Problem Set 6

Due November 11 (Thursday) in Class

Solve each of the problems below. Please show all work. Point totals are given in parentheses after the number of the problem.

1. (10 total - 5 points for each part). Find the inverse of each matrix, if it exists, using the determinant method.
   (a). \[
   \begin{bmatrix}
   9 & 6 \\
   2 & -6 \\
   \end{bmatrix}
   \]
   (b). \[
   \begin{bmatrix}
   4 & 1 \\
   3 & 1 \\
   \end{bmatrix}
   \]

2. (10 total - 5 points for each part). Find the solution to each system of linear equations using the determinant method.
   (a).
   \[
   \begin{align*}
   x_1 + 4x_2 - 3x_3 &= -1 \\
   2x_1 + 5x_2 + 4x_3 &= 2 \\
   x_1 - 3x_2 - 2x_3 &= 4 \\
   \end{align*}
   \]
   (b).
   \[
   \begin{align*}
   x_1 + x_2 - x_3 &= 6 \\
   3x_1 - x_2 &= 8 \\
   2x_1 - 3x_2 + 4x_3 &= -3 \\
   \end{align*}
   \]

3. (15 total - 5 points for each part). Find the eigenvalues and the eigenvectors of the following matrices:
   (a). \[
   \begin{bmatrix}
   \frac{3}{5} & \frac{2}{5} \\
   \frac{3}{10} & \frac{7}{10} \\
   \end{bmatrix}
   \]
   (b). \[
   \begin{bmatrix}
   2 & 2 & 0 \\
   1 & -1 & 2 \\
   0 & 1 & 2 \\
   \end{bmatrix}
   \]
   (c). \[
   \begin{bmatrix}
   2 & 4 & 2 \\
   0 & -3 & -1 \\
   0 & 0 & 0 \\
   \end{bmatrix}
   \]
4. (10 total - 5 points for each part). Given the matrix 
\[
\begin{bmatrix}
1 & 1 \\
-1 & 4 
\end{bmatrix}
\].
(a). Find its largest eigenvalue.
(b). Find the eigenvector associated with the largest eigenvalue.

5. (10 total - 2.5 points for each part). Determine which of the following are stochastic matrices:
(a.) 
\[
A = \begin{bmatrix}
\frac{1}{3} & 1 \\
\frac{1}{2} & 0 \\
\frac{1}{3} & \frac{1}{2}
\end{bmatrix}
\];
(b.) 
\[
B = \begin{bmatrix}
\frac{3}{4} & \frac{3}{4} \\
\frac{1}{4} & \frac{1}{2}
\end{bmatrix}
\];
(c.) 
\[
C = \begin{bmatrix}
\frac{3}{2} & \frac{1}{4} \\
-\frac{1}{2} & \frac{3}{4}
\end{bmatrix}
\];
(d.) 
\[
D = \begin{bmatrix}
\frac{3}{4} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{2}
\end{bmatrix}
\]

6. (10 total). Consider a Markov process with initial probability distribution 
\[
q_0 = \begin{bmatrix}
\frac{1}{3} \\
0 \\
\frac{1}{2}
\end{bmatrix}
\]
and the following matrix: 
\[
M = \begin{bmatrix}
0 & \frac{1}{2} & 0 \\
\frac{1}{6} & \frac{1}{2} & \frac{2}{3} \\
\frac{1}{2} & 0 & 0
\end{bmatrix}
\]
Find the following three probability distributions \(q_1, q_2, \) and \(q_3\).

7. (15 total - 5 points for each part). Find the unique fixed probability vector of each stochastic matrix:
(a.) 
\[
A = \begin{bmatrix}
\frac{1}{3} & 1 \\
2 & 0 \\
\frac{3}{4} & 0
\end{bmatrix}
\]
(b.) 
\[
B = \begin{bmatrix}
\frac{1}{2} & 2 \\
\frac{1}{2} & 1 \\
\frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]
(c.) 
\[
C = \begin{bmatrix}
.7 & .8 \\
.3 & .2
\end{bmatrix}
\]

8. (10 total). Find the unique fixed probability vector of the following regular stochastic matrix:
\[
P = \begin{bmatrix}
0 & \frac{1}{6} & 0 \\
1 & \frac{1}{2} & \frac{2}{3} \\
0 & \frac{1}{3} & \frac{1}{3}
\end{bmatrix}
\]

9. (10 total). Sally’s study habits are as follows. If she studies one night, she is 70 percent sure not to study the next night. On the other hand, if she does not study one night, she is only 60 percent sure not to study the next night as well. Find out how often, in the long run, Sally studies.