STRATEGIC FORM GAMES

Show all steps and calculations in your answers, for proofs justify each step.

QUESTION 1. Some bullets are loaded into a six-shooter. The cylinder is spun and the revolver is pointed at your head. Assuming that you prefer life to death and more money to less (if you die, you don't care how much money you paid), would you be willing to pay more to have one bullet removed when only one bullet was loaded or when three bullets were loaded? Can your preferences be represented by an expected utility function? If not, would you be willing to amend your statement after having worked through the logic?

QUESTION 2. Two roommates each need to choose to clean their apartment, and each can choose an amount of time, $t_i \geq 0$ to clean. If their choices are t_i and t_j, then player i's payoff is $(10 - t_j)t_i - t_i^2$. (This function implies that the more one roommate cleans, the less valuable is cleaning for the other roommate.) Answer each of the following:

(a) What is the best response correspondence for each player i?

(b) Which choices survive one round of elimination of strictly dominated strategies?

(c) Which choices survive the iterated elimination of strictly dominated strategies?

QUESTION 3. Consider the game in Figure 1. Let p be the probability that player 1 chooses C and let q be the probability that player 2 chooses C. The story is that two teenagers are speeding in their race cars toward each other, and each has two choices: Continue or Swerve. If one continues while the other swerves, the player that continues is the winner and the other is the loser (chicken). If both swerve, they are both chicken but since nobody emerged as a winner, each player's payoff is at least better than being the sole chicken. If neither swerves, they collide and die, which is assumed to be worse than being the sole chicken. Answer each of the following:

(a) Derive the best responses graphically by plotting player i's payoff to his two pure strategies as a function of his opponent's mixed strategy.

		Player 2	
		C	S
Player 1	C	-3,-1	10,0
	S	-2,5	1,2

Figure 1: The Game of Chicken.

(b) Plot the two best response correspondences in (p, q) space.

(c) Let $\text{BR}(p, q)$ be the best response correspondence that maps strategy profiles from $[0, 1]^2$ (the (p, q) space) onto itself. (That is, it returns $(\hat{p}, \hat{q})$ such that $\hat{p} \in \text{BR}_1(q)$ and $\hat{q} \in \text{BR}_2(p)$. What is $\text{BR}(0, 0)$? $\text{BR}(1/3, 1/2)$? $\text{BR}(1/2, 10/11)$? $\text{BR}(3/4, 2/3)$? $\text{BR}(1, 10/11)$?

(d) What are the Nash equilibria?

(e) What is the probability distribution over the outcomes in the MSNE?

(f) What are the players' expected payoffs in the MSNE?

QUESTION 4. Consider now a general version of the Game of Chicken with the payoff matrix in Figure 2. The payoffs are $D < L < Q < W$ (disaster is worse than losing, which is worse than quitting, which is worse than winning). Answer each of the following:

		Player 2	
		C	S
Player 1	C	D, D	W, L
	S	L, W	Q, Q

Figure 2: The General Game of Chicken.

(a) What are the Nash equilibria?

(b) What is the probability distribution over the outcomes in the MSNE?

(c) What are the players' expected payoffs in the MSNE?

(d) How does the probability of disaster (the $\langle C, C \rangle$ outcome) in the MSNE change as:

  (i) the value of winning (W) goes up?

  (ii) the value of losing (L) goes down?

  (iii) the value of disaster (D) goes down?

  Do any of these results strike you as counter-intuitive? Why or why not?

(e) Suppose now that player i's payoffs are $D_i < L_i < Q_i < W_i$. How does the probability that a player 1 swerves in the MSNE change as:

  (i) his value of winning (W_1) goes up? his opponent's value of winning (W_2) goes up?

  (ii) his value of losing (L_1) goes down? his opponent's value of losing (L_2) goes down?

  (iii) his value of disaster (D_1) goes down? his opponent's value of disaster (D_2) goes down?

  Do any of these results strike you as counter-intuitive? Why or why not?

QUESTION 5. Show that the two-player game in Figure 3 has a unique equilibrium. (Hint: Show that it has a unique pure strategy equilibrium. Then show that player 1 cannot put positive weight on any combination of pure strategies.)

		Player 2		
		L	C	R
	U	$1, -2$	$-2, 1$	$0, 0$
Player 1	M	$-2, 1$	$1, -2$	$0, 0$
	D	$0, 0$	$0, 0$	$1, 1$

Figure 3: The Unique Equilibrium Game.

QUESTION 6. Two students sign up to prepare an honors thesis with a professor. Each can invest time in his own project: either no time, one week, or two weeks (these are the only three options). The cost of time is 1 unit of payoff for each week spent on the project. The more time a student puts in, the better the work, and if they put in the same amount of time, the quality is the same. The professor will give an A to the higher quality work, a B to the lower quality work, and will toss a fair coin to decide which one gets the A and which one gets the B if the quality is the same. To each student, A is worth 3 units of payoff, and a B is worth 0. Answer each of the following:

(a) Write the game in strategic form.

(b) Are there any strictly dominated strategies? Are there any weekly dominated strategies?

(c) Find all Nash equilibria. Discuss the meaning of the results.

QUESTION 7. THE TRAGEDY OF THE COMMONS. There are $N \geq 2$ players who wish to enjoy a pretty shoreline of a mountain lake. The shoreline has recently been opened to privatization and players simultaneously decide whether to appropriate access by spending effort $e \in [0, 1]$ on such activities. (That is, $e = 0$ means that the player abstains from attempts to privatize access, and $e = 1$ means that he has privatized everything that a single player can privatize.) The payoff for a player is given by $u_i = 1 - 3\overline{e} + 2e_i$, where $\overline{e}$ is the average effort of the population of N players. Find the PSNE for this game. Interpret the result by comparing the equilibrium payoff to the situation where the shoreline is not open to privatization.

QUESTION 8. Player 1 must choose whether to file taxes honestly or to cheat. Player 2 decides how much effort to invest in auditing, and can choose $e \in [0, 1]$; with the cost of effort being $100e^2$. If player 1 is honest, he gets a payoff of 0, whereas player 2 pays the costs of the audit, so her payoff is $-100e^2$. If player 1 cheats, then his payoff depends on whether he is caught. If he is caught, his payoff is -100, and player 2's payoff is $100 - 100e^2$. If he is not caught, his payoff is 50, and player 2's payoff is $-100e^2$. The probability that an audit catches a cheater is simply e, the effort player 2 puts into auditing. Answer each of the following:

(a) What is player 2's best response to player 1 cheating? What is her best response to player 1 being honest?

(b) What is player 2's best response to player 1 cheating with probability p?

(c) Is there a PSNE in this game? Why or why not?

(d) Is there a MSNE in this game? Why or why not?