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When Payoffs Look Like Probabilities: Separating Form and Content in Risky Choice

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Paralleling research in perception, behavioral models of risky choice posit "psychophysical" transformations of material outcomes and probabilities. Prospect theory assumes a value function that is concave for gains and convex for losses, and an inverse S-shaped probability weighting function. But in typical experiments, form and content are confounded: Probabilities are represented on a bounded numerical scale, whereas representations of monetary gains (losses) are unbounded above (below). To unconfound form and content, we conducted experiments using a probability-like representation of outcomes and an outcome-like representation of probability. We show that interchanging numerical representations can interchange the resulting psychophysical functions: A proportional (rather than absolute) representation of outcomes leads to an inverse S-shaped value function for gains. This alternative value function generates novel framing effects, a common ratio effect for bounded gains, and a "framing interaction," where gain-loss framing matters less for proportional outcomes. In addition, we show that an absolute (rather than proportional) representation of probability reduces sensitivity to large probabilities. These findings highlight the deeply constructive nature of the psychophysics of risky choice, and suggest that traditional models may reflect subjective reactions to numerical form rather than substantive content.

Keywords: decision making, framing effects, prospect theory, psychophysics, risk attitudes

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Attempts to map out the transformations that convert objective stimulus dimensions to internal subjective representations have occupied a central place in experimental psychology since its inception (Fechner, 1860). Careful study of psychophysical functions has been especially prominent in perceptual research—for example, in determining the functional form that best captures diminishing sensitivity to objective dimensions such as light intensity (e.g., Stevens, 1957). Inspired by work in perception, researchers in decision making have posited analogous "psychophysical" transformations of abstract quantities like probability and monetary amounts: Prospect theory, the leading behavioral model of risky choice (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), assumes a concave value function for gains relative to a reference point, a convex value function for losses, and an inverse-S shaped weighting function for probability (see Figure 1). The value function captures diminishing sensitivity to monetary magnitudes, while the weighting function captures strong reactions to changes in probability at the extremes (near impossibility and certainty), and a relative insensitivity to increments in intermediate probabilities. Prospect theory's value and weighting functions generate several behavioral "anomalies" that challenge standard economic models of rational choice, such as framing effects and the certainty effect (Kahneman & Tversky, 1979; Tversky & Kahneman, 1981).

It is natural to suppose, and often implicitly assumed, that these functions capture general attitudes toward outcomes (i.e., gains or losses) and probabilities. From this perspective, the difference in shape between the value and weighting functions reflects the fact that outcomes and probabilities are different kinds of entities, and people have accordingly different reactions to them. But in typical decision making experiments, outcomes and probabilities are also provided to participants in different numerical representations that have different formal properties. Outcomes are typically conveyed in an absolute representation that is unbounded in one direction (above for gains, below for losses). Probability, by contrast, is usually cast in a proportional representation, bounded by conspicuous lower (0%) and upper (100%) endpoints. The *form* of the numerical representations (bounded vs. unbounded) is thus confounded with the substantive *content* that the numbers represent

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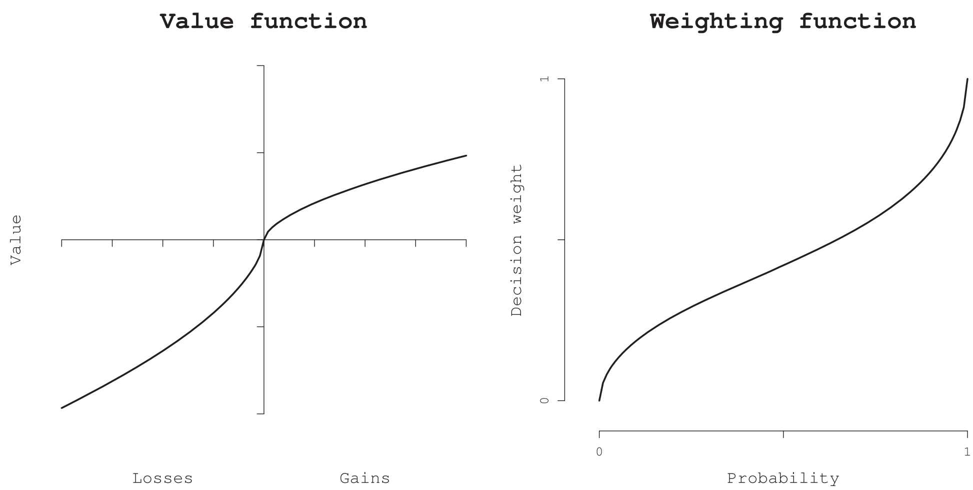

Figure 1. Prospect theory: Representative value and probability weighting functions.

(probabilities vs. outcomes). This common confounding of form and content introduces an ambiguity in the interpretation of prospect theory's value and weighting functions: Do the shapes of the functions reflect different reactions to different objective content, to different formal representations, or to both? A resolution of this ambiguity would have important implications for the psychological underpinnings of prospect theory, and for the scope of the anomalies it predicts. Here, we report studies that unconfound form and content, using a probability-like numerical representation of outcomes and an outcome-like numerical representation of probability.

There are reasons to suspect that numerical form, over and above substantive content, may be important in determining the shape of the value and weighting functions. First, while prospect theory is usually applied to monetary gains and losses, it has also been effectively applied in domains where substantive considerations are very different (such as lives saved or lost; Tversky & Kahneman, 1981) but formal properties are similar (a scale unbounded in one direction). Further, we suspect that, in responding to an absolute numerical representation of an unfamiliar or even meaningless objective dimension, such as "the number of grimbles in a splot," people would exhibit diminishing sensitivity, with the transition from 1 to 6 grimbles having greater subjective impact than the transition from 1821 to 1826. For abstract quantities like probability or grimbles—in contrast to hardwired perceptual inputs like light intensity—people may effectively construct their psychophysical functions on the fly, in which case numerical representations may well be critical. In particular, if discriminability is enhanced in the vicinity of salient landmarks or reference points on arbitrary quantitative scales, one would predict concave (convex) psychophysical functions for numerical representations with a single lower (upper) endpoint, and an inverse S-shape for representations with two endpoints. This principle can account for the shape of prospect theory's value and weighting functions, as noted by Tversky and Kahneman (1992) and Gonzalez and Wu (1999)—provided that outcomes and probabilities are represented in the usual way. But the same principle also implies that different numerical representations of the same objective quantities will lead to very different psychophysical functions. Specifically, it suggests a "reversal hypothesis": For large outcomes or probabilities, decision makers should exhibit diminishing sensitivity in a representation without an upper bound and accelerating sensitivity in a bounded representation. The reversal hypothesis, in turn, predicts a class of novel framing effects in risky choice.

To test these predictions, we investigated proportional representations of outcome and absolute representations of uncertainty.[1] Across five experiments, we found strong support for the reversal hypothesis. When the same probabilities and outcomes are conveyed with different numbers, people behave as if they have very different attitudes to risk. The findings suggest that the standard psychophysical functions for value and uncertainty primarily capture reactions to superficial numerical form rather than substantive content.

Overview of Experiments

Figure 2 illustrates the paradigm used in Experiments 1–3. Participants choose between two gambles offering different probabilities of winning different prizes, drawn from a pot containing a fixed total amount ($3100). The numerical representation of potential prizes is either formally unbounded ($; left panel) or bounded above (%; right panel), and we compare risk attitudes and the shape of the implied value function across the two representations. In Experiment 1, we show that framing large gains in proportional terms leads to a pronounced reduction in risk aversion. Experiment 2 uses psychometric methods to estimate value functions in both representations. The value function for gains that best describes choice behavior appears to be concave in the unbounded representation, but exhibits an inverse S-shape in the bounded representation. In Experiment 3, we demonstrate a novel common ratio effect for bounded gains, which provides further support for the reversal hypothesis. Experiment 4 extends the paradigm to losses as well as gains in a nonmonetary outcome domain

[1] We use the term "uncertainty" in the broad sense, to refer to any situation in which knowledge about future outcomes is incomplete. We note, however, that the term is sometimes used more narrowly, to contrast situations in which probabilities cannot be precisely quantified ("uncertainty") with ones in which probabilities are precisely known ("risk"). In our usage, as in Wakker (2010), "uncertainty" encompasses both types of situations, and "risk" is simply a special case of uncertainty.

Figure 2. Experimental paradigm in Experiments 1–3. The representation format of the monetary amounts, manipulated between participants, was either unbounded (left) or bounded (right) above. See the online article for the color version of this figure.

(lives saved or lost). As predicted by the reversal hypothesis, a well-known effect of gain-loss framing (Tversky & Kahneman, 1981) is markedly attenuated in the bounded representation. Turning to probability, Experiment 5 contrasts the subjective impact of chance in a bounded numerical representation (% of winning balls in an urn of fixed size) and in an unbounded representation (# of winning balls in the urn). When expressed on an unbounded scale, large probabilities appear to have a smaller subjective impact. Taken together, these findings indicate that subjective transformations of probability and outcome strongly depend on the formal properties of their numerical representations.

All experimental tasks consisted of web forms programmed in Qualtrics (Qualtrics, 2016). They generally included several screens with instructions and an example, one or more response screens, and a postexperimental demographic survey. All experiments involved forced choices between hypothetical gambles, with the exception of Experiment 5, in which participants rated the attractiveness of a single gamble. Complete sets of screenshots from two representative experimental tasks (Experiments 1 and 5) are included in the online supplemental materials. None of the experiments included any measures or conditions that are not reported.

Experiment 1

A strictly concave value function (Figure 1, left panel) predicts risk aversion for gains: When expected value is held fixed, a lower probability of a larger gain should be less appealing, because of diminishing sensitivity to increasing gains. Risk aversion for gains (when probabilities are not extreme) has been demonstrated in a wide range of studies using the standard unbounded representation of monetary amounts (e.g., Stott, 2006; Wakker, 2010). How, if at all, will risk attitudes change when gains are represented on a bounded scale? According to the reversal hypothesis, the usual concave value function for gains should become inverse S-shaped when outcomes are represented on a bounded scale. For small to moderate gains, concave and inverse-S shaped value functions are qualitatively similar, as sensitivity diminishes with increasing distance from the lower bound. While this pattern continues for the concave function, it eventually reverses for the inverse-S function, with *accelerating* sensitivity to increasing gains near the upper bound of the scale. This predicts less risk aversion for large gains in the bounded than in the unbounded representation. We tested the predicted reduction in risk aversion in Experiment 1.

Method

A total of 145 undergraduate students at UCSD's Rady School of Management (33.8% female; M_{age} = 21.6 years) participated in Experiment 1 for partial course credit. The stimuli are shown in Figure 2. Participants made a single choice between two hypothetical gambles, and were told that "each gamble offers a chance at winning some money from a pot of prize money" holding $3100. Each participant chose between a relatively risky gamble offering $2976, or 96% of the pot, with probability .2, and a safer gamble offering $2170, or 70% of the pot, with probability .25. (The riskier gamble was designed with an expected value advantage, to balance the attractiveness of the two gambles in the unbounded condition.) Participants in the unbounded condition (n = 72) saw all monetary amounts expressed in dollar terms (Figure 2, left panel) while those in the bounded condition (n = 73) saw all monetary amounts in percentage terms (right panel). The total pot size was prominently displayed in both conditions. The conditions thus differ only in the formal properties of the payoffs' numerical representation (# vs. %).

Results

In the unbounded condition, 51.4% of participants selected the relatively safe gamble, whereas only 24.7% selected the safer gamble in the bounded condition, $\chi^2(1, N = 145) = 11.00$, $p <$ .001. This novel framing effect supports the reversal hypothesis. Risk aversion is markedly reduced when large gains are framed in proportional terms.

Experiment 2

To obtain a more detailed picture of value sensitivity, we estimated value functions for both unbounded and bounded gains in a psychometric study featuring a wide range of gambles. Insofar as the value and weighting functions reflect reactions to numerical form rather than substantive content, the standard concave value function for unbounded gains should turn into an inverse S-shape when gains are framed proportionally. The design also permits an extended conceptual replication of Experiment 1: For gamble pairs involving relatively large gains, risk aversion should be reduced in the bounded representation.

Method

A total of 472 undergraduate students in the UCSD Psychology Department (67.6% female; M_{age} = 20.0 years) participated in Experiment 2 for partial course credit. Each participant was assigned to either the unbounded (n = 234) or bounded (n = 238) condition, and made 10 choices between gambles in the same representation, with a fixed pot size of $3100. The gambles were randomly drawn from a master set that ranged widely in probabilities and monetary amounts. To infer value functions from the choice data, we utilized a semiparametric procedure introduced in Stewart, Reimers, and Harris (2015). The method lends itself to modeling repeated-measures data, which it can accommodate via nonlinear mixed-effects modeling. This approach does not make any a priori assumptions about the value function (i.e., it does not assume a parametric form) and thus allows the function to take any—standard or nonstandard—shape, though (as explained below) it requires auxiliary assumptions about probability weighting.

Stimuli. The stimuli were pairs of hypothetical gambles. Each gamble involved some probability of winning a specified amount from a $3100 pot, and otherwise winning nothing. We constructed individual gambles by crossing the winning probabilities {.1, .3, .5, .7, .9} with the payoffs corresponding to {5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%, 85%, 95%} of the $3100 pot, resulting in 50 distinct gambles.

From the 1225 possible (unordered) pairings of these gambles, we selected 100 gamble pairs to be included in the study (for a complete list, see the online supplemental materials). We excluded pairs in which one gamble dominated the other. We also sought to avoid pairs in which one gamble is clearly superior to (i.e., has a much larger expected value than) the other, while maintaining an equal distribution of probabilities and outcomes. To do so, we used the COIN-OR branch-and-cut routine for integer programming implemented in the open source software OpenSolver (Lougee-Heimer, 2003; Mason, 2012; OpenSolver, 2017) to minimize the total sum of the absolute within-pair expected value (EV) differences, subject to two sets of constraints. First, the five probabilities were required to appear equally often (i.e., in exactly 40 gambles). Second, the payoffs were also required to appear equally often (i.e., in exactly 20 gambles). Because of a programming error, there was a small distortion in computing within-pair EV differences. The error inflated the probabilities for the safe gamble in all pairs by .1 for selection purposes, leading the algorithm to match safe gambles with risky counterparts that tended to have slightly higher EVs. As a result, the final list generated by the algorithm, while satisfying all of the above constraints (equal occurrences of probabilities and outcomes, with dominance excluded), is a non-optimal solution to the selection problem. The list, however, mostly overlaps with an optimal solution, and robustness checks reported in the online supplemental materials indicate that the discrepancy does not affect the findings reported below.

Each participant encountered 10 gamble pairs that were sampled without replacement from the list of 100 pairs. Across all participants, each gamble pair was presented at least 12 times in each condition, and the distribution of gamble pairs in the two conditions did not differ significantly, $\chi^2(99, N = 472) = 100.91$, $p = .43$.

Estimation method. To estimate the value functions, we used a semiparametric procedure developed by Stewart et al. (2015). This approach treats the subjective valuations of each of the 10 payoffs in each of the two conditions as independent parameters and thus allows the functions to take any shape. Formally, it amounts to maximum-likelihood estimation of the value parameters u_c in

$$\Pr(R) = \frac{b_c[w(p)u_c(x)]^{\gamma_c}}{b_c[w(p)u_c(x)]^{\gamma_c} + [w(q)u_c(y)]^{\gamma_c}} \quad (1)$$

where b_c captures an overall bias toward risk-seeking (or risk-aversion) in condition c; $w(p)$ and $w(q)$ are the decision weights associated with the winning probabilities of the riskier and the safer gambles, respectively; x and y are the respective payoffs for the riskier gamble (i.e., the gamble with a lower probability of a larger gain) and the safer gamble (i.e., the gamble with a higher probability of a smaller gain); and γ_c controls how differences in value translate to differences in choice probabilities in condition c.

This approach requires two noteworthy assumptions. First, because the procedure does not allow joint estimation of values and probability weights, auxiliary assumptions about probability weighting are needed. The analyses reported below assume a standard cumulative prospect theory (CPT) weighting function, shown in Figure 1, using the parametrization and median parameter estimate from Tversky and Kahneman (1992). Additional analyses using other weighting functions (a linear function, or a CPT weighting parameter derived from our data in an alternative two-step estimation procedure) are reported in the online supplemental materials. These analyses yield value functions generally similar to those reported below, though the assumption of linear probability weighting results in reduced curvature in both the unbounded and bounded value functions. Second, the procedure accommodates response noise via Luce's choice rule (Luce, 1959). A standard method for incorporating a stochastic component in a deterministic choice model, this assumption implies that the odds with which one option is chosen over another are proportional to the ratio of the options' valuations (for details, see the online supplemental materials).[2]

Results

Figure 3 plots estimated subjective valuations for different payoffs for participants in the unbounded (left panel) and bounded (right panel) conditions, along with best-fitting third-degree polynomials. In the standard unbounded condition, the value function appears to be concave, consistent with previous psychometric studies (Abdellaoui, 2000; Camerer & Ho, 1994; Gonzalez & Wu,

[2] Stewart, Canic, and Mullett (2017) identify a methodological problem that arises in comparing value functions inferred from different choice sets, and that complicates the interpretation of Stewart et al. (2015). The problem does not apply to the present experiment, however, because the choice sets in the two representation conditions are not systematically different.

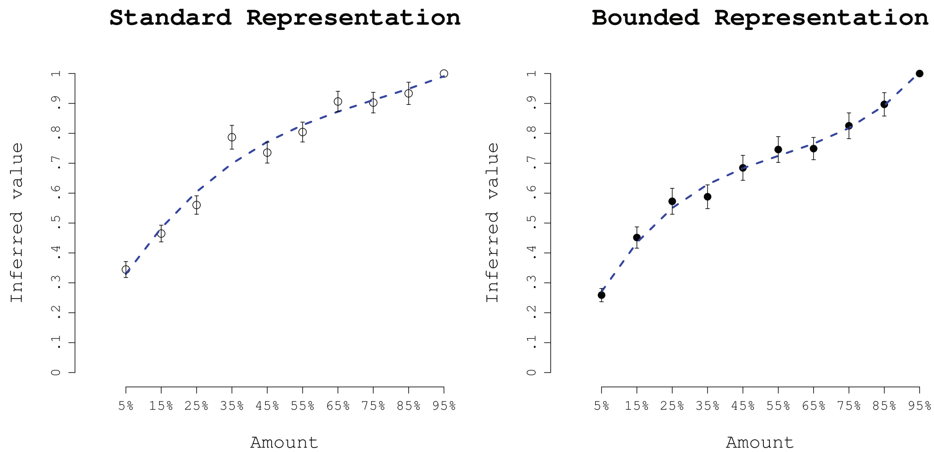

Figure 3. Experiment 2: Value functions inferred from choice behavior for the standard (unbounded above; left panel) and bounded (right panel) representations of monetary gains. Error bars show bootstrap *SE*s. (Note that inferred values and their *SE*s are normalized relative to the highest value within each condition; see the online supplemental materials for details.) See the online article for the color version of this figure.

1999; Stott, 2006; Tversky & Kahneman, 1992; but see Stewart et al., 2015). Subjective valuations in the bounded condition exhibit a different pattern. While the function decelerates for small-to-moderate amounts, it appears to accelerate for amounts near the upper bound—a property usually seen in probability weighting functions but not value functions. This acceleration predicts that, as in Experiment 1, risk aversion in the bounded condition should be reduced for gamble pairs involving at least one gain near the upper bound.

To test this prediction, we examined the 40 gamble pairs featuring either of the two large gains—85% and 95% of the $3100 pot—for which acceleration is apparent in the right panel of Figure 3. Directionally, participants in the unbounded condition selected the safer option more often than their counterparts in the bounded condition in 29 choice pairs, and selected the riskier option more often in only 10 pairs ($p < .01$, sign test; Figure 4). Collapsing across all 40 pairs involving large gains, the risk-averse option was chosen 67.4% of the time in the unbounded condition and only 59.1% of the time in the bounded condition, $\chi^2(1, N = 1881) = 13.92$, $p < .001$. By contrast, for gamble pairs not featuring either of these large gains, the safer option was selected 64.4% of the time in the unbounded condition and 62.7% in the bounded condition, $\chi^2(1, N = 2839) = 0.88$, $p = .35$. Risk attitudes systematically differ across representations only in the vicinity of the upper bound. The reduced risk aversion for bounded gains demonstrated in Experiment 1 is thus not limited to the specific stimuli used there, but generalizes to a broad range of probabilities and outcomes.[3] These findings, together with the value functions in Figure 3, provide strong evidence that value sensitivity is not invariant across representations. When the numerical representation makes an upper bound salient, relatively large amounts are more impactful. Framing gains in proportional terms thus yields a value function that resembles a standard probability weighting function.

Experiment 3

The results of the psychometric study suggest that the value function for bounded gains (Figure 3, right panel) is nonlinear—and, more generally, that it cannot be represented by a power function. More direct evidence for this conclusion comes from a common ratio design. This design, traditionally used to demonstrate nonlinear weighting of *probabilities* (Allais, 1953; Kahneman & Tversky, 1979), here reveals that the bounded-value function is not a power function.

Method

A total of 200 online workers from the general U.S. population at Amazon Mechanical Turk (33.0% female; $M_{age} = 35.5$ years) participated in Experiment 3 in exchange for $0.10. Each participant made a single choice between two gambles in the bounded representation, with a pot size of $3100. The gamble pairs differed across conditions in the prize money at stake (high vs. low), as shown in Table 1. Note that the high-stakes gambles were constructed by multiplying each possible gain in the low-stakes gambles by a common factor of 7. If value sensitivity is captured by a power function—$v(x) = cx^a$, where $a = 1$ in the linear case—then (assuming any fixed probability weighting function) the product of value and (weighted) probability will be higher for the safer option in the low-stakes condition if and only if it is also higher in the high-stakes condition. A linear or other power function for value

[3] In particular, the reduction in risk seeking does not depend on whether the risky gamble has higher expected value (EV), as in Experiment 1, or lower expected value. For gamble pairs involving large gains (85% or 95%) in which the risky gamble has the higher EV, the safe option was chosen 62.6% of the time in the unbounded condition and 54.8% of the time in the bounded condition, $\chi^2(1, N = 1337) = 8.442$, $p < .01$. For gamble pairs involving large gains in which the safe gamble has the higher EV, the safe option was chosen 80.0% of the time in the unbounded condition and 69.0% of the time in the bounded condition, $\chi^2(1, N = 544) = 8.441$, $p < .01$.

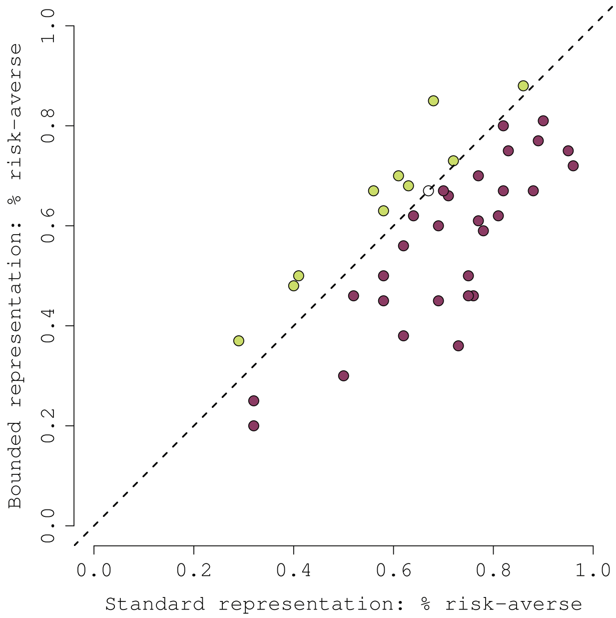

Figure 4. Risk aversion across the two representations for gamble pairs involving a large monetary gain (85% or 95% of the pot) in Experiment 2. Purple (light green) points indicate gamble pairs for which there was less risk aversion in the bounded (unbounded) representation. See the online article for the color version of this figure.

sensitivity would thus imply similar choice behavior in the low- and high-stakes conditions.

Results

Whereas 58.4% of participants chose the safer gamble in the low-stakes condition, only 30.3% chose the safer gamble in the high-stakes condition, $\chi^2(1, N = 200) = 16.00$, $p < .001$. This common ratio effect cannot be captured by a power function. It is, however, compatible with an inverse S-shaped value function, shifting from diminishing to increasing sensitivity across the range of possible bounded gains. For proportional outcomes as for probabilities, scaling up all values by a common factor triggers predictable shifts in risk preference.

Experiment 4

The previous studies have all examined monetary gains, and corroborate prospect theory's concave value function for unbounded, but not for bounded, monetary gains. Prospect theory has also been applied to nonmonetary outcomes, and it posits diminishing sensitivity for losses as well as gains. This pattern generates framing effects when the same outcomes are alternately framed as gains or losses relative to a manipulated reference point, as in the well-known Asian disease problem (Tversky & Kahneman, 1981) and numerous related paradigms (reviewed in Druckman, 2001; Levin, Schneider, & Gaeth, 1998). In Experiment 4, we examined sensitivity to nonmonetary gains and losses, adapting the Asian disease problem to a setting in which outcomes are readily represented on both bounded and unbounded scales.

Method

A total of 1000 workers from Amazon Mechanical Turk (45.5% female; $M_{age} = 34.0$ years) participated in Experiment 4 in exchange for $0.10. We excluded 26 participants who reported being familiar with the Asian disease problem from all further analyses. All findings reported below and their statistical significance remained qualitatively unchanged when these participants are included.

Participants in four framing conditions were asked to:

> Imagine that a remote mountain village is facing a major natural disaster. The disaster is expected to kill the village's 600 inhabitants. Two alternative programs have been proposed in response to the danger. Assume that the exact scientific estimates of the consequences of the programs are as follows.

Participants who received the unbounded-gain frame ($n = 248$) chose between the following two programs [with the unbounded-loss frame ($n = 242$) in brackets].

> If Program A is adopted, 200 inhabitants will be saved [400 inhabitants will die].

> If Program B is adopted, there is 1/3 probability that 600 inhabitants will be saved [0 inhabitants will die], and 2/3 probability that 0 inhabitants will be saved [600 inhabitants will die].

Though the programs have numerically equivalent outcomes in the unbounded-gain and -loss frames, diminishing sensitivity predicts different choices in the two conditions. Participants should be risk-averse (selecting Program A) in the gain ("saved") frame, because gaining three times as much (600 vs. 200) is not three times as good. But participants should be risk-seeking (selecting B) in the loss ("die") frame, because losing 1.5 times as much (600 vs. 400) is not 1.5 times as bad.

In the remaining conditions of Experiment 4, gain or loss outcomes were represented on a bounded scale. Participants in the bounded-gain condition ($n = 239$) saw the following two programs [bounded-loss condition ($n = 245$) in brackets].

> If Program A is adopted, 33% of the inhabitants will be saved [67% of the inhabitants will die].

> If Program B is adopted, there is 1/3 probability that 100% of the inhabitants will be saved [0% of the inhabitants will die], and 2/3 probability that 0% of the inhabitants will be saved [100% of the inhabitants will die].

If value functions for bounded gains and losses are inverse S-shaped, then the framing effect should be reduced in the bounded representation. This is because diminishing sensitivity for small-to-moderate amounts is counteracted by accelerating sensi-

Table 1
Experiment 3: Common Ratio Effect for Monetary Amounts in a Bounded Representation (Pot Size = $3100)

Condition	Safe gamble	Risky gamble	Choices: n (%) safe
Low stakes	$p = .39$ to win 11%	$p = .35$ to win 14%	59/101 (58.4%)
High stakes	$p = .39$ to win 77%	$p = .35$ to win 98%	30/99 (30.3%)

tivity near the upper bound. The reversal hypothesis thus predicts reduced risk aversion in the gain fame and reduced risk seeking in the loss frame.

Results

Consistent with previous research, we found a strong framing effect in the standard unbounded conditions, with clear risk aversion in the gain frame (69.4% selecting Program A) and clear risk seeking in the loss frame (only 36.8% selecting A). In the bounded conditions, the framing effect was markedly attenuated, with a majority of participants selecting Program A in both the gain frame (65.3%) and the loss frame (51.8%). A logistic regression confirms a significant frame-by-representation interaction ($b = -.80$, $SE = .27$, $z = -3.0$, $p < .01$). While the framing effect was significant in both the unbounded ($p < .001$) and bounded ($p < .01$) conditions, note that risk seeking for losses was eliminated in the bounded representation. Comparing the bounded and unbounded representations, the predicted reduction in risk seeking was significant for the loss frame, $\chi^2(1, N = 487) = 11.19$, $p < .001$, while the predicted reduction in risk aversion fell short of significance for the gain frame ($p = .34$). Numerical representation thus matters for nonmonetary as well as monetary outcomes, and for losses as well as gains. Going beyond the standard framing effect, these findings demonstrate a "framing interaction." The effect of gain-loss framing depends on whether the gains and losses are themselves framed in absolute or proportional terms.

Experiment 5

Experiments 1–4 indicate that when a bounded representation is imposed on gains and losses, the value function changes, with increased sensitivity to large amounts. In Experiment 5, we ask whether diminishing sensitivity to probability will emerge when an unbounded representation makes the natural upper bound for probability less salient. We used ball-and-urn gambles, in which probability may be expressed by the percentage (bounded representation) or number (unbounded representation) of winning balls in an urn of fixed size.

Method

A total of 303 undergraduate students in the UCSD Psychology Department (65.0% female; $M_{age} = 20.3$ years) participated in Experiment 5 for partial course credit. Each participant rated the attractiveness of a single gamble on a 1 to 9 scale. The gamble could be either relatively risky or relatively safe, with the winning probability represented on either a bounded or an unbounded scale, as shown in Table 2. We opted for a between-subjects rating task to avoid the complexity of a choice problem involving two different urns with different numbers of winning marbles and different prizes.[4] For both gambles in both representations, a visual depiction of the urn, indicating the total number of "marbles" in the urn (1373), was prominently displayed (for screenshots, see the online supplemental materials). In the standard bounded representation, the difference in probabilities (82% vs. 94%) near the upper bound should be heavily weighted, enhancing the relative attractiveness of the safe gamble. If the unbounded representation leads to diminishing sensitivity for probability, the difference in the number of winning marbles (1126 vs. 1291) should be less heavily weighted, reducing the relative attractiveness of the safe gamble.

Table 2
Experiment 5: Attractiveness Ratings of Relatively Safe and Risky Gambles Across Numerical Representations

Representation	Gamble	Description	n	Rating: Mean (SD)
Bounded	Safe	94% winning marbles, $15 prize	78	8.2 (1.2)
	Risky	82% winning marbles, $21 prize	73	7.3 (1.7)
Unbounded	Safe	1291 winning marbles, $15 prize	81	7.6 (1.6)
	Risky	1126 winning marbles, $21 prize	71	7.5 (1.5)

Results

Mean attractiveness ratings are shown in Table 2. We conducted a 2 × 2 analysis of variance (ANOVA) with gamble (safe vs. risky) and representation (bounded vs. unbounded) as independent variables. There was a main effect of gamble, with higher ratings overall for the safe gamble, $F(1, 299) = 8.24$, $p < .01$, and no main effect of representation, $F(1, 299) = 1.68$, $p = .20$. Importantly, the ANOVA confirmed the predicted interaction, $F(1, 299) = 4.42$, $p = .04$. The safe gamble received significantly higher attractiveness ratings than the risky gamble in the bounded representation ($p < .001$) but not in the unbounded representation ($p = .59$). This finding suggests that sensitivity to probabilities, like sensitivity to outcomes, depends on numerical framing. When the numerical representation is unbounded, large values have less subjective impact—an effect observed regardless of whether the numbers represent outcomes (Experiment 1) or probabilities (Experiment 5).

General Discussion

Prospect theory posits diminishing sensitivity for gains and losses, and an inverse S-shaped weighting function for probability. These "psychophysical" principles have received considerable empirical support across a range of studies, using standard numerical representations of outcomes (unbounded) and uncertainty (bounded). Here, we explored bounded representations of outcome and unbounded representations of uncertainty, and found evidence for very different psychophysical relationships. For gambles involving gains near the upper bound of a bounded numerical representation, risk aversion is markedly reduced, consistent with accelerating sensitivity at the upper end of the scale. This novel framing effect was found in Experiment 1 and across a range of gambles in an aggregate analysis in Experiment 2, where psychometric modeling suggested an inverse S-shaped value function in the bounded representation. An inverse S-shape predicts a com-

[4] A potential problem in interpreting the ordering of between-subjects evaluations is that different stimuli may evoke different reference sets for comparison (Birnbaum, 1999; but see Leong, McKenzie, Sher, & Müller-Trede, 2017). In the present study, this would be a problem only if different numbers were to evoke different reference sets *within* a given representation. There is no reason to believe that this would be the case for the numbers used here (82% vs. 94% in the bounded condition; 1126 vs. 1291 in the unbounded condition).

mon ratio effect, which has previously been confirmed for probability (in its usual bounded representation). In Experiment 3, we demonstrated a new common ratio effect for *outcomes*, when these are represented on a bounded scale. This finding is incompatible with a linear or power value function. Experiment 4 adapted Tversky and Kahneman's (1981) Asian disease problem to examine nonmonetary gains and losses. We found a pronounced attenuation of the classic framing effect in the bounded representation, consistent with accelerating sensitivity for the highest gains (reducing risk aversion in the "saved" frame) and for the lowest losses (reducing risk seeking in the "die" frame). A final study looked at an unbounded representation of probability, in a conceptual mirror image of the first experiment: In Experiment 1, moving from an unbounded to a bounded representation led to increased sensitivity to large gains. In Experiment 5, moving from a bounded to an unbounded representation led to reduced sensitivity to large probabilities.

These findings suggest that swapping the form of their numerical representations can make payoffs look psychometrically like probabilities and probabilities look like payoffs. In other words, numerical form matters. But this does not imply that content is irrelevant. For example, people are likely to exhibit robust risk aversion when extreme outcomes are at stake, because a fixed change in income matters less for the very wealthy. As Bernoulli (1738/1954, p. 24) noted, "There is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount." This insight into the psychology of value is unlikely to be an artifact of a particular numerical representation. Nonetheless, for the gambles and stakes studied here (and in most experiments on risky choice), the shapes of the psychophysical functions may be as much about numbers as about material outcomes.

In light of these effects of representation on risk attitudes, it is natural to ask whether specific default representations are systematically preferred for specific content domains. The foregoing studies used pot ($3100) and urn (1373) sizes that presumably made conversion between representations difficult. If there are strongly preferred default representations (e.g., absolute amounts for money), one might expect effects on risk attitudes to disappear when conversion between representations is trivial. In the online supplemental materials, we report several studies that provide evidence to the contrary. These studies use the same paradigm as Experiment 1, but with pot sizes ($1000 and $100) that make conversion straightforward (e.g., 96% becomes $960 or $96). Representation effects, while somewhat less robust, persist even in these transparent cases. That is, people do not appear to revert to a strongly preferred default representation for gains. Furthermore, effects on risk attitudes are observed even when computational complexity is strictly matched across representations.

In some respects, the present findings suggest limitations on prospect theory's scope. The validity of its value and weighting functions appears to depend on how probabilities and outcomes are described. This observation has important practical consequences, because outside the laboratory, uncertainty is not always communicated in percentages (Beyth-Marom, 1982; Phillips & Edwards, 1966; Wallsten, Budescu, & Zwick, 1993), and outcomes are sometimes represented proportionally (e.g., in budgets) or in terms of progress toward a goal (which can serve as a salient reference point; Bonezzi, Brendl, & De Angelis, 2011; Heath, Larrick, & Wu, 1999). More fundamentally, in most everyday choice problems, probabilities and outcomes are not explicitly quantified at all. Traditionally, behavioral decision research has relied on simple gambles as an experimental "fruit fly" (Lopes, 1983), under the assumption that systematic responses to numerical probabilities and outcomes will generalize to their more nebulous real-world counterparts. The finding that standard psychophysical functions depend on a specific numerical representation reinforces questions (e.g., Hertwig & Erev, 2009) about whether simple gambles are viable "model organisms" for the study of everyday choice under uncertainty.

Our findings also complement recent work indicating that the shape of the value and weighting functions may depend on the context of gambles under consideration (Stewart et al., 2015; Walasek & Stewart, 2015). In addition, a substantial literature has documented departures from prospect theory when probabilities are learned experientially rather than descriptively, resulting in a "description-experience gap" (Hertwig, Barron, Weber, & Erev, 2004; Hertwig & Erev, 2009; for psychometric modeling, see Lejarraga & Müller-Trede, 2016). The present findings show that the psychophysical functions depend not only on which gambles we consider (context) and how we encounter them (description vs. experience), but also on the specific numerical representations used to descriptively convey probabilities and outcomes. Nonetheless, we do not believe that the present work contradicts prospect theory. Rather, it calls attention to an ambiguity implicit in the theory, and provides experimental evidence contributing to its resolution. In particular, our experimental findings suggest that prospect theory's value and weighting functions are best understood as being defined over specific numerical representations of uncertainty and outcome. This conclusion is likely to generalize to other models of choice under uncertainty that posit subjective transformations of outcomes and/or probabilities (e.g., Köszegi & Rabin, 2006; Quiggin, 1982).

More generally, the findings reported here suggest that psychophysical functions for probability and value are in effect constructed, and depend on the numerical framing of probabilities and outcomes. This work adds to a sizable literature on "constructed preference," which documents the many ways in which choices and attitudes depend on the context, framing, and history of choice problems (Payne, Bettman, & Schkade, 1999; Slovic, 1995). While illustrating the fertility of psychophysical ideas in choice under risk, the present studies also point to an important difference between psychophysical transformations at the "perceptual" level (e.g., light intensity) and the "conceptual" level (e.g., abstract quantities like money or probability). Psychophysical transformations in perception are relatively permanent and inflexible, presumably because they have been wired in, down to the receptor level, in evolution and development. Psychophysical functions for abstract quantities, by contrast, likely depend on idiosyncratic learning, and must emerge from conceptual systems that are subject to flexible control. We can freely modify our conceptual representations and reference points, but we cannot voluntarily rewire our sensory receptors or choose our perceptual adaptation levels. Psychometric investigation thus reveals deep commonalities as well as telling differences between perceptual and conceptual psychophysics.

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