Lecture Notes: Equivalent Circuit

I. Model only one ion \( \rightarrow K^+ \)

Set up

- Lipid bilayer membrane
- \( K^+ \) channel
- \( K^+ \) concentration gradient
  - Outside: Extracellular \( [K^+] \rightarrow 20 \text{ mM} \)
  - Inside: Intracellular \( [K^+] \rightarrow 400 \text{ mM} \)

I consider what happens to a \( K^+ \) ion
- The probability of it getting through the channel

\( K^+ \) ion flows through channel \( \rightarrow \) the electrical potential is "pushing" \( K^+ \) through the channel

- A channel can be electrically modeled as a resistor (how easy or difficult to go
through the Channel = resistance.)

- **RESISTANCE**
  - it is measured electrically in ohms ($\Omega$)
  - a resistor is represented by $\frac{1}{R}$ (symbol)
    - units: siemens (S)
  - **CONDUCTANCE** ($g$) = $\frac{1}{R}$

- **BATTERY** is the potential across the membrane
  - units are in Volts (V)
  - a battery symbol is: $\frac{1}{V}$

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**K^+ ion flows through channel**: the electrical potential is "pushing" $K^+$ through the channel

- a channel can be electrically modeled as a resistor (how easy or difficult to go
difference in charge → creates a voltage across memb.

a potential to "do work"
separation of charge

The lipid bilayer is represented as a capacitor ← it stores charge

capacitor (F) Farads

\[
\text{current} - \text{you need to have a change in charge (Q)}
\text{to have a current (I)}
\]

\[\Delta Q \alpha I\]

outside

inside

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Important relationships

1. $V \propto I$ (the higher the voltage, the higher the current)

2. If there is very high resistance

   $\rightarrow$ How much current will flow?

   $\checkmark$ Not very much

Ohm's Law: $V = IR$

In terms of conductance ($g$): $V = \frac{I}{g}$$

Recall: $g = \frac{1}{R}$ or $R = \frac{1}{g}$
voltage across the membrane \((V_m)\)

\[ V_m = \frac{I_{ion}}{g_{ion}} + E_{ion} \]

**Solve for Current**

\[ I_{ion} = g_{ion} (V_m - E_{ion}) \]

- The resistor equation
- The current = the conductance * the driving force
What is the relationship between:

a) voltage & capacitance

b) charge & current

Recall:

The relationship between voltage & charge is proportional (v) (q)

Q ∝ V

→ for example:

- Larger plates
  - This will store more charge
  - e.g. larger cell or larger patch of membrane of surface area

- Smaller plates
Compare the distance between the plates.

The larger the distance, the less charge can be stored.

\[ Q = CV \]  \rightarrow  \text{Fundamental capacitor equation}

Currents are changes in charge over time.

\[ \Delta Q = C \cdot \Delta V \]  \rightarrow  \text{A change of charge will change voltage}

Small change in charge
now consider it as a rate:

\[ \frac{\Delta Q}{\Delta t} = C \times \frac{\Delta V}{\Delta t} \]

the change in charge with time = C time change of voltage with time

\[ \text{the change in charge with time} \]

recall: \( \frac{\Delta Q}{\Delta t} \) is current! (I)

so,

\[ I = C \times \frac{\Delta V}{\Delta t} \]

this is the capacitor equation

for very small time increments:

\[ I = C \times \frac{dv}{dt} \]
In summary:

Capacitor eq.:

\[ I = \frac{dV}{dt} \]

Resistor eq.:

\[ I_{ion} = g_{ion} (V_m - E_{ion}) \]