Problem Set 1

Due September 30 (Thursday) in Class

Solve each of the problems below. Please show all work. Point totals are given in parentheses after the number of the problem.

1. (10 total - 5 points for each part). Construct a truth table for each of these two logical propositions:
   (a.) \( p \Leftrightarrow \sim (\sim p) \).
   (b.) \( \sim (p \land q) \Leftrightarrow \sim p \lor \sim q \).

2. (15 total - 5 points for each part). Using quantifiers, deny the following propositions:
   (a.) \( \forall x \in R : \exists y \in R / x \leq y \).
   (b.) \( \forall x : (x \in R \Rightarrow x \in Q) \).
   (c.) \( \exists x \in N / x + 1 \leq x \).

3. (10 total - 2.5 points for each part). Rewrite explicitly giving the elements in each set:
   (a.) \( A = \{ x : x^2 - x - 2 = 0 \} \).
   (b.) \( B = \{ x : x \text{ is a letter in the word "follow"} \} \).
   (c.) \( C = \{ x : x^2 = 9, x - 3 = 5 \} \).
   (d.) \( E = \{ x : x \text{ is a digit in the number 2324} \} \).

4. (10 total - 2.5 points for each part). Let \( A = \{ x : x \in N, 0 < x < 10 \} \), \( B = \{2, 4, 6, 8\} \), \( C = \{1, 3, 5, 7, 9\} \), \( D = \{3, 4, 5\} \), and \( E = \{3, 5\} \). Which sets can equal \( X \) if we are given the following information:
   (a.) \( X \) and \( B \) are disjoint.
   (b.) \( X \subseteq D \) but \( X \not\subseteq B \).
   (c.) \( X \subseteq A \) but \( X \not\subseteq C \).
   (d.) \( X \subseteq C \) but \( X \not\subseteq A \).
5. (15 total - 5 points for each part). Decide in each of the following cases whether or not the given set is bounded. For those sets which are bounded, write down the smallest upper bound and the largest lowest bound. Decide which of the sets have a maximum and, where this exists, write down its value. Similarly, decide which of the sets have a minimum and, where this exists, write down its value.

   (a.) \( A = \{ x/|x| < 3 \} \).

   (b.) \( C = \{ x/|x| > 4 \} \).

   (c.) \( F = \{ x/|x + 1| < 4 \} \).

6. (5 total). Prove that \( A = \{2, 3, 4, 5\} \) is not a subset of \( B = \{ x : x \in \mathbb{N}, x \text{ is even} \} \).

7. (5 total). Prove: \( B \setminus A = B \cap A' \).

8. (10 total). If \( a \) and \( b \) are real numbers, prove that:

\[
 a \cdot (-b) = -(a \cdot b).
\]

9. (10 total). If \( a, b, \) and \( c \) are real numbers and \( a > b \), prove that:

If \( ac \leq bc \) then \( c \leq 0 \).

Please, use an indirect proof strategy.

10. (10 total). If \( x \) and \( y \) are nonnegative real numbers that satisfy \( x + y = 0 \), prove that:

\( x = 0 \) and \( y = 0 \).