War.'' *American Journal of Political Science* 44 (3): 469–484.

WALTER, BARBARA F. 2002. *Committing to Peace: The Successful Settlement of Civil Wars*. Princeton: Princeton University Press.

WALTZ, KENNETH. 1979. *Theory of International Politics*. New York: McGraw-Hill.

WITTMAN, DONALD. 1979. ''How a War Ends: A Rational Model Approach.'' *Journal of Conflict Resolution* 23 (4): 743–763.

Technical Supplement (proofs of propositions) to "Does Private Information Lead to Delay or War in Crisis Bargaining?"

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1 Proofs of Propositions

1.1 SPE in Powell's Model

The following proposition characterizes SPE in a generalization of Powell's (1996a, 1996b, 1999: Chapter 3) model in which D and S are allowed to have different discount factors, δ_D and δ_S. The SPE that Powell (1996a, 263) characterizes is the one in which $\delta_S = \delta_D$ and D chooses to always satisfy S's minimal demand when making a proposal.

Proposition 10 *The following are SPE when D is dissatisfied, i.e., when $q < p - c_D$.*
(a) In any period in which D makes a proposal, he (i) proposes $x^* = q(1-\delta_S) + \delta_S(p - c_D)$ if $\delta_D < \delta_S$, (ii) proposes some $x > x^*$ if $\delta_D > \delta_S$ (this proposal is rejected and agreement is reached on y^* in the next period), and (iii) is indifferent between proposing x^* and proposing some $x > x^*$ if $\delta_D = \delta_S$ (this latter proposal is rejected and agreement is reached on y^* in the next period), and hence can be choosing either (or mixing). He always accepts any offer $(y, 1-y)$ such that $y \geq p - c_D$. In any period in which he gets a lower offer than this, he fights (does not continue to the next period).
(b) S always proposes $(y^*, 1-y^*)$ where $y^* = p - c_D$, and always accepts any offer $(x, 1-x)$ such that $x \leq q(1-\delta_S) + \delta_S(p - c_D)$. In any period in which she gets a worse offer than this, she continues to the next period (does not fight).

Proof: First consider S's decisions. Given D's acceptance rule, S is strictly best off proposing $y^* = p - c_D$ whenever she makes a proposal (if S proposes a lower y, D chooses war, which is strictly worse for S). Now consider periods in which D makes an offer. We have just shown that S's optimal continuation value for moving to the next period is $(1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}$. If she goes to war instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. It is easy to show that the former is strictly greater than the latter, and hence S cannot credibly reject any proposal $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq (1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}$, or $x \leq q(1-\delta_S) + \delta_S(p - c_D)$, and she must move to the next period rather than fight if she gets a worse offer.

Now consider D's decisions. Given S's acceptance rule, the best (for himself) acceptable proposal that D can make in a period in which he makes a proposal is $x^* = q(1-\delta_S) + \delta_S(p - c_D)$. (Because this is strictly greater than q, if that agreement will eventually be reached, it is strictly better to reach it now rather than in a later period.) D's other option is to propose some $x > x^*$, which is rejected, and in the next period, S proposes $y^* = p - c_D$, which D can (i) accept, (ii) reject it and fight, or (iii) reject it and make a counteroffer. In case (iii), D is back in the same scenario that he is in the current period, except that he has received q for two periods. Since $x^* > q$, offering x^* in the current period is strictly better than ending up in case (iii). Cases (i) and (ii) give D the same total payoff of $q + \frac{\delta_D(p-c_D)}{1-\delta_D}$. Therefore, D is strictly best off offering x^* in the current period rather than some $x > x^*$ if and only if $\frac{x^*}{1-\delta_D} > q + \frac{\delta_D(p-c_D)}{1-\delta_D}$, which can be shown to hold if and only if $\delta_D < \delta_S$. If $\delta_D > \delta_S$, then D strictly prefer to offer some $x > x^*$ (which is rejected and leads to agreement being reached on $y^* = p - c_D$ in the next period), and if $\delta_D = \delta_S$, then D is indifferent between proposing x^* and some $x > x^*$, and hence can be choosing either, or mixing.

Now consider a period in which S makes an offer. If D chooses to fight upon receiving a low offer, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to move to the next period instead, he either (depending on the relative values of δ_S and δ_D) (i) finds it optimal to offer x^* (if $\delta_D \leq \delta_S$), which is accepted, or (ii) finds it optimal to propose some $x > x^*$ (if $\delta_D \geq \delta_S$), which is rejected. In case (ii), D is back in the same position that he is in the current period (in which S makes an offer), except that he has received q for two periods, and he can get at best $y^* = p - c_D$ in the period that he now finds himself in. Since $p - c_D > q$ and S's strategy does not change, if case (ii) holds, D is strictly better fighting rather than saying no if gets a low offer in the current period, and therefore he cannot credibly reject any offer $(y, 1-y)$ such that $y \geq p - c_D$. If case (i) holds, then D's optimal continuation value for moving to the next period is $q + \frac{\delta_D x^*}{1-\delta_D}$. It is easy to show that $\frac{p-c_D}{1-\delta_D}$ is strictly greater than this, and hence D cannot credibly reject any proposal $(y, 1-y)$ such that $y \geq p - c_D$, and must choose to fight if he gets a lower offer. Q.E.D.

1.2 Proof of Proposition 1

First consider D's decisions. Given S's acceptance rule, D is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal, as this is the best possible payoff he can effectively get in the model (given that S can choose to fight instead of accepting a worse offer, and if

S chooses to fight, D's payoff is $p - c_D$, which is strictly worse than $p + c_S$). Now consider a period in which S makes an offer. If she makes a low offer and D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to say no instead, we have just shown that his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that if D says no, S chooses to pass rather than fight). For the upper bound on δ_D in this equilibrium, the former is greater than the latter, and hence D cannot credibly reject any offer $(y, 1-y)$ such that $y \geq p - c_D$, and must be choosing to fight rather than say no if he gets a worse offer. Now suppose D has made an offer and S has said no. If D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to pass instead, we have just shown that his payoff is $q + \frac{\delta_D y^*}{1-\delta_D}$. The former is strictly greater than the latter, and hence D must be choosing to fight.

Now consider S's decisions. Given D's acceptance rule, S is strictly best off proposing $y^* = p - c_D$ (if S makes a lower offer, D chooses to fight, in which case S is strictly worse off). Now consider a period in which D makes an offer. Since D is choosing to fight if S says no to his offer, S cannot credibly reject any offer than gives her at least her utility from war, i.e., she cannot credibly reject any offer $(x, 1-x)$ such that $x \leq p + c_S$. If she gets a worse offer, she is indifferent between fighting and saying no, since in the latter case, D chooses to fight anyway. Therefore, she can be doing either, or she can be mixing. Now suppose S has made an offer and D has said no. If S chooses to fight, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. If she chooses to pass instead, we have just shown that her payoff is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. The latter is strictly greater than the former, and hence S must be choosing to pass. Q.E.D.

1.3 Proof of Proposition 2

First consider D's decisions. The same argument as above shows that D is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal, given S's acceptance rule. Now consider a period in which S makes an offer. If she makes a low offer and D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to say no instead, we have just shown that his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that if D says no, S chooses to pass rather than fight). For the lower bound on δ_D in this equilibrium, the latter is greater than the former, and hence D cannot credibly reject any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq q(1-\delta_D) + \delta_D(p + c_S)$, and must choose to say no rather than fight if he gets a worse offer. Now suppose D has made an offer and S has said no. If D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to pass instead, his payoff is $q + \frac{\delta_D y^*}{1-\delta_D}$. (This is based on the assumption that S proposes y^* in the next period. As we show below, depending on the size of δ_S relative to δ_D, S may or may not find it optimal to offer y^* in the next period — however, whatever S chooses to do, D's average per-period payoff in the subgame beginning in the next period is y^* (this is because y^* is the offer that makes him just indifferent between accepting it and moving to the next period and reaching agreement on x^* therein), and hence this argument is fine.) For the upper bound on δ_D in this equilibrium, the former is greater than the latter, and hence D chooses to fight rather than pass.

Now consider S's decisions. S cannot credibly reject any offer that gives her at least her utility from war, i.e., any offer $(x, 1-x)$ such that $x \leq p + c_S$. This is because D chooses to fight if S says no to his offer. If S gets a worse offer, she is indifferent between fighting and saying no (since in the latter case D fights anyway), and hence can be doing either, or mixing. Now suppose D has said no to S's offer. We have just shown that S's continuation value for passing is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. This is strictly greater than her payoff $\frac{1-p-c_S}{1-\delta_S}$ for fighting, and hence S must be passing rather than fighting. Now consider periods in which S makes an offer. Given D's acceptance rule, the best possible (for herself) acceptable agreement that S can propose in the current period is $y^* = q(1-\delta_D) + \delta_D(p + c_S)$, for a total payoff of $\frac{1-y^*}{1-\delta_S}$. Setting this greater than her payoff for proposing a lower y that is rejected and leads to agreement being reached in the next period, $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$, and simplifying, we obtain $\delta_S > \delta_D$. Hence, if $\delta_S > \delta_D$, S is strictly best off proposing y^*. If $\delta_S < \delta_D$, S is strictly best off proposing some $y < y^*$ which is rejected and leads to agreement being reached on x^* in the next period. If $\delta_S = \delta_D$, then S is indifferent between proposing y^* and some $y < y^*$, and hence can be choosing either, or mixing. Q.E.D.

1.4 Proof of Proposition 3

Note that in this proof, we use the "one-stage-deviation principle," henceforth OSDP, for infinite horizon games with discounting of future payoffs (Fudenberg and Tirole 1991, 108-110). This principle states that, to verify that a profile of strategies comprises a SPE, one just has to verify that, given the other players' strategies, no player can improve her payoff at any history at which it is her turn to move by deviating from her equilibrium strategy at that history and then reverting to her equilibrium strategy afterwards.

We want to look for a SPE in which D is mixing between passing and fighting, at any decision node at which S has said no to D's offer.[1] Suppose that in this (supposed) SPE, D's average per-period payoff for the subgame beginning in the next period (in which S makes an offer) is y'. Then, for mixing to be okay, it must be the case that D is indifferent between fighting and passing, i.e., $\frac{p-c_D}{1-\delta_D} = q + \frac{\delta_D y'}{1-\delta_D}$, or $y' = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$. It is easy to verify that, given S's strategy and D's strategy for the rest of the game, D's average per-period payoff for the subgame beginning in the next period is indeed y', and hence D's strategy of mixing at this stage satisfies the OSDP. Therefore, suppose that D is choosing to fight with some probability $\beta \in (0,1)$ and pass with probability $1-\beta$.

Now consider when D has to make a proposal. Given S's acceptance rule, the best D can do if he wants an agreement to be reached in the current period is to propose $x^* = \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$. If he proposes some bigger x, S says no and D's expected payoff (if he then uses his equilibrium strategy for the rest of the game) is $\beta(\frac{p-c_D}{1-\delta_D}) + (1-\beta)[q + \frac{\delta_D y'}{1-\delta_D}]$. Setting $\frac{x^*}{1-\delta_D}$ strictly greater than the latter and simplifying, we obtain $q < p - c_D$, which is true (note that β drops out of the simplification, so this is true for any value of β). Therefore, D cannot profitably deviate from proposing x^*, and then revert to his equilibrium strategy, and hence D's strategy satisfies the OSDP at histories at which D makes a proposal.

Now suppose S has just made an offer to D. If D fights, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he says no instead, then, according to his equilibrium strategy for the rest of the game, his overall payoff will be $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that S's strategy is to pass if D says no, and so the next period will be reached). Setting the latter strictly greater than the former and simplifying,

[1] A natural way to establish continuity with Propositions 1 and 2 would be if, when δ_D is high, as in Proposition 3, D chooses to pass with certainty even when S rejects his offer. In Proposition 2, δ_D is high enough that D chooses to say no (rather than fight) if S makes too low an offer, but still low enough that he prefers to fight if S rejects D's offer (his continuation value for moving to the next period is higher in the former case than in the latter, since in the former case he gets to make the proposal in the next period). So, it would be natural to expect that when δ_D is even higher, as in Proposition 3, D would choose to pass with certainty even when S rejects his offer. However, it turns out that this behavior cannot be supported as part of a stationary SPE, and instead D starts passing with positive probability, and this probability begins from zero and approaches one as the players' discount factors approach one. So, Proposition 3 *is* a natural continuation of Propositions 1 and 2, but uses a mixed strategy.

we obtain $q < p - c_D$, which is true. Therefore, D is strictly better off saying no rather than fighting, if S's offer is too small. Therefore, he cannot do any better (assuming that he uses his equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, and say no (rather than fight) if he gets a lower offer.[2] We have thus verified that D's strategy satisfies the OSDP, i.e., there exists no history at which D can profitably deviate from his equilibrium strategy at that stage and then revert back to his equilibrium strategy. Now we have to verify that the same is true for S.

Suppose D has just said no to S's offer. If S fights, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. If she passes instead and follows her equilibrium strategy in the future, her payoff is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Setting the latter strictly greater than the former and simplifying, we obtain $\delta_D^2 > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$, and hence S's strategy satisfies the OSDP at this stage.

Now consider periods in which S makes a proposal. Given D's acceptance rule, the most favorable (for herself) acceptable agreement that S can propose is $y^* = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, leaving her with an overall payoff of $\frac{1-y^*}{1-\delta_S}$. If she instead proposes some $y < y^*$, D rejects it and agreement is reached on x^* in the next period (assuming that S follows her equilibrium strategy), giving S an overall payoff of $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_S > \delta_D$. Hence, (i) when $\delta_S > \delta_D$, S is strictly better off proposing y^* rather than doing something else and then reverting to her equilibrium strategy, (ii) when $\delta_S < \delta_D$, she is strictly better off proposing some $y < y^*$ rather than doing something else and then reverting to her equilibrium strategy, and (iii) when $\delta_S = \delta_D$, she is indifferent between proposing y^* and some $y < y^*$, and hence can be choosing either, or mixing. (And, this is strictly preferred to doing something else, namely proposing some $y > y^*$, and then reverting to her equilibrium strategy, although the latter point is moot because the agreement will be accepted and the game will end.) Therefore, we have verified that S's strategy satisfies the OSDP at histories at which she makes a proposal.

[2] Note that the only acceptance rule which is as good as this one for *any* offer by S, i.e., at *any* history at which S has just made an offer to D, is to accept any proposal $(y, 1-y)$ such that $y > \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, and say no (rather than fight) if he gets a lower offer.

Finally, we need to verify that S's acceptance rule satisfies the OSDP. We consider the three cases in turn.

Case (i): $\delta_S > \delta_D$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, in the next period, agreement would be reached on y^* (since $\delta_S > \delta_D$). Therefore, in the current period, S's continuation value for saying no (if she uses her equilibrium strategy in the future) is $\beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\frac{\delta_S(1-y^*)}{1-\delta_S}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_D > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\frac{\delta_S(1-y^*)}{1-\delta_S}]$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition (namely, $x \leq \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$) and solving for β, we obtain $\beta = \frac{(1-\delta_S\delta_D)[(p-c_D)-q]}{\delta_D\{\delta_D[(p+c_S)-q]-\delta_S[(p-c_D)-q]\}} \in (0,1)$. That is, when β takes on this value, S's acceptance rule as described in the proposition satisfies the OSDP at any history at which D has just made an offer to S. Note that $\beta \to 1$ (from below) as $\delta_D \to \frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]}$ (from above). That is, this equilibrium converges to that of Proposition 2. Also note that our requirement in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ means that $\beta > 0$ always. As $\delta_S, \delta_D \to 1$ (from below), $\beta \to 0$ (from above).

Case (ii): $\delta_S < \delta_D$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, in the next period, S will propose some $y < y^*$, which D rejects, and agreement will be reached on x^* in the following period. Therefore, in the current period, S's continuation value for saying no (if she uses her equilibrium strategy in the future) is $\beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\delta_S(1-q)+\frac{\delta_S^2(1-x^*)}{1-\delta_S}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_D^2 > \frac{\delta_S^2[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \beta[\frac{1-p-c_S}{1-\delta_S}] + (1-\beta)[(1-q) + \delta_S(1-q) + \frac{\delta_S^2(1-x^*)}{1-\delta_S}]$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition and solving for β, we obtain $\beta = \frac{(1-\delta_S^2)[(p-c_D)-q]}{\delta_D^2[(p+c_S)-q]-\delta_S^2[(p-c_D)-q]} \in (0,1)$. Note that $\beta \to 1$ (from below) as $\delta_D \to \frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]}$ (from above). That is, this equilibrium converges to that of Proposition 2. Also note that $\beta > 0$ always, since $\delta_D > \delta_S$. As $\delta_S, \delta_D \to 1$ (from below), $\beta \to 0$ (from above).

Case (iii): $\delta_S = \delta_D = \delta$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, S's continuation value for saying no in the current period (regardless of whether she chooses to propose y^* or some $y < y^*$, or mix, in the next period) is $\beta[\frac{1-p-c_S}{1-\delta}]+(1-\beta)[(1-q) + \frac{\delta(1-y^*)}{1-\delta}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta}$. Setting the former strictly greater than the latter and simplifying, we obtain $c_S + c_D > 0$, which is true. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta} \geq \beta[\frac{1-p-c_S}{1-\delta}] + (1-\beta)[(1-q) + \frac{\delta(1-y^*)}{1-\delta}]$, or $x \leq \beta(p+c_S) + (1-\beta)(p-c_D)$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition and solving for β, we obtain $\beta = \frac{(1-\delta^2)[(p-c_D)-q]}{\delta^2(c_D+c_S)} \in (0,1)$. Note than $\beta \to 1$ (from below) as $\delta \to \frac{(p-c_D)-q}{\delta[(p+c_S)-q]}$ (from above), and hence this equilibrium converges to that of Proposition 2. Also note that $\beta \to 0$ (from above) as $\delta \to 1$ (from below).

Therefore, we have verified that D and S's strategies satisfy the OSDP at any history at which it is their turn to move. Q.E.D.

1.5 Proof of Proposition 4

The SPE characterized in Proposition 3 are stationary, except that when $\delta_S \leq \delta_D$, S can be choosing different actions (among which she is indifferent) at different histories (but that lead to structurally identical subgames) at which it is her turn to make an offer, and this allows for non-stationarity (but D and S's payoffs are the same in all of these SPE).

It turns out that when δ_D is high, there are also SPE that are non-stationary in a more genuine sense. Suppose that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$, so that Proposition 3 holds. Consider the model in which D makes the first offer. Suppose that, in the subgame beginning in the second period, D and S use the strategies of Proposition 3, which we already know are best responses to each other. Then, in the last decision node of the first period, D is indifferent between passing and fighting, and so suppose D is choosing to fight with certainty (as opposed to fighting with probability β, as in Proposition 3). Then, in the first period, S's acceptance rule must be to accept any proposal $(x, 1-x)$ such that $x \leq p + c_S$, and so in the first period, D optimally proposes $x^* = p + c_S$. Hence, in Figure 6, there exist non-stationary SPE when δ_D is high in which x^* remains at $p + c_S$, rather than gradually decreasing to $p - c_D$, as in the stationary SPE.

In fact, we can suppose that in the last decision node of the first period, D fights with some probability λ and passes with probability $1 - \lambda$. When $\lambda = 1$, we are in the SPE described above, and when $\lambda = \beta$, we are in the stationary SPE of Proposition 3. As λ decreases, D's proposal for himself in the first period, x^*, decreases. When $\delta_S \geq \delta_D$ (so that agreement will be reached on $y^* = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$ in the second period — see Proposition 3), then in the first period, S accepts all agreements $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \lambda[\frac{1-p-c_S}{1-\delta_S}] + (1-\lambda)[(1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}]$, and says no (rather than fight) for any worse offer.

This can be simplified to obtain that in the first period, S accepts all offers $(x, 1-x)$ such that $x \leq x^*$, where $x^* = \lambda[(p + c_S) - q] + \delta_S y^*(1 - \lambda) + q(1 - \delta_S) + \lambda q \delta_S$. When $\lambda = 1$, $x^* = p + c_S$, and when $\lambda = 0$, $x^* = \delta_S y^* + q(1 - \delta_S)$. (And, since x^* is a continuous function of λ, any value of x^* in between these two extreme values can be obtained, for the right value of λ.) $\frac{\partial x^*}{\partial \lambda} > 0$ can be simplified to obtain $\delta_D > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our stipulation that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, what S allows D to keep for himself in the first period is increasing in λ, which makes intuitive sense. (Also note that $\frac{\partial x^*}{\partial \delta_S} = 0$ when $\lambda = 1$ and $\frac{\partial x^*}{\partial \delta_S} > 0$ when $\lambda < 1$.)

Therefore, in the model in which D makes the first offer, there exist non-stationary SPE when $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ (the binding condition of Proposition 3) and $\delta_S \geq \delta_D$ in which D's offer to himself (which S accepts) in the first period ranges from a minimum of $x^* = \delta_S y^* + q(1-\delta_S)$

to a maximum of $x^* = p + c_S$. Note that when $\lambda = 0$ and $\delta_S = \delta_D$, then $x^* = p - c_D$, i.e., D offers himself just his payoff from war. Of course, x^* can never be lower than $p - c_D$ in a SPE.

Now consider the model in which S makes the first offer. Suppose that, beginning in the third period and forever afterwards, both players use the strategies of Proposition 3. Then, in the last stage of the second period, D can be choosing to fight with any probability $\lambda \in [0, 1]$, and hence his payoff in the second period ranges from a minimum of $x^* = \delta_S y^* + q(1 - \delta_S)$ to a maximum of $x^* = p + c_S$. Now consider the first period. If $x^* = p + c_S$ in the second period (i.e., $\lambda = 1$), then our restriction on δ_D means that, in the first period, D must be accepting any offer $(y, 1 - y)$ such that $y \geq q(1 - \delta_D) + \delta_D(p + c_S)$, and saying no (rather than fight) for any lower y. Since we have been assuming that $\delta_S \geq \delta_D$, S chooses to offer $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ in the first period. Thus, there exist non-stationary SPE (in which $\lambda = 1$) when $\delta_S \geq \delta_D$ in which, in Figure 6, the trend continues as δ_D moves from medium to large, i.e., $x^* = p + c_S$ and $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ even when δ_D becomes very large. Note that, in these equilibria, as $\delta_D \to 1$, the agreement reached approaches $p + c_S$, i.e., D gets *all* of the gains from avoiding war, regardless of who gets to make the first proposal.

Now suppose that $\lambda = 0$ and $\delta_S = \delta_D$ (the worse possible case for D). Then, in the second period, D will propose $x^* = p - c_D$, and hence, in the first period, D's acceptance rule must be to accept any offer $(y, 1 - y)$ such that $y \geq p - c_D$, and fight (rather than say no) for any lower y. Thus, S proposes $y^* = p - c_D$, and hence S gets *all* of the gains from avoiding war.

Thus, when $\delta_S \geq \delta_D$ and S makes the first offer, the agreement reached in a non-stationary SPE can range from a minimum (for D) of $y^* = p - c_D$ (when $\lambda = 0$ and $\delta_S = \delta_D$) to a maximum of $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ (when $\lambda = 1$; as $\delta_D \to 1$, this converges to $p + c_S$).

We can generate additional non-stationary SPE. Suppose that in the model in which D makes the first proposal, the two players begin using the strategies of Proposition 3 beginning in the fourth period. Then, in the last stage of the third period, D can be choosing to fight with any probability $\lambda \in [0, 1]$. We can continue to build non-stationary SPE like this. The key is to stipulate that the players eventually start using the strategies of Proposition 3 in some period in which S makes an offer, and forever afterwards. In the last stage of the period just before then, D can be choosing to fight with any probability $\lambda \in [0, 1]$, and the choice of λ uniquely determines what has to occur previous to that stage (i.e., everything before then can be solved using backwards induction). Thus, we can create non-stationary SPE for any value of λ and any number of non-stationary initial periods. Thus, we have a folk-theorem type result when $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ (the binding condition of Proposition 3), in which a whole lot of payoff combinations can be supported as SPE (these payoffs lie between the upper and lower bounds identified earlier, since those bounds are determined by the most and least favorable agreements that D can possibly get in the first period in which he makes a proposal).

1.6 Proof of Proposition 5

We want to construct a PBE in which neither type of D rejects S's initial offer in order to make a counteroffer. Each type accepts all initial offers $(y, 1-y)$ such that y is at least as great as its expected utility from war, and fights (rather than says no) if it gets a lower offer. We also want that if the second period is reached (this is off-the-equilibrium path behavior), the strategies of the players are such that agreement is reached on $x^* = p + c_S$, i.e., D gets all of the gains from avoiding war. First note that if such an agreement were to be reached, then S would be strictly best off passing rather than fighting if D says no to S's initial offer, i.e., $\frac{1-p-c_S}{1-\delta_S} < (1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Then, for type c_{D_l} to be fighting rather than saying no if he gets a low initial offer, it must be that $\frac{p-c_{D_l}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$. This also ensures that type c_{D_l} cannot credibly reject any initial offer $(y, 1-y)$ such that $y \geq p - c_{D_l}$. Similarly, for type c_{D_h}'s acceptance rule to be to accept any initial offer $(y, 1-y)$ such that $y \geq p - c_{D_h}$ and go to war (rather than say no) for a lower y, it must be that $\frac{p-c_{D_h}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$. Since $c_{D_h} > c_{D_l}$, the latter is the binding restriction on δ_D.

Now we need to construct a PBE of the subgame beginning in the second period, which is in never reached in equilibrium, in which agreement is reached on $x^* = p + c_S$. The simplest way to do this is to stipulate that if this subgame is reached, S believes that it is facing type c_{D_l} (the low-cost, or highly resolved, type) with certainty (and that this belief never changes later on), and that S and type c_{D_l} therefore use the complete information strategies of Proposition 1, which are best responses to each other (note that we could also stipulate that S believes she is facing type c_{D_h} with certainty, and this belief never changes; the argument below would require only minor modifications). This requires the binding condition of Proposition 1 to hold (when D's cost of war is c_{D_l}), namely that $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, which is already implied by our binding condition in the previous paragraph.

Now we need to construct a strategy (in this subgame) for type c_{D_h} that is a best response to S's strategy (which is given by Proposition 1). Given S's acceptance rule, type c_{D_h} is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal. Now suppose S has just made a low offer to type c_{D_h}. We have just shown that if type c_{D_h} says no, his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that S's strategy is to pass rather than fight if D says no, and hence the next period will be reached). Given the upper bound on δ_D that we have derived earlier, namely that $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, type c_{D_h}'s payoff from war, $\frac{p-c_{D_h}}{1-\delta_D}$, is greater than this, and hence type c_{D_h}'s acceptance rule must be to always accept any offer $(y, 1-y)$ such that $y \geq p - c_{D_h}$, and fight if he gets a lower offer. Finally, suppose S has said no to type c_{D_h}'s offer. Given S's proposal and the acceptance rule we have just derived for type c_{D_h}, the latter's continuation value for passing is $q + \frac{\delta_D(p-c_{D_l})}{1-\delta_D}$. Given the upper bound on δ_D that we have derived earlier, namely that $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, type c_{D_h}'s payoff from war, $\frac{p-c_{D_h}}{1-\delta_D}$, is strictly greater than this, and hence type c_{D_h} must always be choosing to fight rather than pass. This completes the description of type c_{D_h}'s best response to S's strategy.

All that remains is to specify the optimal offer that S makes in the first period of the game. Given the acceptance rules of types c_{D_l} and c_{D_h}, S's best response is either to make the big offer $y^* = p - c_{D_l}$, which both types accept (and so war is avoided with certainty), or to make the lower offer $y^* = p - c_{D_h}$, which only type c_{D_h} accepts. Type c_{D_l} rejects it and goes to war. It is easy to see that no other proposal can be a best response. If $0 < s < 1$ is the prior probability that D is of type c_{D_l}, then making the big offer is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-(p-c_{D_h})}{1-\delta_S}]$, or $s \geq \frac{c_{D_h}-c_{D_l}}{c_{D_h}+c_S} \in (0,1)$. Q.E.D.

1.7 Proof of Proposition 6

This equilibrium is similar to the previous one in that, if the second period is reached, agreement is reached on $x^* = p+c_S$. However, because we have stipulated in this proposition that $\delta_D \geq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, in the first period, if type c_{D_h} gets a low initial offer, he prefers to move to the second period and get x^* rather than fight. Thus, his acceptance rule in the first period must be to accept any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq q(1-\delta_D)+\delta_D(p+c_S)$, and say no (rather than fight) for any lower y. Because we have stipulated in this proposition that $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, type c_{D_l} prefers to go to war if he gets a low initial offer rather than get x^* in the next period, and so his acceptance must be to accept all initial offers $(y, 1-y)$ such that $y \geq p - c_{D_l}$, and go to war for any lower y. Because agreement will be reached on x^* in the next period, if D says no to S's initial offer, S is strictly better off passing rather than fighting.

If the second period is reached, we stipulate that S believes with certainty that she is facing type c_{D_h}, and this belief never changes. (If the second period is reached on-the-equilibrium path, this belief follows from Bayes' rule, and if it is reached off-the-equilibrium path, we as the analyst stipulate that this is S's belief, since Bayes' rule does not apply. This off-the-equilibrium path belief is quite reasonable, because type c_{D_h}'s payoff from war is lower and hence he is more likely to say no rather than go to war than type c_{D_l}.) Since we have stipulated in this proposition that $\frac{(p-c_{D_h})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, the conditions for Proposition 2 are satisfied (when D is of type c_{D_h}), and hence we stipulate that, beginning in the second period, S and type c_{D_h} play the strategies of Proposition 2, which are best responses to each other. Because S is using the strategy of Proposition 2, agreement will indeed be reached on $x^* = p + c_S$, which we have been assuming. It is easy to construct type c_{D_l}'s best response to S's strategy, in the subgame beginning in the second period.

All that remains is to determine S's optimal offer in the first period. She can either make the large offer $y^* = p - c_{D_l}$, which both types of D accept, or make a smaller offer $y \leq q(1-\delta_D) + \delta_D(p+c_S)$, which type c_{D_l} rejects and goes to war. It is easy to see that no other offer can be a best response. We know from our proof of Proposition 2 that if $\delta_S \geq \delta_D$, then if S prefers to make the small offer, she prefers to offer exactly $y^* = q(1-\delta_D)+\delta_D(p+c_S)$, whereas if $\delta_S \leq \delta_D$, then if S prefers to make the small offer, she prefers to offer some $y < y^*$, so that agreement is reached on x^* in the next period (if D turns out to be type c_{D_h}). We consider the two cases in turn.

If $\delta_S \geq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-q(1-\delta_D)-\delta_D(p+c_S)}{1-\delta_S}]$, or $s \geq \frac{(p-c_{D_l})-[q(1-\delta_D)+\delta_D(p+c_S)]}{(p+c_S)-[q(1-\delta_D)+\delta_D(p+c_S)]} \in [0,1)$.

If $\delta_S \leq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}]$, or $s \geq \frac{(p-c_{D_l})-[q(1-\delta_S)+\delta_S(p+c_S)]}{(p+c_S)-[q(1-\delta_S)+\delta_S(p+c_S)]} \in [0,1)$. Q.E.D.

1.8 Proof of Proposition 7

We want to construct a risk-return tradeoff equilibrium even when δ_D is high. That is, type c_{D_l} goes to war rather than saying no if he gets a low initial offer. We stipulate that if the second period is reached, S believes that it is facing type c_{D_h} with certainty (on-the-equilibrium path, this will follow from Bayes' rule, and off-the-equilibrium path, this is a reasonable stipulation, for the same reason as in the previous proof), and that this belief never changes. Therefore, since we want to allow δ_D to be high, we stipulate that in the subgame beginning in the second period, S and type c_{D_h} use the strategies of Proposition 3, which are best responses to each other if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, which we therefore stipulate to hold in this proposition. Therefore, we know from the proof of Proposition 3 that type c_{D_h}'s acceptance rule in the first period is optimal, given that he expects agreement to be reached on $x^* = \frac{(p-c_{D_h})-q(1-\delta_D^2)}{\delta_D^2}$ if the second period is reached.

Now consider type c_{D_l}'s behavior in the first period. He knows that if the second period is reached, S adopts the strategy of Proposition 3, treating D as if he is of type c_{D_h} (since S believes this with certainty). What is type c_{D_l}'s best response to this (in the subgame beginning in the second period)? It is either to propose $x^* = \frac{(p-c_{D_h})-q(1-\delta_D^2)}{\delta_D^2}$, or to propose some $x > x^*$ which S rejects, and then go to war. (Since type c_{D_h} is indifferent between passing and fighting if S rejects D's offer in the second period, type c_{D_l}, whose payoff from war is strictly higher, strictly prefers to fight rather than pass.) He prefers to offer x^* if $\frac{x^*}{1-\delta_D} \geq \frac{p-c_{D_l}}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$. On the other hand, if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$, then c_{D_l} prefers to instigate war in the second period rather than offer x^*.

Note that $\frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]} > \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$. Therefore, suppose that $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$, so that c_{D_l} is best off instigating war in the subgame beginning in the second period. He is strictly better off going to war in the first period than in the second period, and therefore his acceptance rule in the first period is fine. Now suppose that $\frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]} \geq \delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, so that c_{D_l} is best off proposing x^* in the subgame beginning in the second period. Then, c_{D_l}'s acceptance rule in the first period is fine as long as $\frac{p-c_{D_l}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \geq \frac{(p-c_{D_h})-q}{(p-c_{D_l})-q}$. Therefore, as long as $\delta_D \geq max\{\frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}, \frac{(p-c_{D_h})-q}{(p-c_{D_l})-q}\}$, type c_{D_l}'s acceptance rule in the first period is fine, and therefore we stipulate this to hold in this proposition.

Note that, if D says no to S's initial offer, the proof of Proposition 3 shows that S is strictly better off passing rather than fighting, since she expects agreement to be reached on x^* in the next period.

All that remains is to determine S's optimal offer in the first period. She can either make the large offer $y^* = p - c_{D_l}$, which both types of D accept, or make a smaller offer $y \leq \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$, which type c_{D_l} rejects and goes to war. It is easy to see that no other offer can be a best response. We know from the proof of Proposition 3 that if $\delta_S \geq \delta_D$, then if S prefers to make the small offer, she prefers to offer exactly $y^* = \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$, whereas if $\delta_S \leq \delta_D$, then if S prefers to make the small offer, she prefers to offer some $y < y^*$, so that agreement is reached on x^* in the next period (if D turns out to be type c_{D_h}). We consider the two cases in turn.

If $\delta_S \geq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-\frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}}{1-\delta_S}]$, or $s \geq \frac{[q(1-\delta_D)+\delta_D(p-c_{D_l})]-(p-c_{D_h})}{[q(1-\delta_D)+\delta_D(p+c_S)]-(p-c_{D_h})} \in [0,1)$.

If $\delta_S \leq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}]$, or $s \geq \frac{\delta_D^2[(p-c_{D_l})-q]-\delta_S[(p-c_{D_h})-q]}{\delta_D^2[(p+c_S)-q]-\delta_S[(p-c_{D_h})-q]} \in [0,1)$. Q.E.D.

1.9 Proof of Proposition 8

We want to construct a PBE in which agreement is reached on $x^* = p + c_S$ if the second period is reached, and in which both types of D make counteroffers rather than go to war if

S's initial offer is too small. The natural way to do this is to use the results of Proposition 2, in which agreement is reached on $x^* = p + c_S$ whenever D makes a proposal, and D says no rather than fights, if S makes a small offer. The stipulation in this proposition that $\frac{(p-c_{D_l})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$ means that the conditions of Proposition 2 hold for both types of D, i.e., it means that $\frac{(p-c_{D_l})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_l})-q}{\delta_D[(p+c_S)-q]}$ and $\frac{(p-c_{D_h})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$.

Thus, we simply specify that, beginning from the very first period and continuing forever after, both types of D use the strategy of Proposition 2. Note that this strategy does not depend on D's cost of war in any way, and hence both types are adopting identical strategies. Since they are adopting identical strategies, S's best response is to adopt the strategy of Proposition 2, regardless of the value of s. And, given that S is adopting the strategy of Proposition 2, the best response of both types of D is to use the strategy of Proposition 2.

If the second period is reached on-the-equilibrium path, S's belief will remain at s, by Bayes' rule. If it is reached off-the-equilibrium path, then we specify that S's belief can be anything, and that S as well as both types of D continue to use the strategies of Proposition 2. If the third period is reached (this can only happen off-the-equilibrium path, since both types of D fight if S rejects D's offer in the second period), then the belief can be anything, and everyone continues to use the strategies of Proposition 2, and so on. Q.E.D.

1.10 Proof of Proposition 9

We want to construct a PBE in which both type of D make counteroffers rather than fight, if S's initial offer is too small, and in which δ_D can be very high. Because we want to allow δ_D to be high, we use the results of Proposition 3. In particular, we stipulate that if the third period is reached (it will turn out that this can only happen off-the-equilibrium path), then S believes with certainty that she is facing type c_{D_h} (which, as we have been discussing earlier, is a sensible belief), and this belief never changes. Thus, we stipulate that, beginning in the third period, S and c_{D_h} use the strategies of Proposition 3 (with S treating D as though it is type c_{D_h} with certainty), which are best responses to each other if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, which we therefore stipulate to hold in this proposition. Type c_{D_l}'s best response to S's strategy is easy to construct.

Now consider the last decision node of the second period, in which D has to decide whether to pass or fight. We know from Proposition 3 that type c_{D_h} is indifferent between fighting and passing, since he expects his payoff from Proposition 3 to be obtained in the next period. Since he is indifferent, we stipulate that he chooses to fight with certainty (as opposed to fighting with probability β, as in Proposition 3). Since c_{D_h} is indifferent, type c_{D_l}, whose payoff from war is strictly higher, strictly prefers to fight. Therefore, since both types of D are choosing to fight if S rejects D's offer in the second period, S's acceptance rule in the second period must be to accept any offer $(x, 1-x)$ such that $x \leq p + c_S$, regardless of what her belief about D's type is at that point. Therefore, both types of D are strictly best off proposing $x^* = p + c_S$ in the second period.

Now consider the first period. Our previously derived stipulation that $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$ ensures that type c_{D_h} strictly prefers to say no (and get x^* in the next period) rather than fight if S's initial offer is too small, and therefore c_{D_h}'s acceptance rule in the first period is fine. Type c_{D_l} prefers to say no and obtain x^* in the next period rather than fight if S's initial offer is too low as long as $\delta_D \geq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, which we therefore stipulate to hold in this proposition, and hence c_{D_l}'s acceptance rule in the first period is fine. The last thing to note is that if D says no to S's initial offer, S is strictly better of passing rather than fighting, since she expects agreement to be reached on x^* in the next period. Q.E.D.

References

[1] Fudenberg, Drew, and Jean Tirole. 1991. *Game Theory*. Cambridge: MIT Press.

[2] Powell, Robert. 1996a. "Bargaining in the Shadow of Power." *Games and Economic Behavior* 15(August):255-89.

[3] Powell, Robert. 1996b. "Stability and the Distribution of Power." *World Politics* 48(January):239-67.

[4] Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton: Princeton University Press.