ELSEVIER Journal of Banking & Finance 21 (1998) 1721–1742

Journal of BANKING & FINANCE

# Credit risk measurement: Developments over the last 20 years

Edward I. Altman, Anthony Saunders *

*Salomon Brothers Center, Leonard Stern School of Business, New York University, 44 West 4th street, New York, NY 10012, USA*

**Abstractz**

This paper traces developments in the credit risk measurement literature over the last 20 years. The paper is essentially divided into two parts. In the first part the evolution of the literature on the credit-risk measurement of individual loans and portfolios of loans is traced by way of reference to articles appearing in relevant issues of the *Journal of Banking and Finance* and other publications. In the second part, a new approach built around a mortality risk framework to measuring the risk and returns on loans and bonds is presented. This model is shown to offer some promise in analyzing the risk-return structures of portfolios of credit-risk exposed debt instruments. © 1998 Elsevier Science B.V. All rights reserved.

*JEL classification:* G21; G28

*Keywords:* Banking; Credit risk; Default

## 1. Introduction

Credit risk measurement has evolved dramatically over the last 20 years in response to a number of secular forces that have made its measurement more

* Corresponding author. Tel.: +1 212 998 0711; fax: +1 212 995 4220; e-mail: asaunder@stern.nyu.edu.


*PII* S 0 3 7 8 - 4 2 6 6 ( 9 7 ) 0 0 0 3 6 - 8

important than ever before. Among these forces have been: (i) a worldwide structural increase in the number of bankruptcies, (ii) a trend towards disintermediation by the highest quality and largest borrowers, (iii) more competitive margins on loans, (iv) a declining value of real assets (and thus collateral) in many markets and (v) a dramatic growth of off-balance sheet instruments with inherent default risk exposure (see, e.g. McKinsey, 1993), including credit risk derivatives.

In response to these forces academics and practitioners alike have responded by: (i) developing new and more sophisticated credit-scoring/early-warning systems, (ii) moved away from only analyzing the credit risk of individual loans and securities towards developing measures of credit concentration risk (such as the measurement of portfolio risk of fixed income securities), where the assessment of credit risk plays a central role (iii) developing new models to price credit risk (such as the – risk adjusted return on capital models (RAROC)) and (iv) developing models to measure better the credit risk of off-balance sheet instruments.

In this paper we trace key developments in credit risk measurement over the past two decades and show how many of these developments have been reflected in papers that have been published in the *Journal of Banking and Finance* over this period. In addition, we explore a new approach, and provide some empirical examples to measure the credit risk of risky debt portfolios (or credit concentration risk).

## 2. Credit risk measurement

### 2.1. Expert systems and subjective analysis

It is probably fair to say that 20 years ago most financial institutions (FIs) relied virtually exclusively on subjective analysis or so-called banker "expert" systems to assess the credit risk on corporate loans. Essentially, bankers used information on various borrower characteristics – such as borrower character (reputation), capital (leverage), capacity (volatility of earnings) and collateral, the so-called 4 "Cs" of credit, to reach a largely subjective judgement (i.e., that of an expert) as to whether or not to grant credit. In a recent paper, Sommerville and Taffler (1995) show that in the context of the *Institutional Investor*'s rating of LDC indebtedness (based on bankers' subjective ratings), that: (a) bankers tend to be overly pessimistic about the credit risk of LDCs and (b) multivariate credit-scoring systems (see below) tend to outperform such expert systems. Perhaps, not surprisingly, FIs themselves have increasingly moved away from subjective/expert systems over the past 20 years towards systems that are more objectively based.

### 2.2. Accounting based credit-scoring systems

In univariate accounting based credit-scoring systems, the FI decision-maker compares various key accounting ratios of potential borrowers with industry or group norms. When using multivariate models, the key accounting variables are combined and weighted to produce either a credit risk score or a probability of default measure. If the credit risk score, or probability, attains a value above a critical benchmark, a loan applicant is either rejected or subjected to increased scrutiny.

In terms of sheer number of articles, developments and tests of models in this area have dominated the credit risk measurement literature in the JBF and in other scholarly journals. In addition to a significant number of individual articles on the subject, the JBF published two special issues (*Journal of Banking and Finance,* 1984, 1988) on the application of distress prediction models internationally. Indeed, international models have been developed in over 25 countries, see Altman and Narayanan (1997).

There are at least four methodological approaches to developing multivariate credit-scoring systems: (i) the linear probability model, (ii) the logit model, (iii) the probit model, and (iv) the discriminant analysis model. By far the dominant methodologies, in terms of JBF publications, have been discriminant analysis followed by logit analysis. In our inaugural issue (JBF, June 1977), Altman et al. (1977) developed the now commonly used and referenced ZETA® discriminant model. Stripped to its bare essentials, the most common form of discriminant analysis seeks to find a linear function of accounting and market variables that best distinguishes between two loan borrower classification groups – repayment and non-repayment. This requires an analysis of a set of variables to maximize the between group variance while minimizing the within group variance among these variables. Similarly, logit analysis uses a set of accounting variables to predict the probability of borrower default, assuming that the probability of default is logistically distributed i.e., the cumulative probability of default takes a logistic functional form and is, by definition, constrained to fall between 0 and 1.

Martin (1977) used both logit and discriminant analysis to predict bank failures in the 1975–1976 period, when 23 banks failed. Both models gave similar classifications in terms of identifying failures/non-failures. West (1985) used the logit model (along with factor analysis) to measure the financial condition of FIs and to assign to them a probability of being a problem bank. Interestingly, the factors identified by the logit model are similar to the CAMEL rating components used by bank examiners. Platt and Platt (1991a) use the logit model to test whether industry relative accounting ratios, rather than simple firm specific accounting ratios, are better predictors of corporate bankruptcy. In general, the industry relative accounting ratio model outperformed the unadjusted model. (Similar findings to this have been found in the context of relative accounting ratio based discriminant analysis models (see Izan, 1984).) Lawrence et al. (1992) use the logit model to predict the probability of default on mobile home loans. They find that payment history is by far the most important predictor of default. Smith and Lawrence (1995) use a logit model to find the variables that offer the best prediction of a loan moving into a default state (calculated from a Markov model of default probabilities).

Finally, as noted earlier, by far the largest number of multivariate accounting based credit-scoring models have been based on discriminant analysis models. Altman et al. (1977) investigate the predictive performance of a seven variable discriminant analysis model (that includes the market value of equity as one variable). A private firm version of this model also exists. In general, the seven variable model – the so-called "Zeta model" – is shown to improve upon Altman's (Altman, 1968) earlier five variable model. Also, Scott (1981) compares a number of these empirical models with a theoretically sound approach. He concludes that the ZETA model most closely approximates his theoretical bankruptcy construct. A large number of other mainly international applications of discriminant analysis credit related models are to be found in the two special JBF issues on credit risk, mentioned above.

### *2.3. Other (newer) models of credit risk measurement*

While in many cases multivariate accounting based credit-scoring models have been shown to perform quite well over many different time periods and across many different countries, they have been subject to at least three criticisms. First, that being predominantly based on book value accounting data (which in turn is measured at discrete intervals), these models may fail to pick up more subtle and fast-moving changes in borrower conditions, i.e., those that would be reflected in capital market data and values. Second, the world is inherently non-linear, such that linear discriminant analysis and the linear probability models may fail to forecast as accurately as those that relax the underlying assumption of linearity among explanatory variables. Third, the credit-scoring bankruptcy prediction models, described in Section 2.2, are often only tenuously linked to an underlying theoretical model. As such, there have been a number of new approaches – most of an exploratory nature, that have been proposed as alternatives to traditional credit-scoring and bankruptcy prediction models.

A class of bankruptcy models with a strong theoretical underpinning are "risk of ruin" models. At its most simple level, a firm goes bankrupt when the market (liquidation) value of its assets ($A$) falls below its debt obligations to outside creditors ($B$). Models of this type can be found in (Wilcox, 1973; Scott, 1981; Santomero and Vinso, 1977). As was recognized by Scott, the risk of ruin model is in many respects similar to the option pricing models (OPM) of Black and Scholes (1973), as well as those of Merton (1974) and Hull and

White (1995). In the Black–Scholes–Merton model, the probability of a firm going bankrupt depends crucially on the beginning period market value of that firm's assets ($A$) relative to its outside debt ($B$), as well as the volatility of the market value of a firm's assets ($\sigma_A$). The ideas of the risk of ruin/OPM models have gained increasing credence in the commercial area. A current example is the KMV (1993) and Kealhofer (1996) model. In the KMV model, crucial inputs into the estimation of the probability of default are $A$ and $\sigma_A$, both of which have to be estimated. The underlying constructs are two theoretical relationships. First is the OPM model, where the value of equity can be viewed as a call option on the value of a firm's assets. Second, is the theoretical link between the observable volatility of a firm's equity value and its (unobservable) asset value volatility. Implied values for both $A$ and $\sigma_A$ can therefore be imputed for all publicly traded companies with adequate stock return data. Moreover, given any initial values of $A$ and $B$ (short-term debt outstanding), and a calculated value for the diffusion of asset values overtime ($\sigma_A$), an expected default frequency (EDF) can be calculated for each borrowing firm. That is, default occurs in some future period when (or if) the value of a firm's assets falls below its outstanding (short-term) debt obligations. That is, the normalized area of the future distribution of asset values which falls below $B$. In actual practice, KMV uses an empirically based "distance from default" measure based on how many standard deviations $A$ values are currently above $B$, and what percent of firms actually went bankrupt within one-year with $A$ values that many standard deviations above $B$.

Major concerns of the OPM type default models are (i) whether the volatility of a firm's stock price can be used as an accurate proxy to derive the expected or implied variability in asset values and (ii) the efficacy of using a comparable, or proxy, analysis necessary for non-publicly traded equity companies.

A second, newer class of models, with strong theoretical underpinnings, are those that seek to impute implied probabilities of default from the term structure of yield spreads between default free and risky corporate securities. An early version of this approach can be found in Jonkhart (1979) with a more elaborate version being presented by Iben and Litterman (1989). These models derive implied forward rates on risk-free and risky bonds and use these rates to extract the "markets" expectation of default at different times in the future. Important assumptions underlying this approach include: (i) that the expectations theory of interest rates holds, (ii) transaction costs are small, (iii) calls, sinking fund and other option features are absent and (iv) discount bond yield curves exist or can be extracted from coupon bearing yield curves. Many of these assumptions are questionable.

A third, capital market based model is the mortality rate model of Altman (1988, 1989) and the aging approach of Asquith et al. (1989). These mortality-default rate models seek to derive actuarial-type probabilities of default from past data on bond defaults by credit grade and years to maturity. All of the rating agencies have adopted and modified the mortality approach (e.g., Moody's, 1990; Standard and Poor's, 1991) and now routinely utilize it in their structured financial instrument analyses (e.g., Duff and Phelps, McElravey and Shah, 1996).

Such models have the potential to be extended to an analysis of the default/mortality of loans, but have been hampered by the lack of a loan default data base of sufficient size. For example, McAllister and Mingo (1994) estimate that to develop very stable estimates of default probabilities, an FI would need some 20,000–30,000 "names" in its data base. Very few FIs worldwide come even remotely close to approaching this number of potential borrowers. This may explain a number of current initiatives in the USA, among the larger banks, to develop a shared national data base of historic mortality loss rates on loans (a current project of Robert Morris Associates, Philadelphia, PA).

A fourth, newer approach is the application of neural network analysis to the credit risk classification problem. Essentially, neural network analysis is similar to non-linear discriminant analysis, in that it drops the assumption that variables entering into the bankruptcy prediction function are linearly and independently related. Specifically, neural network models of credit risk explore potentially "hidden" correlations among the predictive variables which are then entered as additional explanatory variables in the non-linear bankruptcy prediction function. Applications of neural networks in distress prediction analysis include Altman et al. (1994) application to corporate distress prediction in Italy, Coats and Fant's (Coats and Fant, 1993) application to corporate distress prediction in the US and several studies summarized in Trippi and Turban (1996). A commercial model of rating replication using neural networks is available from *Finance FX* (Atlanta, GA).

The major criticism of the neural network approach is its ad hoc theoretical foundation and the "fishing expedition" nature by which hidden correlations among the explanatory variables are identified. Also, in a comparison test, Altman et al. (1994) concluded that the neural network approach did not materially improve upon the linear discriminant structure.

### *2.4. Measures of the credit risk of off-balance sheet instruments*

Perhaps one of the most profound developments over the past 20 years has been the expansion in off-balance sheet instruments – such as swaps, options, forwards, futures, etc. – in FIs portfolios (see Jagtiani et al., 1995; Brewer and Koppenhaver, 1992; Saunders, 1997) as well as credit risk derivatives (default insurance). Along with the expansion of these instruments has come concerns regarding default risk properties. This has in turn been reflected in the BIS risk-based capital ratios finally imposed in 1992, requiring banks to hold capital reserves to cover both the current and future replacement costs of such instruments, should default occur.

The probability of default on off-balance sheet instruments issued by a counter-party can, in principle, be measured in the same fashion as on-balance sheet loans since a necessary condition for default by a counter-party to an off-balance sheet contract is that the party is in financial distress, i.e., the models of Sections 2.1–2.3 can be applied.

However, there are a number of subtle differences between the default risk on loans and over-the counter (OTC), off-balance sheet instruments. First, even if the counter-party is in financial distress, it will only default on out-of-the-money contracts. That is, it will seek to enforce all in-the-money contracts. This potential "cherry picking" incentive has been recognized by the market through increased use of master netting agreements, where losses on defaulted contracts can be offset against contracts that are in the money to the defaulting counter-party. Second, for any given probability of default, the amount lost on default is usually less for off-balance sheet instruments than for loans. A lender can lose all the principal and interest on a loan, while by comparison for an interest rate swap of the same notional principal size, losses are confined to the present-value difference between the fixed and expected future cash flows on the swap (e.g., as implied by the forward rate curve).

### *2.5. Measures of credit concentration risk*

Increasingly FIs have recognized the need to measure credit concentration risk as well as the credit risk on individual loans. The early approaches to concentration risk analysis were based either on: (1) subjective analysis (the expert's feel as to a maximum percent of loans to allocate to an economic sector or geographic location, e.g., an SIC code or Latin America, (2) on limiting exposure in an area to a certain percent of capital (e.g., 10%) or (3) on migration analysis, measuring the transition probabilities of relatively homogenous loans, in a given pool, moving from current to any number of possible default states, varying from 30 days overdue to charge-off. With respect to migration analysis, the usual methodology employed to estimate transition probabilities has been the Markovian stable or unstable model (see, Altman and Kao, 1992). In an earlier JBF article, Bennett (1984) presented rating migration of bank assets in a pioneering portfolio risk discussion. He emphasized the need for a common risk rating system for all bank assets, including corporate, country, consumer loans and loans to other banks. Migration analysis plays a critical role in the recent Credit Metrics® (1997) approach.

More recently, the potential for applying modern portfolio theory (MPT) to loans and other fixed income instruments has been recognized. One attempt at applying MPT was that of Chirinko and Guill (1991). Their approach required the use of a macro econometric model of the US economy to generate future possible states of the world and thus SIC sector loan payoffs (loss rates). From the distribution of such loss rates, means, variances and covariances could be calculated and an efficient loan portfolio constructed (defined at the level of SIC code aggregation).

In the remainder of this paper, we discuss an alternative portfolio theory based approach for analyzing the optimal composition of fixed income (either bond or loan) portfolios.

## 3. Fixed income portfolio analysis

Since the pioneering work of Markowitz (1959), portfolio theory has been applied to common stocks. The traditional objectives of maximizing returns for given levels of risk or minimizing risk for given levels of return have guided efforts to achieve effective diversification of portfolios. Such concepts as individual stock and portfolio betas to indicate risk levels and to calculate efficient frontiers, with optimal weightings of the portfolio's member stocks, are now common parlance among investment professionals and in textbooks (e.g., Elton and Gruber, 1995). This is not to say that these concepts are widely used to the exclusion of more traditional industrial sector, geographical location, size, or some other diversification strategy. The necessary data in terms of historical returns and correlations of returns between individual stocks are usually available to perform the portfolio optimization analysis.

One might expect that these very same techniques would (and could) be applied to the fixed income area involving corporate and government bonds and even to bank loans. There has been, however, very little published work in the bond area [1] and a recent survey of practices by commercial banks found fragmented and untested efforts. [2] The objective of effective risk reducing methods is, however, a major pre-occupation of FIs, with bank loan research departments and regulators spending considerable resources to reduce the likelihood of major loan losses that jeopardize the very existence of the lending institution. Recent bank failures attributed to huge loan losses in the US, Japan, Europe and Latin America have raised the level of concern. Still, conceptually sound diversification techniques have eluded most bank and bond portfolio managers, probably for valid reasons. And, despite recent analytical attempts

[1] Platt and Platt (1991b) did some preliminary work for high yield "junk bond" portfolios by introducing a linear programming algorithm which maximized yield-to-maturity subject to a constraint as to the level of default risk and the degree of diversification. To our knowledge, however, corporate bond portfolio managers have not utilized this concept and continue to invest based on traditional industry, size, and credit rating criteria.

[2] The survey of McAllister and Mingo (1994) concluded that commercial banks were experimenting with a number of different techniques but few had been implemented or had impacted corporate lending practices.

(e.g. Credit Metrics®, 1997), effective portfolio management techniques of loans/bonds is still, in our opinion, an unresolved area.

It is the objective of this section of our paper to outline a method that will avoid the major data and analytical pitfalls that have plagued fixed income portfolio efforts and to provide a sound and empirically feasible portfolio approach. Our empirical examples will involve corporate bonds but we feel confident that the methodology is applicable as well to commercial and industrial loans.

### 3.1. Return-risk framework

The classic mean variance of return framework is not valid for long-term, fixed income portfolio strategies. The problem does not lie in the expected return measure on individual assets, but in the distribution of possible returns. While the fixed income investor can lose all or most of the investment in the event of default, positive returns are limited. This problem is mitigated when the measurement period of returns is relatively short, e.g., monthly, and the likely variance of returns is small and more normal. We will return to measures of portfolio risk both for short-term returns and the more challenging buy-and-hold, long-term strategy.

### 3.2. Return measurement

The measurement of expected portfolio return is actually quite straightforward for fixed income bond and loan assets. The investor (or FI) is promised a fixed return (yield-to-maturity or yield-to-worst) over time and should subtract, from this promised yield, the expected losses from default of the issuer. For certain measurement periods, the return will also be influenced by changes in interest rates but we will assume, for purposes of exposition, that these changes are random with an expected capital gain of zero. Likewise, we acknowledge that investors can infer capital gains or losses from the yield curve and also from whether the bonds are trading at a premium or discount from par.

The expected annual return is therefore

$$\mathrm{EAR} = \mathrm{YTM} - \mathrm{EAL}, \tag{1}$$

where EAR is the Expected annual return, YTM the Yield-to-Maturity (or Yield-to-Worst) and EAL the Expected Annual Loss.

We derive the EAL from prior work on bond mortality rates and losses (Altman, 1988, 1989). Each bond is analyzed based on its initial (or existing) [3]

[3] The measurement of expected defaults for existing bonds compared to newly issued ones is essentially the same for bonds with maturities of at least five years. Moody's and S&P publish data on existing baskets of bonds by rating without regard to age. Their results and ours essentially converge after year four (see Altman, 1992).

Table 1
Mortality rates by original rating: [a] 1971–1994 (years after issuance)

| Rating | | 1 (%) | 2 (%) | 3 (%) | 4 (%) | 5 (%) | 6 (%) | 7 (%) | 8 (%) | 9 (%) | 10 (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AAA | Yearly | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | cumulative | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 |
| AA | Yearly | 0.00 | 0.05 | 1.06 | 0.09 | 0.00 | 0.00 | 0.01 | 0.00 | 0.06 | 0.04 |
| | cumulative | 0.00 | 0.05 | 1.11 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 | 1.26 | 1.30 |
| *A* | Yearly | 0.00 | 0.19 | 0.07 | 0.21 | 0.06 | 0.06 | 0.20 | 0.19 | 0.00 | 0.00 |
| | cumulative | 0.00 | 0.19 | 0.26 | 0.47 | 0.53 | 0.59 | 0.78 | 0.98 | 0.98 | 0.98 |
| BBB | Yearly | 0.41 | 0.25 | 0.32 | 0.55 | 0.89 | 0.39 | 0.09 | 0.00 | 0.59 | 0.23 |
| | cumulative | 0.41 | 0.66 | 0.97 | 1.51 | 2.39 | 2.77 | 2.86 | 2.86 | 3.44 | 3.66 |
| BB | Yearly | 0.50 | 0.58 | 4.15 | 4.84 | 1.13 | 0.33 | 0.94 | 0.23 | 0.64 | 0.58 |
| | cumulative | 0.50 | 1.08 | 5.19 | 9.78 | 10.79 | 11.26 | 13.64 | 13.87 | 14.55 | 15.21 |
| *B* | Yearly | 1.59 | 7.12 | 6.80 | 7.29 | 3.40 | 3.40 | 2.80 | 2.13 | 2.83 | 3.43 |
| | cumulative | 1.59 | 8.60 | 14.82 | 21.02 | 23.71 | 28.21 | 30.22 | 31.70 | 33.63 | 35.91 |
| CCC | Yearly | 8.32 | 10.69 | 18.53 | 10.26 | 9.18 | 5.56 | 2.49 | 2.97 | 12.28 | 1.35 |
| | cumulative | 8.32 | 18.13 | 33.30 | 40.14 | 45.63 | 48.66 | 49.94 | 51.42 | 57.39 | 58.31 |

[a] Rated by S&P at issuance.
*Source*: Altman and Kishore (1995).

bond rating which implies an expected rate of default for up to ten (or longer) years after issuance. Tables 1 and 2 list cumulative mortality rates and cumulative mortality losses, respectively, covering the period 1971–1994. [4] Table 3 annualizes these mortality rates and losses. So, for example, a 10-year BB (S&P rated) bond has an expected annual loss of 91 basis points per year. If the newly issued BB rated bond has a promised yield of 9.0% with a spread of 2.0% over 7.0% risk-free US Treasury bonds, then the expected return is 8.09% per year, or a risk premium of 109 basis points over the risk-free rate. If our measurement periods were quarterly returns instead of annual, then the expected return would be about 2.025% per quarter. Again, our expected return measure is focused primarily on credit risk changes and not on yield curve implications.

The latter is obviously more relevant to government bond portfolios.

The problem of measuring expected returns for commercial loans is a bit more complex. Since most loans do not have a risk rating attached by the rating agencies, [5] the loan portfolio analyst must utilize a proxy measure. We advocate using the bank's own risk rating system, or a rating replication system, as long as each of the internal ratings is linked with the public bond ratings, e.g., those used by Altman, Moody's or S&P in their cumulative default studies.

[4] For updated data through 1996, see Altman and Kishore (1997).

[5] The rating agencies will rate loans by their private placement service but these are relatively few in number.

Table 2
Mortality losses by original rating: [a] 1971–1994 (years after issuance)

| Rating | | 1 (%) | 2 (%) | 3 (%) | 4 (%) | 5 (%) | 6 (%) | 7 (%) | 8 (%) | 9 (%) | 10 (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AAA | Yearly | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | cumulative | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 |
| AA | Yearly | 0.00 | 0.02 | 0.21 | 0.03 | 0.00 | 0.00 | 0.01 | 0.00 | 0.04 | 0.02 |
| | cumulative | 0.00 | 0.02 | 0.23 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.30 | 0.32 |
| *A* | Yearly | 0.00 | 0.03 | 0.02 | 0.15 | 0.06 | 0.03 | 0.11 | 0.13 | 0.00 | 0.00 |
| | cumulative | 0.00 | 0.03 | 0.05 | 0.20 | 0.26 | 0.29 | 0.40 | 0.52 | 0.52 | 0.52 |
| BBB | Yearly | 0.27 | 0.10 | 0.21 | 0.26 | 0.36 | 0.30 | 0.06 | 0.00 | 0.41 | 0.14 |
| | cumulative | 0.27 | 0.37 | 0.58 | 0.84 | 1.19 | 1.49 | 1.55 | 1.55 | 1.95 | 2.08 |
| BB | Yearly | 0.26 | 0.26 | 3.34 | 2.14 | 0.70 | 0.33 | 0.94 | 0.23 | 0.64 | 0.58 |
| | cumulative | 0.26 | 0.51 | 3.84 | 5.90 | 6.56 | 6.86 | 7.74 | 7.95 | 8.54 | 9.07 |
| *B* | Yearly | 0.83 | 5.12 | 5.02 | 5.95 | 2.44 | 3.93 | 2.06 | 1.64 | 1.98 | 1.59 |
| | cumulative | 0.83 | 5.90 | 10.63 | 15.95 | 18.00 | 21.22 | 22.84 | 24.11 | 25.61 | 26.79 |
| CCC | Yearly | 7.22 | 8.87 | 15.30 | 6.82 | 6.76 | 3.29 | 2.49 | 0.91 | 8.35 | 1.25 |
| | cumulative | 7.22 | 15.45 | 28.39 | 33.27 | 37.78 | 39.83 | 41.33 | 41.87 | 47.47 | 47.61 |

[a] Rated by S&P at issuance.
*Source*: Altman and Kishore (1995).

We will also show that these proxy risk measures, either from internal systems or from commercially available systems, [6] are critical ingredients in the compilation of historical correlations of risk and return measures between assets in the portfolio. The expected portfolio return ($R_{\mathrm{p}}$) is therefore based on each asset's expected annual return, weighted by the proportion ($X_i$) of each loan/bond relative to the total portfolio, where

$$R_{\mathrm{p}} = \sum_{i=1}^{N} X_i \, \mathrm{EAR}_i. \tag{2}$$

### 3.3. Portfolio risk and efficient frontiers using returns

The classic mean return-variance portfolio framework is given in Eq. (3) when we utilize a short holding period, e.g., monthly or quarterly, and historical data exist for the requisite period to calculate correlation of returns among the loans/bonds

[6] Such systems as ZETA Services (Hoboken, NJ), KMV Corporation (San Francisco, CA), and Finance FX (Atlanta, GA) are available to assign ratings and expected defaults to all companies, whether or not they have public debt outstanding. See our earlier discussions of these models in Sections 2.2 and 2.3.

Table 3
Annualized cumulative default rates and annualized cumulative mortality loss rates (1971–1994)

| Original rating/Year | 1 (%) | 2 (%) | 3 (%) | 4 (%) | 5 (%) | 6 (%) | 7 (%) | 8 (%) | 9 (%) | 10 (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| *Annualized cumulative default rates* | | | | | | | | | | |
| AAA | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| AA | 0.00 | 0.00 | 0.27 | 0.27 | 0.22 | 0.19 | 0.16 | 0.14 | 0.13 | 0.12 |
| *A* | 0.00 | 0.05 | 0.08 | 0.11 | 0.10 | 0.09 | 0.10 | 0.11 | 0.10 | 0.09 |
| BBB | 0.04 | 0.27 | 0.26 | 0.33 | 0.37 | 0.40 | 0.44 | 0.39 | 0.35 | 0.37 |
| BB | 0.00 | 0.35 | 1.26 | 1.44 | 2.10 | 1.91 | 2.02 | 1.81 | 1.68 | 1.59 |
| *B* | 0.99 | 2.14 | 4.61 | 5.01 | 5.14 | 4.71 | 4.58 | 4.25 | 3.97 | 4.09 |
| CCC | 2.24 | 8.35 | 11.75 | 10.50 | 9.87 | 9.78 | 8.82 | 8.07 | 7.21 | 8.35 |
| *Annualized cumulative mortality loss rates* | | | | | | | | | | |
| AAA | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AA | 0.00 | 0.00 | 0.05 | 0.06 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 |
| *A* | 0.00 | 0.01 | 0.01 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| BBB | 0.03 | 0.15 | 0.15 | 0.20 | 0.19 | 0.20 | 0.24 | 0.22 | 0.19 | 0.21 |
| BB | 0.00 | 0.20 | 0.86 | 1.01 | 1.22 | 1.11 | 1.09 | 0.98 | 0.94 | 0.91 |
| *B* | 0.42 | 1.23 | 3.29 | 3.64 | 3.81 | 3.46 | 3.36 | 3.12 | 2.91 | 2.89 |
| CCC | 1.51 | 7.19 | 9.79 | 8.69 | 7.82 | 7.57 | 6.87 | 6.13 | 7.06 | 7.25 |

*Source*: Calculation on data from Tables 1 and 2.

$$V_{\mathrm{p}} = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i \, X_j \, \sigma_i \, \sigma_j \, \rho_{ij}, \tag{3}$$

where $V_{\mathrm{p}}$ is the variance (risk) of the portfolio, $X_i$ the proportion of the portfolio invested in bond issue $i$, $\sigma_i$ the standard deviation of the return for the sample period for bond issue $i$, and $\rho_{ij}$ the correlation coefficient of the quarterly returns for bonds $i$ and $j$.

For example, if returns on all assets exist for 60 months or 20 quarters, then the correlations are meaningful and the classic efficient frontier can be calculated. Fig. 1 shows an efficient frontier, i.e., maximization of expected return for given levels of risk or minimization of risk (variance of returns) for given levels of return, for a hypothetical high yield bond portfolio. The objective is to maximize the High Yield Portfolio Ratio (HYPR) for given levels of risk or return. Note that an existing portfolio with a HYPR of 5.0 can be improved to 6.67 holding risk constant or to 10.0 holding return constant.

Our HYPR is a variation on the so-called Sharpe ratio, first introduced as a reward-to-variability ratio by Sharpe (1966), later popularized as the Sharpe Index or Sharpe ratio by many, e.g., Morningstar (1993), and finally generalized and expanded to cover a broader range of applications by Sharpe (1994). Most often applied to measuring the performance of equity mutual funds, this ratio captures the average differential return ($\overline{d}$) between a fund's return ($R_{\mathrm{F}}$) and an appropriate benchmark ($R_{\mathrm{B}}$) and the standard deviation

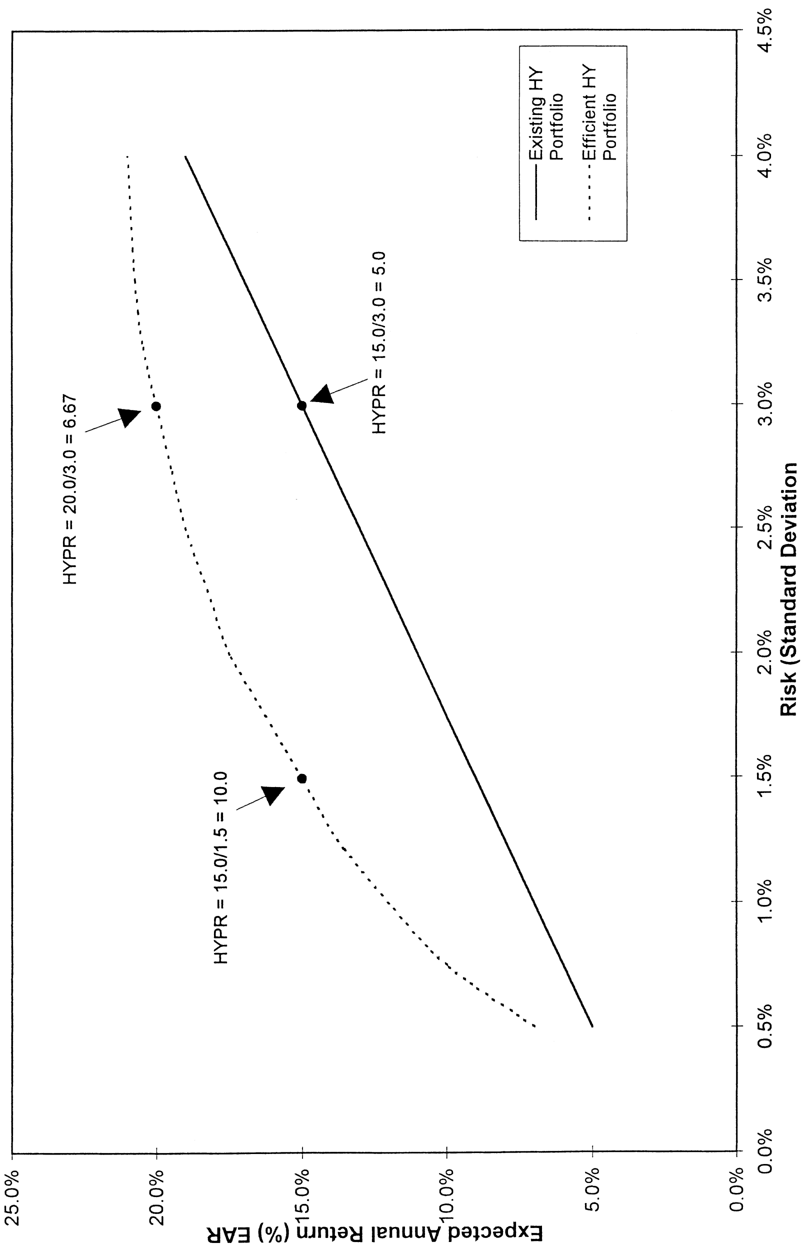


Fig. 1. The HYPR approach for risk-return assessment.

($\sigma_d$) of the differences over the period. As such, it captures the average differential return per unit of risk (standard deviation), assuming the appropriate risk measure is the variance of returns.

The only other applications of a version of the Sharpe ratio to fixed income asset portfolios and derivatives were proposed in unpublished manuscripts by McQuown (1994) and Kealhofer (1996). They utilize a risk of default model developed by KMV (see Section 2.3) which itself is based (indirectly) on the level, variability and correlations of the stock prices of the existing and potential companies in the portfolio. Our fixed income asset portfolio model has many similarities to that of McQuown, with the major difference being the measure of default risk in the model (see our earlier discussion of the $Z$ and Zeta risk measures and KMV's EDF approach).

We agree with McQuown and Kealhofer that the risk of any individual bond/loan as well as the entire portfolio itself is a measure that incorporates the *unexpected loss*. We will return to the concept of unexpected losses shortly.

Fig. 2 shows an efficient frontier based on a potential portfolio of 10 high yield corporate bonds utilizing actual quarterly returns from the five year period 1991–1995. The efficient portfolio compared to the equally weighted one shows considerable improvement in the return-risk tradeoff. For example, the HYPR goes from about 0.67 (2.0/3.0) to 1.14 (2.0/1.75) for the same expected return and to 1.0 (3.0/3.0) for the same variance of return. Note also the link between the risk-free rate at about 1.5% per quarter and the tangent line to the efficient frontier, indicating various proportions of risky vs. risk-free fixed income assets. The efficient frontier, calculated without any constraint as to the number of issues in the portfolio, involved eight of the possible ten high yield bonds. And, when we constrain the model such that no issue can be greater than 15% of the portfolio, the actual number of issues was either seven or eight depending upon the different expected returns (shown in Table 5).

### *3.4. Portfolio risk and efficient frontiers using an alternative risk measure*

The reality of the bond and loan markets is that even if one was comfortable with the distribution qualities of returns, the need to analyze a reasonably large number of potential assets precludes the use of the classic mean-variance of return framework. Specifically, there simply is insufficient historical high yield bond return and loan return data to compute correlations. The same problem would be true if, instead of using return correlations, which can vary due to maturity differences between bonds, we utilized the correlation of the duration of each bond with other bonds and with the overall index of bonds to calculate the (i) correlation between bonds and (ii) variance of the portfolio. Other sample selection problems include the change in maturities of individual bonds over the measurement period and the exclusion of bonds that defaulted in the past.

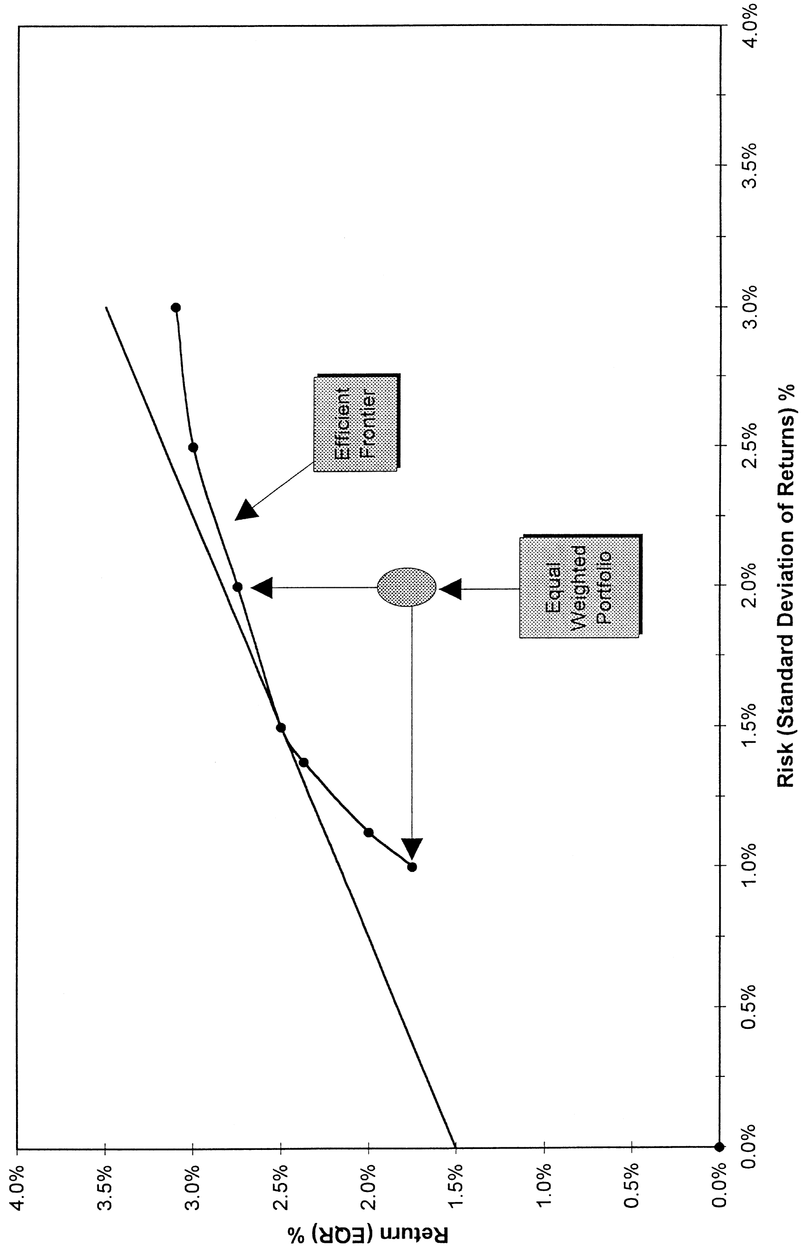


Fig. 2. Efficient high yield bond portfolio, using EMS 10 issues.

We analyzed the potential to use returns or durations in the high yield corporate debt market and out of almost 600 bond issues that existed as of year-end 1995, less than forty had 20 quarters of historical data. If we add to this scenario our other conceptual concerns, as indicated above, it is simply not appropriate (theoretically or empirically) to utilize the variance of return as the measure of the portfolio's risk.

An alternative risk measure, one that is critical to most bank and fixed income portfolio managers, is *unexpected loss* from defaults. Recall that we adjusted the promised yield for expected losses. Therefore, the risk is the downside in the event that the expected losses underestimate actual losses. [7] In addition, unexpected losses are the cornerstone measure in the determination of appropriate reserves against bank capital in the RAROC approach adopted by many banks.

Our suggested approach for determining unexpected losses is to utilize a variation of the $Z$-Score model, called the $Z''$-Score model (Altman, 1993) to assign a bond rating equivalent to each of the loans/bonds that could possibly enter the portfolio. [8] As noted earlier, these scores and rating equivalents can then be used to estimate expected losses over time. If we then observe the standard deviation around the expected losses, we have a procedure to estimate unexpected losses. For example, the expected loss on a BB rated equivalent 10 year bond is 91 basis points per year (Table 3). The standard deviation around this expected value was computed to be 2.65%, or 265 basis points per year. The standard deviation is computed from the individual issuance years' (independent) observations that were used to calculate the cumulative mortality losses. For example, there are 24 one-year default losses, for bonds issued in a certain rating class, over the 1971–1995 period, i.e., 1971 issued bonds defaulting in 1972, 1972 issued bonds defaulting in 1973, etc. In the same way, there are 23 two-year cumulative loss data points, 22 three-year loss observations, etc., up to 15 ten-year observations.

As noted above, the model used here is the $Z''$-Score, risk rating model, indicated in Eq. (4) with the bond rating equivalents shown in Table 4. [9]

[7] This idea is similar to the use of the semi-variance measure of returns, whereby the analyst is concerned only with the return below the mean.

[8] The $Z''$-Score model is a four variable version of the $Z$-Score approach. It was designed to reduce distortions in credit scores for firms in different industries. We have also found this model extremely effective in assessing the credit risk of corporate bonds in the emerging market arena (see Altman et al., 1995).

[9] In order to standardize our bond rating equivalent analysis, we add a constant term of 3.25 to the model; scores of zero (0) indicating a D (default) rating and positive scores indicating ratings above D. The actual bond rating equivalents are derived from a sample of over 750 US corporate bonds with average scores for each rating category (shown in Table 6).

Table 4
US bond rating equivalent, based on $Z''$-Score

| US equivalent rating | Average Z″-Score | Sample size |
|---|---|---|
| AAA | 8.15 | 8 |
| AA+ | 7.60 | – |
| AA | 7.30 | 18 |
| AA– | 7.00 | 15 |
| A+ | 6.85 | 24 |
| A | 6.65 | 42 |
| A– | 6.40 | 38 |
| BBB+ | 6.25 | 38 |
| BBB | 5.85 | 59 |
| BBB– | 5.65 | 52 |
| BB+ | 5.25 | 34 |
| BB | 4.95 | 25 |
| BB– | 4.75 | 65 |
| B+ | 4.50 | 78 |
| B | 4.15 | 115 |
| B– | 3.75 | 95 |
| CCC+ | 3.20 | 23 |
| CCC | 2.50 | 10 |
| CCC– | 1.75 | 6 |
| D | 0.00 | 14 |

Average based on over 750 US industrial corporates with rated debt outstanding; 1994 data.
*Source:* In-Depth Data Corporation.

$$Z''\text{-Score} = 6.56\,(X_1) + 3.26\,(X_2) + 6.72\,(X_3) + 1.05\,(X_4) + 3.25, \tag{4}$$

where $X_1$ is the working capital/total assets, $X_2$ the retained earnings/total assets, $X_3$ the EBIT/total assets and $X_4$ the equity (book value)/total liabilities.

### 3.5. Portfolio risk

The formula for our portfolio risk measure is given by

$$\mathrm{UAL_p} = \sum_{i=1}^{N}\sum_{j=1}^{N} X_i\, X_j\, \sigma_i\, \sigma_j\, \rho_{ij}. \tag{5}$$

The measure $\mathrm{UAL_p}$ is the unexpected loss on the portfolio consisting of measures of individual asset unexpected losses ($\sigma_i$,$\sigma_j$) and the correlation ($\rho_{ij}$) of unexpected losses over the sample measurement period. Again, these unexpected losses are based on the standard deviation of annual expected losses for the bond rating equivalents calculated at each quarterly interval. [10]

All that is necessary is that the issuing firm (or borrower) was operating for the entire sample period, e.g., five years, and had quarterly financial statements. The actual bonds/loans did not have to be outstanding in the period, as is necessary when returns and variance of returns are used. Since the actual debt issue may not have been outstanding during the entire measurement period, leverage measures will likely also vary over time. Still, we expect to capture most of the covariance of default risk between firms.

### 3.6. Empirical results

We ran the portfolio optimizer program [11] on the same ten bond portfolio analyzed earlier, this time using the $Z''$-Score bond rating equivalents and their associated expected and unexpected losses, instead of returns. Fig. 3 shows the efficient frontier compared to an equal weighted portfolio. As we observed earlier, the efficient frontier indicates considerably improved HYPRs. For example, the return/risk ratio of just above 0.50 for the equal weighted 10-bond portfolio can be improved to 1.60 (2.00/1.25) at the 2.00% quarterly return level and to about 1.00 for the same risk (3.75%) level.

Table 5 shows the portfolio weights for the efficient frontier portfolio using both returns and risk (unexpected losses) when the individual weights are constrained at a maximum of 15% of the portfolio. [12] This is for the 1.75% quarterly expected return. Note that both portfolios utilize eight of the ten bonds and very similar weightings. Indeed, seven of the eight bonds appear in both portfolios. These results are comforting in that the unexpected loss derived from the $Z''$-Score is an alternative risk measure. Our small sample test results are encouraging and indicate that this type of portfolio approach is potentially

---

[10] We do recognize that our measure of covariance is potentially biased in two ways. First, estimates of individual firms' debt unexpected losses are derived from empirical data on bonds from a given bond rating class and as such will probably understate the risk of loss from individual firm defaults. On the other hand, the covariance of default losses between two firms' debt is based on the joint probability of both defaulting at the same time. If the default decision of each firm is viewed as 0,1, i.e., as a binomial distribution, then the appropriate covariance or correlation should be calculated from a joint density function of two underlying binomial distributions. Our measure, however, assumes a normal density function for returns and thus returns are jointly, normally distributed for each firm which could result in a higher aggregate measure of portfolio risk. As such, the two biases neutralize each other to some extent although it is difficult to assess the relative magnitude of each.

[11] Using a double precision, linear constrained optimization program (DLCONG).

[12] The unconstrained weighting results yielded efficient portfolios of between five and eight individual bonds with some weightings of over 30%. These high weights would not be prudent for most portfolio managers.

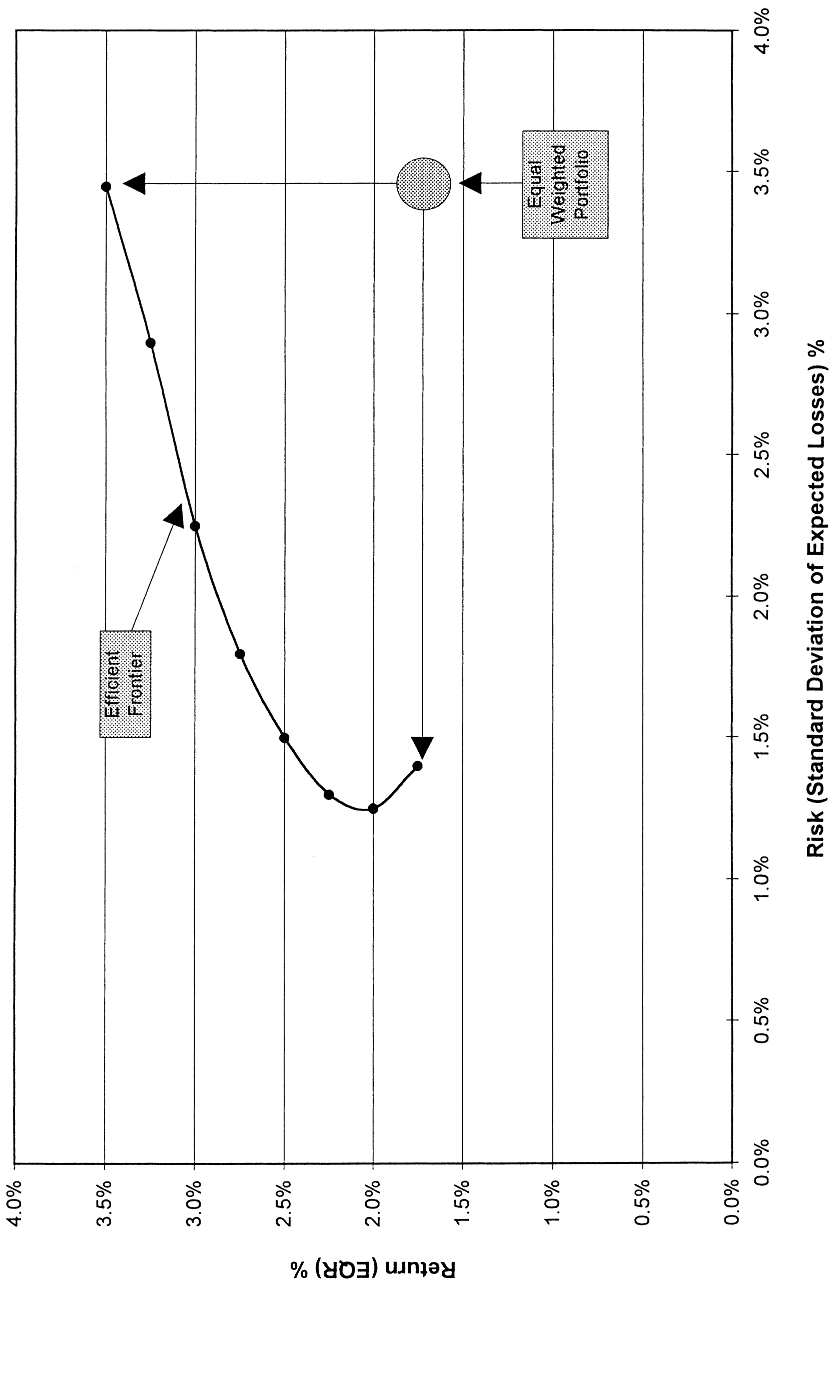


Fig. 3. Efficient high yield bond portfolio, using EMS scores 10 issues.

Table 5
Portfolio weights using two different measures of risk and return

| Company ticker | Weights using $Z''$-Scores | Weights using returns (quarterly) |
|---|---|---|
| AS | 0.0000 | 0.1065 |
| BOR | 0.0776 | 0.0000 |
| CGP | 0.1500 | 0.1500 |
| CQB | 0.1500 | 0.1500 |
| FA | 0.0000 | 0.0000 |
| IMD | 0.1500 | 0.1351 |
| RHR | 0.1500 | 0.1209 |
| STO | 0.1500 | 0.1500 |
| USG | 0.1500 | 0.1500 |
| WS | 0.0224 | 0.0376 |

Return = 1.75%, constrained to 15% maximum weights.

quite feasible for fixed income assets. The important factor in our analysis is that credit risk management plays a critical role in the process.

We should note clearly that these are preliminary findings. Subsequent conceptual refinements and larger sample empirical tests are necessary to gain experience and confidence with this portfolio technique for fixed income assets (including loans).

## 4. Summary and conclusion

In this paper we have sought to accomplish two objectives. In Sections 1 and 2 we traced the development of credit risk measurement techniques over the past 20 years and showed how many of these developments have been mirrored in published articles in the JBF. In Section 3, we developed a new approach to measuring the return risk trade-off in portfolios of risky debt instruments, whether bonds or loans. In particular, we showed that this new approach added much promise to the complex problem of estimating the optimal composition of loan/bond portfolios.

Clearly, over the next 20 years one can foresee significant improvements in data bases on historical default rates and loan returns. With the development of such data bases will come new and exciting approaches to measuring the ever present credit risk problems facing FI managers.

## Acknowledgements

We acknowledge the helpful comments of our colleagues at NYU Edwin Elton and Martin Gruber.

## References

Altman, E.I., 1968. Financial ratios discriminant analysis and the prediction of corporate bankruptcy. Journal of Finance 589-609.

Altman, E.I., 1988. Default Risk, Mortality Rates, and the Performance of Corporate Bonds. Research Foundation, Institute of Chartered Financial Analysts, Charlottesville, VA.

Altman, E.I., 1989. Measuring corporate bond mortality and performance. Journal of Finance September, 909–922.

Altman, E.I., 1992. Revisiting the high yield debt market. Financial Management Summer, 78–92.

Altman, E.I., 1993. Corporate Financial Distress and Bankruptcy, 2nd ed. Wiley, New York.

Altman, E.I., Haldeman, R., Narayanan, P., 1977. Zeta analysis: A new model to identify bankruptcy risk of corporations. Journal of Banking and Finance 29–54.

Altman, E.I., Hartzell, J., Peck, M., 1995. A Scoring System for Emerging Market Corporate Debt. Salomon Brothers, 15 May.

Altman, E.I., Kao, D.L., 1992. The implications of corporate bond rating drift. Financial Analysts Journal June, 64–75.

Altman, E.I., Kishore, V., 1995. Default and returns in the high yield debt market, 1991–1995, NYU Salomon Center Special Report.

Altman, E.I., Kishore, V., 1997. Default and returns in the high yield debt market, 1991–1996, NYU Salomon Center Special Report.

Altman, E.I., Marco, G., Varetto, F., 1994. Corporate distress diagnosis: Comparisons using linear discriminant analysis and neural networks (The Italian Experience), Journal of Banking and Finance 505–529.

Altman, E.I., Narayanan, P., 1997. Business Failure Classification Models: An International Survey. In: Choi, F. (Ed.), International Accounting, 2nd ed. Wiley, New York.

Asquith, P., Mullins Jr., D.W., Wolff, E.D., 1989. Original issue high yield bonds: Aging analysis of defaults, exchanges and calls. Journal of Finance 923–953.

Bennett, P., 1984. Applying portfolio theory to global bank lending. Journal of Banking and Finance 153–169.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 637–659.

Brewer, E., Koppenhaver, G.D., 1992. The impact of standby letters of credit on bank risk: A note. Journal of Banking and Finance 1037–1046.

Chirinko, R.S., Guill, G.D., 1991. A framework for assessing credit risk in depository institutions: Toward regulatory reform. Journal of Banking and Finance 785–804.

Coats, P., Fant, L., 1993. Recognizing financial distress patterns using a neural network tool. Financial Management 142–155.

Credit Metrics®, 1997. J.P. Morgan & Co., New York (April 3).

Elton, E., Gruber, M., 1995. Modern Portfolio Theory and Investment Analysis, 5th ed. Wiley, New York.

Hull, J., White, A., 1995. The impact of default risk on the prices of options and other derivative securities. Journal of Banking and Finance 299–322.

Iben, T., Litterman, R., 1989. Corporate bond valuation and the term structure of credit spreads. Journal of Portfolio Management 52–64.

Izan, H.Y., 1984. Corporate distress in Australia. Journal of Banking and Finance 303–320.

Jagtiani, J., Saunders, A., Udell, G., 1995. The effect of bank capital requirements on bank off-balance sheet financing. Journal of Banking and Finance 647–658.

Jonkhart, M., 1979. On the term structure of interest rates and the risk of default. Journal of Banking and Finance 253–262.

Journal of Banking and Finance, 1984. Special Issue on "Company and Country Risk Models" 151–387.

Journal of Banking and Finance, 1988. Supplement Studies in Banking and Finance, "International Business Failure Prediction Models".

Kealhofer, S., 1996. Measuring Default Risk in Portfolios of Derivatives. Mimeo KMV Corporation, San Francisco, CA.

KMV Corporation, 1993. Credit Monitor Overview, San Francisco, Ca, USA.

Lawrence, E.L., Smith, S., Rhoades, M., 1992. An analysis of default risk in mobile home credit. Journal of Banking and Finance 299–312.

McAllister, P., Mingo, J.J., 1994. Commercial loan risk management, credit-scoring and pricing: The need for a new shared data base. Journal of Commercial Bank Lending 6–20.

McElravey, J.N., Shah, V., 1996. Rating Cash Flow Collateralized Bond Obligations. Special Report, Asset Backed Securities, Duff and Phelps Credit Rating Co., Chicago, IL, USA.

McKinsey, 1993. Special report on "The new world of financial services" The McKinsey Quarterly, Number 2.

McQuown, J., 1994. All that counts is diversification: In bank asset portfolio management. IMI Bank Loan Portfolio Management Conference, May 11.

Markowitz, H., 1959. Portfolio Selection: Efficient Diversification of Investments, Wiley, New York.

Martin, D., 1977. Early warning of bank failure: A logit regression approach. Journal of Banking and Finance 249–276.

Merton, R., 1974. On the pricing of corporate debt. Journal of Finance 449–470.

Moody's Special Report, 1990. Corporate Bond Defaults and Default Rates, 1970–1989, April.

Morningstar Mutual Funds User's Guide, 1993. Chicago, Morningstar, Inc.

Platt, H.D., Platt, M.B., 1991a. A note on the use of industry-relative ratios in bankruptcy prediction. Journal of Banking and Finance 1183–1194.

Platt, H.D., Platt, M.B., 1991b. A linear programming approach to bond portfolio selection. Economic Financial Computing, Spring 71–84.

Santomero, A., Vinso, J., 1977. Estimating the probability of failure for firms in the banking system. Journal of Banking and Finance 185–206.

Saunders, A., 1997. Financial Institutions Management: A Modern Perspective, 2nd ed. Irwin, Homewood, IL.

Scott, J., 1981. The probability of bankruptcy: A comparison of empirical predictions and theoretical models. Journal of Banking and Finance. September, 317–344.

Sharpe, W., 1966. Mutual fund performance. Journal of Business January, 111–138.

Sharpe, W., 1994. The Sharpe Ratio. Journal of Portfolio Management Fall, 49–58.

Smith, L.D., Lawrence, E., 1995. Forecasting losses on a liquidating long-term loan portfolio. Journal of Banking and Finance 959–985.

Sommerville, R.A., Taffler, R.J., 1995. Banker judgement versus formal forecasting models: The case of country risk assessment. Journal of Banking and Finance 281–297.

Standard and Poor's, 1991. Corporate Bond Default Study. Credit Week, September 16.

Trippi, R., Turban, E., 1996. Neural Networks in Finance and Investing, revised ed. Irwin, Homewood, IL.

West, R.C., 1985. A factor-analytic approach to bank condition. Journal of Banking and Finance 253–266.

Wilcox, J.W., 1973. A Prediction of business failure using accounting data. Journal of Accounting Research, Vol. 2.