Discounting of delayed rewards across the life span: age differences in individual discounting functions

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Abstract

The present effort addressed both the issue of the generality of choice models and the issue of possible qualitative developmental change in temporal discounting by examining behavior at the individual level across the life span. Data from individual children, young adults, and older adults who participated in two previous studies were analyzed [Green, L., Fry, A.F., Myerson, J., 1994. Discounting of delayed rewards: a life-span comparison. Psychol. Sci. 5, 33–36; Green, L., Myerson, J., Lichtman, D., Rosen, S., Fry, A., 1996. Temporal discounting in choice between delayed rewards: the role of age and income. Psychol. Aging 11, 79–84]. At all ages, a hyperbola-like function originally proposed by Green et al. (1994) based on group data, provided the best description of individual discounting functions. Two developmental trends were observed. The rate at which individuals discounted the value of delayed rewards decreased with age, and there was a systematic change in the shape of the discounting function. Each of these trends was reflected in a separate parameter of the model. The fact that the same mathematical model described the behavior of individuals of different ages suggests that age and individual differences in the discounting of delayed rewards are primarily quantitative in nature and reflect variations on fundamentally similar choice processes. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

People tend to discount the value of delayed rewards. For example, given a choice between immediate and delayed monetary rewards that are equal in amount, people usually choose the immediate reward. In fact, people will typically choose
a delayed over an immediate reward only if the amount of the delayed reward is greater than the amount of the immediate reward. This behavior is often interpreted as resulting from a decrease in the subjective value of a reward as the delay until its receipt increases. Thus, people may be said to choose immediate over delayed rewards of equal amount because the value of a delayed reward is discounted. Similar behavior in situations involving choice between immediate and delayed rewards has been reported in rats and pigeons as well as humans (Green et al., 1981, 1994a; Mazur, 1987; Rachlin et al., 1991).

Discounting is central to accounts of impulsivity and self control (Ainslie, 1992) and has been used to explain addiction, gambling, and risk taking in general (Rachlin, 1990, 1992). Because discounting plays such a fundamental role in behavior in many different situations, it is important to understand how age and individual differences influence discounting. A prerequisite to understanding such influences is the development of models and measurement techniques that describe discounting behavior at the individual level across the life span.

In a previous study comparing the discounting behavior of children and young and older adults, participants were given a choice between immediate and delayed hypothetical monetary rewards (Green et al., 1994b). As the time until receipt of the delayed reward increased, the present value of the reward decreased monotonically in all three age groups, but the rate of decrease was greatest for the child group (mean age 12 years), intermediate for the young adults (mean age 20 years), and least for the older adults (mean age 68 years). Although consistent with the notion of age-related increases in self control, the results are also important because they show no qualitative differences among the three age groups. For all three groups, discounting was well described ($R^2 > 0.99$) by the same mathematical function:

$$ V = \frac{A}{1 + kD} $$

where $V$ and $A$ represent the subjective or present value of the delayed reward and its amount, respectively, and $D$ is the delay. The parameters $k$ and $s$, respectively, govern the rate of discounting and the scaling of amount and/or delay.

It is well known, however, that the form of the function that best describes group average data may not be the same as that which best describes data at the individual level (Sidman, 1952; Estes, 1956). In a previous paper (Myerson and Green, 1995) we reanalyzed the young adult data from Green et al. (1994b) and verified that functions of the form of Eq. (1) (with $s < 1.0$ in nine out of 12 cases) accurately described discounting at the individual level and did so better than either simple hyperbolas (i.e. Eq. (1) with $s = 1.0$) or exponential functions. The focus of the Myerson and Green paper was on providing empirical and theoretical justification for mathematical models of individual discounting and, accordingly, we did not reanalyze the child and older adult data.

Thus, it remains to be determined whether Eq. (1) provides a more accurate description of discounting behavior in children and older adults than the other models that have been proposed (i.e. hyperbola or exponential decay). The hyperbola

$$ V = \frac{A}{1 + kD} $$

has been used by a number of psychologists to describe discounting (Mazur, 1987; Rachlin, 1989; Ainslie, 1992). In contrast, economists have traditionally used exponential functions of the form

$$ V = A e^{-kD} $$

when considering behavior in situations involving what they refer to as ‘intertemporal choice’ (for a historical overview, see Loewenstein, 1992).

It has been shown that both Eqs. (1) and (2) are better descriptive models than exponential decay functions for young adult behavior (Myerson and Green, 1995). Choice theorists often implicitly assume that their models are extremely general and apply to different populations if not to different species. However, such generality is an empirical question, and the issue of whether the best model of discounting in young adults is also the best for children and older adults has yet to be addressed. In addition, it is possible that there are qualitative changes in choice behavior across the life span, as might be expected by a developmental stage theory of choice and decision making. By examining behavior at the individual level across
the life span, the present investigation addresses both the issue of the generality of choice models and the issue of possible qualitative developmental change. In order to address these issues, we reanalyze individual data from the child and older samples studied by Green et al. (1994b, 1996) and compare these data with those for young adults.

2. Method

2.1. Participants

The present study analyzes the data from individual sixth-grade children \( (n = 12; \text{mean age} = 12.1 \text{ years}) \) and older adults \( (n = 12; \text{mean age} = 67.9 \text{ years}) \) whose group median data were reported in Green et al. (1994b) and the individual upper-income older adults \( (n = 20; \text{mean age} = 70.7 \text{ years}) \) whose group median data were reported in Green et al. (1996) and whose median discounting rate closely matched that of the older adults studied by Green et al. (see Fig. 2 in Green et al., 1996). Because the children were studied with delayed rewards of $100 and $1000 whereas the young and older adults were studied with $1000 and $10000, the present analyses focused on the one delayed reward amount (i.e. $1000) that was used with all three age groups.

2.2. Procedure

Participants were asked to make a series of choices between a hypothetical $1000 reward available after a delay and a smaller amount available immediately. They viewed two sets of 4 × 6-inch cards. The card on the left indicated the immediate amount and the card on the right indicated the $1000 reward as well as the delay until its receipt. For example, participants were asked to choose between $600 now or $1000 in 3 years. There were eight delays for the $1000 reward ranging from 1 week to 25 years and 30 immediate amounts ranging from $1 to $1000. For each of the eight delays used for the $1000 reward, the immediate amounts were presented in both ascending and descending orders. In the ascending series, the amount of the immediate reward was increased until the participant switched preference from the delayed to the immediate reward. In the descending series, the amount of the immediate reward was decreased until the participant switched preference from the immediate to the delayed reward. The subjective value of the delayed reward \( (V \text{ in Eqs. (1)–(3)}) \) was defined as the average of the immediate amounts preceding and following preference reversal. Detailed descriptions of the procedure may be found in Green et al. (1994b, 1996).

3. Results

Consistent with previous reports comparing discounting functions (Rachlin et al., 1991; Green et al., 1994b; Kirby and Marakovic, 1995), the group median data for all three of the present samples (children, young adults, and older adults) were better described by a simple hyperbola (Eq. (2)) than by an exponential decay function (Eq. (3)). The first set of analyses examined whether this was also true at the individual level for each of these age groups.

For the individual children, the hyperbola accounted for more of the variance than the exponential in nine out of 12 cases whereas the exponential accounted for more of the variance in only one case. (In the remaining two cases, neither function was able to fit the data; that is, the \( R^2 \)'s were less than or equal to 0.) The difference between the fits of the hyperbola and exponential to individual data was statistically reliable; Wilcoxon signed-rank test on the \( R^2 \)'s; \( Z = 2.70, P < 0.01 \).

Similar findings were obtained with respect to the other two age groups. For the young adults, the hyperbola accounted for more of the variance than the exponential in seven cases and the exponential accounted for more of the variance in three cases (two cases could not be fit by either function). This difference in the fits to individual data was statistically reliable; Wilcoxon signed-rank test; \( Z = 2.09, P < 0.05 \). For the older adults, the hyperbola accounted for more of the variance than the exponential in 20 cases and the exponential accounted for more of the variance in ten
cases (two cases could not be fit by either function). Again, the difference in the fits to individual data was statistically reliable; Wilcoxon signed-rank test; $Z = 2.79$, $P < 0.01$.

The second set of analyses examined whether the fit of the hyperbola to individual data was significantly improved by adding an exponent. This was done by testing whether the exponent in Eq. (1) differed significantly from 1.0 based on a $t$-statistic in which the numerator was 1.0 minus the estimated value of $s$ and the denominator was the standard error of the estimate (Gallant, 1987). The degrees of freedom for such a test is equal to the number of data points (i.e. eight) minus the number of parameters in the model (i.e. two).

For 11 of the 12 children, the estimated value of $s$ was less than 1.0, and significantly so in nine cases; all $t$s > 4.6, all $P$s < 0.01. Exemplary and representative fits to individual data are shown in Fig. 1. The symbols represent the immediate amount judged subjectively equivalent to the $1000 reward at each delay. The curves represent the fits of Eq. (1) to these data. The children whose data are depicted in the upper two panels are those whose data were best fit by Eq. (1) (i.e. the two for whom the $R^2$ was highest). The children whose data are depicted in the lower two panels are those with the median $R^2$s for their age group.

For the young adults, the estimated value of $s$ was less than 1.0 in 8 out of 12 cases, and in six of these, the difference was significant; all $t$s > 4.5, all $P$s < 0.01. The upper and lower left panels of Fig. 2 show exemplary and representative fits of Eq. (2) to data from individual young adults for whom $s$ did not differ significantly from 1.0. The upper and lower right panels show exemplary and representative fits of Eq. (1) to data from individuals for whom $s$ was significantly less than 1.0.

Of the 32 older adults, the estimated value of $s$ was less than 1.0 in 17 cases, and in 13 of these, this difference was significant; all $t$s > 3.5, all $P$s < 0.02. Exemplary and representative fits of...
Eqs. (1) and (2) to data from individual older adults are shown in Fig. 3. As in Fig. 2, the upper left panel shows the results for the individual with the highest $R^2$ of those for whom $s$ did not differ from 1.0, the lower left panel shows the results for the individual with the median $R^2$, and the right panels show the results for the corresponding individuals from those for whom $s$ was significantly less than 1.0.

The preceding analyses indicate that the data from individuals in all three age groups was well described by a hyperbola-like function (Eq. (1) or Eq. (2)). In addition, these analyses show that raising the denominator of the function to a power less than 1.0 significantly improves the fit for some, although not all, individuals. Importantly, for no individual of any age was the fit significantly improved by raising the denominator to a power greater than 1.0. As may be seen in Table 1, Eq. (1) accounted for an extremely high proportion of the variance in both the group median data and the individual data for all three age groups.

Additional analyses examined changes in the parameters of individual discounting functions across the life span. Notably, the median of the estimated values of $s$ for individual participants increased systematically with age from 0.25 in children to 0.80 in older adults, and the proportion of individuals for whom $s$ was less than 1.0 was significantly larger for the child group than for the older adult group; Fisher’s exact test, $P < 0.05$. The median estimated value of $s$ for young adults was intermediate between that for the younger and older groups, and the proportion of young adults for whom $s$ was less than 1.0 did not differ significantly from that for either of the other groups. Further evidence of a life span trend in the $s$ parameter is provided by the systematic decrease with age in both the proportion of individuals for whom $s$ was less than 1.0 and the proportion for whom $s$ was significantly less than 1.0 (Table 1).

<table>
<thead>
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<th>Group</th>
<th>Parameters for fits to group medians</th>
<th>Parameters for individual fits</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$k$</td>
<td>$s$</td>
</tr>
<tr>
<td>Children</td>
<td>0.618</td>
<td>0.368</td>
</tr>
<tr>
<td>Young adults</td>
<td>0.075</td>
<td>0.724</td>
</tr>
<tr>
<td>Older adults</td>
<td>0.010</td>
<td>0.957</td>
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</tbody>
</table>

$^a$ This column reports the percentage of individuals for whom $s$ was less than 1.0, regardless of statistical significance.  
$^b$ This column reports the percentage of individuals for whom $s$ was significantly less than 1.0 (all $Ps < 0.02$).
As may be seen in Table 1, there is also clear evidence of a life span trend in the \( k \) parameter. The median of the estimated values of \( k \) for individual participants decreased systematically with age from 1.026 in children to 0.016 in older adults. The median estimated \( k \) for the individual children was significantly greater than that for the individual older adults; Mann–Whitney rank sum test, \( T = 391, P < 0.01 \). The median estimated \( k \) for the individual young adults was intermediate between that for the children and the older adults and did not differ significantly from either.

4. Discussion

The results of the present analyses demonstrate that mathematical models of temporal discounting (Eqs. (1) and (2)) accurately describe the behavior of individuals ranging in age from 11 to 79 years. At all ages, a simple hyperbola (Eq. (2)) provided a better description of individual discounting behavior than an exponential decay function (Eq. (3)). Raising the denominator of the hyperbola to a power less than 1 (Eq. (1)) produced a statistically significant increase in the accuracy of the fit for 75% of the children, 50% of the young adults, and approximately 40% of the older adults. For each age group, the median proportion of variance in individual data that was accounted for by Eq. (1) was greater than 0.90.

There are two developmental trends in the discounting behavior, and each is captured by a separate parameter of Eq. (1) (Table 1). As individuals get older, they appear to discount the value of delayed rewards less steeply and, in addition, the shape of the discounting function changes systematically. The first of these developmental trends (i.e. the decrease in discounting rate with age) was reflected in a systematic decrease in the \( k \) parameter with age. This trend is not simply explained by the fact that as one gets older, the same absolute amount of time represents a smaller and smaller percentage of one’s life span. The data show a similar trend when the subjective value of a delayed reward for different age groups is compared at delays that represent the same relative amount of time, given the age of the group, rather than the same absolute amount of time. For example, at delays of 10% of the mean age of the group (i.e. ~1 year for the child group, 2 years for the young adult group, and 7 years for the older adult group), the subjective value of the $1000 reward was still lowest for the children and highest for the older adults.

The second developmental trend (i.e. the systematic change in the shape of the discounting function) is reflected in an increase in the \( s \) parameter with age. More specifically, although the subjective value of a reward showed a negatively accelerated decrease with delay for all individuals, for some the negative acceleration was even more pronounced than predicted by a simple hyperbola. For these individuals, the value of \( s \) was less than 1.0. The proportion of such individuals decreased with age. In addition, the median value of individual \( s \) parameters increased from children to young adults, and again from young to older adults. Notably, even for older individuals the median value of \( s \) was still only 0.8 and was never significantly greater than 1.0 for any individual.

Previous research indicates that Eq. (1) provides a better description of temporal discounting at the group level than a simple hyperbola (Green et al., 1994b; Ostaszewski et al., 1998). However, fits to group data, although they may provide useful descriptive information, are of uncertain theoretical significance for understanding individual behavior. One could even argue that fits to group data are of uncertain theoretical significance for understanding group differences. After all, what is important about group differences is what they tell us about differences between individuals, albeit differences between individuals who belong to different groups. Consequently, regardless of whether the focus is on individual behavior or group differences, one needs to know whether fits to group data are representative of fits to individual data. A major goal of the present effort was to address this need. The present findings demonstrate that Eq. (1) provides a better description of discounting at both the individual and group levels than the two other models that have been proposed.
With respect to these other models (i.e. the simple hyperbola and the exponential decay function), the hyperbola has been shown to be superior to the exponential at both the group and individual levels in adults (Rachlin et al., 1991; Myerson and Green, 1995; Kirby, 1997), and the present study shows that this is true for children as well. What distinguishes a simple hyperbola (Eq. (2)) from Eq. (1) is the value of the exponent. For the majority of individuals examined in the present analyses, the exponent $s$ in Eq. (1) was less than 1.0, suggesting that Eq. (1) provides a better model of individual behavior. Of course, when the exponent equals 1.0, Eq. (1) reduces to the simple hyperbola, revealing that the latter is a special case of the former. Thus, although there may be cases where the hyperbola works as well as Eq. (1), the latter provides a more general model of discounting.

There are at least two reasons why it is important to use the best mathematical model to describe data. For theoretical purposes, the mathematical form of the model has implications for the mechanisms underlying the observed behavior, and a better model is more likely to provide insight into such mechanisms. For descriptive purposes, the parameters of a mathematical model may help characterize behavior, and the parameters of a better model may provide a better basis for comparing either different individuals or behavior in different situations. The focus of the present effort has been primarily on the descriptive aspect, and we have shown that both parameters of Eq. (1) serve to characterize life-span developmental changes in individual behavior. These changes might have been missed using other approaches or models, particularly those that attempt to describe discounting using only one parameter.

The decreases in the $k$ parameter of Eq. (1) with age suggest that the ability to delay gratification increases as one ages. This has been shown most clearly for children (Mischel et al., 1989), but there is also considerable evidence that risk-taking decreases with age during adulthood (Zuckerman et al., 1978; Ball et al., 1984; Wilson and Herrnstein, 1985, Chapter 5), suggesting that there are differences in impulsivity between young and older adults. The increases in the $s$ parameter with age observed in the present analyses suggest developmental changes in the way time is scaled. To children, for example, delays greater than a few years may be relatively equivalent, and this is reflected in the small values of the $s$ parameter. Young adults have larger values of $s$ than children but smaller values than older adults. Whether these age differences reflect maturational changes or differences in experience with long-delayed rewards remains to be determined.

By examining behavior at the individual level across the life span, the present study addressed both the issue of the generality of choice models and the issue of possible qualitative developmental change in temporal discounting. The present findings demonstrate that the model represented by Eq. (1) is extremely general in that it describes the discounting of delayed rewards in individuals from childhood to old age. The fact that the same mathematical model captures these changes suggests that they are primarily quantitative in nature and argues against qualitative changes of the sort that might be predicted by developmental stage theories of choice and decision-making. So, too, although individuals may report using different decision making strategies, the fact that the same model describes their behavior suggests that individual differences in discounting are primarily quantitative in nature and reflect variations on fundamentally similar choice processes.

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References